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Probing Exotic Leptons through Oblique Parameters

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ABSTRACT

We show that the oblique electroweak parameter S may serve as a discriminant between singlet neutrinos, vector doublet leptons, and vector triplet leptons (no matter how heavy), provided they mix with their sequential counterparts.

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The Standard Model (SM) of electroweak interactions reigns supreme. The precision tests at LEP, based on the 1990 data of $\sim$ 5,50,000 Z events, have further squeezed possibilities of physics beyond the SM. Nonetheless, it is perhaps prudent to still keep an open mind and continue to examine extensions of it by confronting them with the high-statistics data that have now become available. One sensitive meter for testing such extensions is provided by the oblique electroweak parameters S, T, and U [1, 2] which capture the effect of new physics on gauge boson self-energies. In this letter we consider the impact of additional leptons on these variables. If the extra leptons are chiral – left- and right-handed (LH and RH) leptons transforming differently – then there already exist tight constraints on their masses from the oblique parameters. In addition, such leptons make non-vanishing contributions to the anomaly and the theory needs additional fermions to be well-behaved. For these reasons, we choose only anomaly-free additional multiplets, which are also unconstrained by the S, T, and U variables. These, for example, could be singlet (i.e. sterile) neutrinos, or leptons appearing in vector multiplets. After spontaneous symmetry breaking these fermions can mix with the sequential fermions and give non-zero contributions to S, T, and U. We consider the mixing with the third-generation leptons only which happen to be the heaviest and check whether from the present (or anticipated) experimental results one can get any handle on these particles.

In general, exotic leptons of arbitrary representations may be allowed to mix with the corresponding SM counterparts by postulating the existence of appropriate Higgs representations in addition to the scalar doublet of the SM. In this letter we restrict ourselves to minimal extensions of the SM whilst only incorporating extra lepton generations, keeping the scalar sector just as in the SM. This limits our choice of the exotic lepton generations to singlet, doublet and triplet only. These multiplets are, in fact, present in many popular extensions of the SM. The neutral singlet and the vector doublet leptons are contained in the fundamental $\underline{27}$ of E_6, whilst the vector triplet appears in a supersymmetrized left-right model where triplet scalars are employed for left-right symmetry breaking and/or neutrino mass generation. Severe constraints [3] exist for the tree-level mixing angles of the singlet and the vector doublet leptons with their sequential counterparts from the line shape analyses around the Z peak.

In the present analysis we examine the loop effects of these leptons on the gauge boson self-energies. We put our emphasis on the parameter S, which is an isospin-conserving contribution and as a result is non-zero even for a degenerate chiral multiplet. A vector multiplet or a sterile singlet does not contribute to S in the unmixed situation. In terms of the gauge boson self-energies, S is defined as [1]

$$S = \frac{8\pi}{M_Z^2}\left[\Pi_{3Y}(0) - \Pi_{3Y}(M_Z^2)\right], \tag{1}$$

where $Q = t_3 + Y/2$. The Π functions receive contributions from the SM as well as from physics beyond it. In the spirit of one of our previous analyses [4], we work with a variant of S, namely $\widetilde{S}$, which indicates only physics beyond the SM for a given SM reference point. Using the full 1990 sample of LEP data on the cross-sections and asymmetries on and around the Z peak, it has been shown in ref. [4] that $\widetilde{S} = -0.76 \pm 0.71$ for $m_t = 140$ GeV, $M_H = 100$ GeV and $\alpha_s = 0.12$. While at the moment it is rather premature to assign a definite sign to $\widetilde{S}$ in view of its large error, accumulation of more statistics and a better control over the systematics in the coming year would cause the errors to shrink significantly. In that event a statistically significant inclination of $\widetilde{S}$ towards a negative value might provide an indicator for discriminating different possibilities of extension beyond the SM.

In every case we also briefly remark in passing about the parameter T:

$$T = \frac{4\pi}{s^2\, c^2\, M_Z^2} \left[\Pi_{11}(0) - \Pi_{33}(0)\right], \tag{2}$$

which we find is not useful in this context.

We now discuss the three cases of singlet neutrinos, vector doublet leptons and vector triplet leptons in turn. In each case we observe that there are two types of mass terms; first an $SU(2)$-breaking mass term (m) driven by the doublet Higgs and then an $SU(2)$-invariant mass term (M) and we take $M \gg m$.

