

# QCD phase diagram using PNJL model with eight-quark interactions

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## Abstract

We present the phase diagram and the fluctuations of different conserved charges like quark number, charge and strangeness at vanishing chemical potential for the 2+1 flavor Polyakov Loop extended Nambu–Jona-Lasinio model with eight-quark interaction terms using three-momentum cutoff regularisation. The main effect of the higher order interaction term is to shift the critical end point to the lower value of the chemical potential and higher value of the temperature. The fluctuations show good qualitative agreement with the lattice data.

*Key words:* Phase Diagram, CEP, Fluctuations, Kurtosis  
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## 1. Introduction

Novel phases of strongly interacting matter at high temperature and density are very interesting area to study. The properties of the strongly interacting matter can be studied through effective models of QCD. Polyakov loop enhanced Nambu–Jona-Lasinio model is one such model that successfully captures various properties of strongly interacting matter [1–4]. The 2+1 flavor PNJL and NJL model have been studied with four-quark and six-quark interaction terms in the Lagrangian [4]. However the six-quark interaction imposes a serious problem of vacuum instability in the model which can be solved by introducing higher order eight-quark interaction terms in the Lagrangian [5,6]. The search for the possible location of the critical end point (CEP) is one of the main issue of strong

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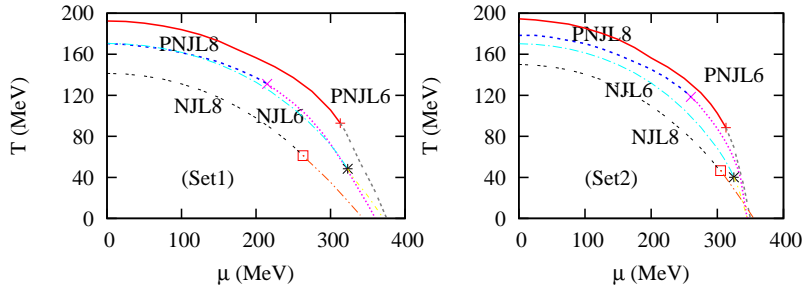


Fig. 1. Phase diagram in  $\mu$  with  $T$  for NJL and PNJL mode.

interaction physics. Also, fluctuations and the correlations of conserved charges and their higher order cumulants provide information about the degrees of freedom of strongly interacting matter. The study of diagonal and off-diagonal susceptibilities from the PNJL model can also provide the information about the existence of critical behavior [7,8]. Here we give a detailed analysis of the CEP and the fluctuations of different conserved charges at zero chemical potential.

## 2. Phase diagram

The phase diagram and the location of the critical end point (CEP) are the most important issue of strong interaction physics. CEP is the point which separate the cross-over transition from the first order phase transition. Here we study the phase diagram of strongly interacting matter using both NJL and PNJL model. We have used the nomenclature NJL6 and PNJL6 for the six-quark (6q) and NJL8 and PNJL8 for the eight-quark (8q) interactions. The details of the model and parameter set may be obtained in ref. [9]. In fig. 1 we have shown the phase diagrams of NJL and PNJL model with 6q and 8q type interactions. The parameter set are taken from [9]. The introduction of the eight-quark interaction terms shift the CEP to lower chemical potential and the higher temperature value. The recent lattice results show that the possible region of CEP should lie in  $\mu_C/T_C \leq 2.5$  [11], which can only be satisfied if the eight-quark interaction is added in the Lagrangian. The values of the CEP for

$$\begin{aligned}
 (NJL6)(\mu_C, T_C)_{(set1)} \text{ MeV} &= (323, 48.5), (\mu_C, T_C)_{(set2)} \text{ MeV} = (325, 40.15), \\
 (NJL8)(\mu_C, T_C)_{(set1)} \text{ MeV} &= (263, 61.2), (\mu_C, T_C)_{(set2)} \text{ MeV} = (305, 46.55), \\
 (PNJL6)(\mu_C, T_C)_{(set1)} \text{ MeV} &= (313, 92.85), (\mu_C, T_C)_{(set2)} \text{ MeV} = (313, 88.5), \\
 (PNJL8)(\mu_C, T_C)_{(set1)} \text{ MeV} &= (260, 118.5), (\mu_C, T_C)_{(set1)} \text{ MeV} = (237, 122.05).
 \end{aligned}
 \tag{1}$$

## 3. Correlation of conserved charges and Specific heat

The fluctuations and the correlation of conserved charges are sensitive indicators of the transition from hadronic matter to the quark-gluon phase. The definition of different cumulants may be obtained in ref. [7]. Here we will use the expansion around  $\mu_X = 0$ ,

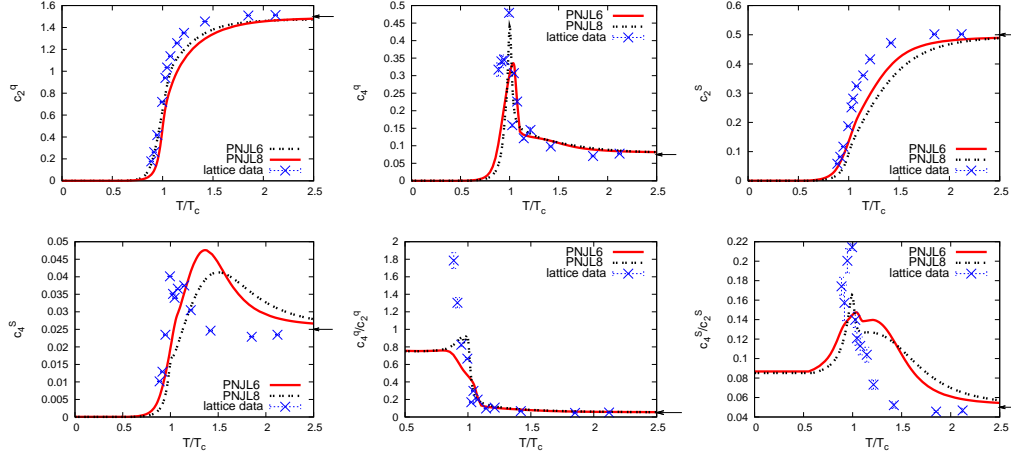


Fig. 2. Variation of  $c_2$ ,  $c_4$  and Kurtosis with  $T/T_C$ , for  $\mu_X = \mu_q, \mu_S$  for 6q and 8q case. The lattice data taken from Ref. [12].

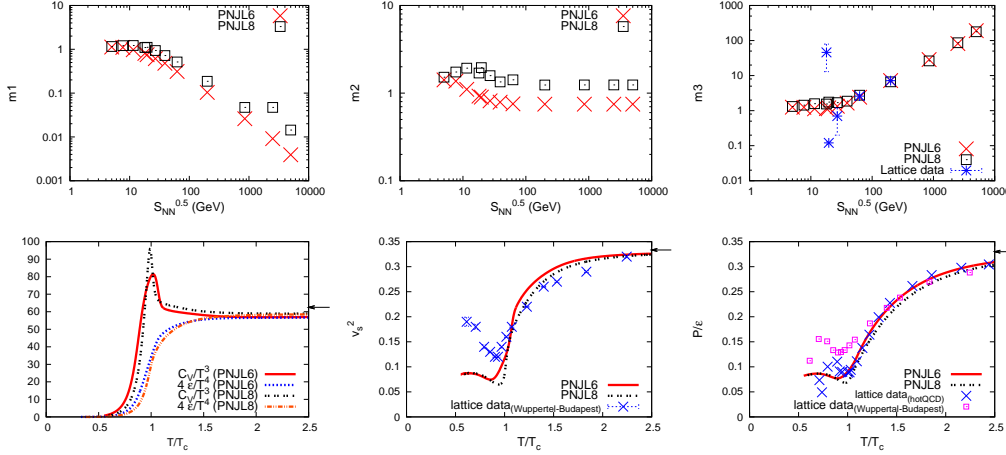


Fig. 3. Variation of  $m_1$ ,  $m_2$  and  $m_3$  with  $T/T_C$ , for 6q and 8q case. The lattice data taken from Ref. [13].  $C_V/T^3$ ,  $v_s^2$  and  $p/\epsilon$  as a function of  $T/T_C$  are also shown. The lattice data is given by [14,15].

where the odd terms vanish due to CP symmetry. In figure (2) we show the variation of  $c_2$ ,  $c_4$  with  $T/T_C$  for  $\mu_q$  for both models and lattice data. It can be seen that QNS ( $c_2^q$ ) shows an order parameter like behavior. The high temperature value reaches almost 99% of the ideal gas value. The fourth order derivative  $c_4^q$  shows a peak near the transition temperature  $T_C$ . The  $c_4^S$  has a similar behavior as the  $c_4^q$ . However the peak appears at much higher  $T$  than  $T_C$  and coincide with the temperature at which  $d\sigma_s/dT$  of heavy strange quark shows a maximum. The charge susceptibilities show similar behavior as quark number susceptibility [7]. We have plotted the kurtosis *i.e.* the ratio of  $c_4^X/c_2^X$  for both type of potential (where  $X = q, Q$  or S) and compared with the lattice data. The plot for  $c_4^q/c_2^q$  for 8q interaction shows more fluctuation near  $T_C$  than 6q interaction. For the strangeness fluctuation, first peak occurs at chiral transition for light flavors

and second peak occurs when chiral transition occurs in strange sector. At intermediate temperatures PNJL model overestimates the ratio than LQCD result. Since the volume of the fireball is unknown, we have constructed the ratios of these cumulants, so that the volume is cancelled out. We now plot the moments for both 6q and 8q interactions. Our results show monotonous behavior in the plots. However the lattice result show nonmonotonous behavior for certain range of chemical potential. This difference may be due to the fact that our CEP is at much higher chemical potential region than the Lattice value [13]. The thermodynamic quantities like specific heat ( $C_V$ ) and the speed of sound ( $v_s$ ) are shown in fig 3. The softest point of the equation of state is found to be  $(p/\epsilon)_{min} \approx 0.07$  for PNJL6 and  $(p/\epsilon)_{min} \approx 0.06$  for PNJL8. PNJL8 model gives better agreement with lattice data, which has its softest point of equation of state as  $(p/\epsilon)_{min} \approx 0.05$  [14]. However the softest point in other lattice data comes out to be  $\sim 0.13$  in [15].

#### 4. Conclusion

To conclude, we have studied the phase diagram and the fluctuations of different conserved charges of the PNJL and NJL model with eight-quark interaction. The eight-quark interaction term shifts the CEP to the low  $\mu$  and high  $T$  value. Furthermore our study concludes that the inclusion of eight-quark interaction is essential to limit  $\mu_C/T_C$  below 2.5, as suggested by the recent lattice data. Fluctuations of different conserved charges show significant behavior near the phase transition temperature.

#### 5. Acknowledgement

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