The standard sequential lepton multiplet is:

$$\psi_L = \begin{pmatrix} \nu \\ l^- \end{pmatrix}_L ; \quad l_L^{c+}. \tag{3}$$

Introducing a sterile LH neutrino, N, is possibly one of the simplest extensions of the SM. Its mixing with the SM neutrino produces a light state which is almost massless and a heavy state whose mass can be arbitrarily large and certainly greater than $M_Z/2$ in view of the LEP results. It may be noted that the light state gets a predominant contribution from ν while the heavy state corresponds mainly to N. This is the standard see-saw mechanism where the mixing angle (ϕ) is proportional to m/M. The contribution to $\widetilde{S}$ is found to be:

$$\widetilde{S} = \frac{1}{3\pi}\left[\frac{4}{5}\ln\frac{M}{M_Z} + 1\right]\sin^2\phi. \tag{4}$$

From the above we find that for $M = 1$ TeV (10 TeV), $\widetilde{S} = 1 \times 10^{-6}$ (1.6×10^{-8}) and the contribution falls off with increasing M and is always positive. Turning now to the parameter $\widetilde{T}$, which is an analogous variant of T, we use the experimental bound $\widetilde{T} = -0.70 \pm 0.49$ [4] to obtain:

$$M \sin\phi \leq 167 \;\; \text{GeV} \tag{5}$$

which is trivially satisfied.

Vector doublet leptons transform as $SU(2)$ doublets both in the LH and RH sectors, e.g.:

$$\begin{pmatrix} N_1 \\ E^- \end{pmatrix}_L ; \quad \begin{pmatrix} E^{c+} \\ N_2 \end{pmatrix}_L . \tag{6}$$

The neutrinos of the vector doublet cannot acquire Majorana masses due to the absence of an appropriate Higgs. In general, there can be two $SU(2)$-invariant mass terms involving the fields in eqs.(3) and (6). However, by a suitable redefinition, it is possible to arrange that only the doublets of eq.(6) are coupled. This forbids any possibility of mixing in the neutrino sector. The situation with the charged leptons is different, owing to the presence of the singlet in eq.(3). The effective (2×2) charged lepton mass matrix in this redefined basis is:

$$\begin{array}{c c} & \begin{array}{cc} e_L & E_L \end{array} \\ \begin{array}{c} e_R \\ E_R \end{array} & \begin{pmatrix} m & m' \\ 0 & M \end{pmatrix} \end{array} \tag{7}$$

where $M \gg m \simeq m'$. It is rather easy to check from the structure of the above mass matrix that the mixing angle (ϕ) in the RH sector is $\sim (m/M)$, whilst in the LH sector it is $\sim (m/M)^2$. In our calculations we retain terms $\sim (m/M)^2$ and find that at this level the LH mixing does not contribute. It is found that to a good approximation the heavy charged and neutral states have a common mass M and $\tilde{S}$ can be expressed as:

$$\begin{aligned} \tilde{S} &= \frac{8\pi \sin^2\phi}{M_Z^2} [\{\Pi_{LL}(0,0,0) - \Pi_{LL}(0,0,M_Z)\} \\ &\quad - \{\Pi_{LL}(0,M,0) - \Pi_{LL}(0,M,M_Z)\}] . \end{aligned} \tag{8}$$

From this equation we find that for $M = 1$ TeV (10 TeV), $\tilde{S} = 1\times 10^{-4}$ (1×10^{-2}) and the contribution grows with increasing M and is always positive. The contribution to $\tilde{T}$ vanishes identically at the level of our approximation.

We now focus our attention on the case of vector triplets under $SU(2)$:

$$\begin{pmatrix} F^+ \\ N_1 \\ E^- \end{pmatrix}_L ; \quad \begin{pmatrix} E^{c+} \\ N_2 \\ F^{c-} \end{pmatrix}_L \tag{9}$$

We assign a lepton number 1 (-1) for the first (second) multiplet. Demanding lepton number to be a good symmetry of the theory and recalling the absence of any Higgs multiplet apart from the SM doublet, we write down the following mass matrices for the neutral and charged lepton sectors, respectively:

$$\begin{array}{c c} & \begin{array}{ccc} \nu & N_1 & N_2 \end{array} \\ \begin{array}{c} \nu \\ N_1 \\ N_2 \end{array} & \begin{pmatrix} 0 & m & 0 \\ m & 0 & M \\ 0 & M & 0 \end{pmatrix} \end{array} \tag{10}$$

and

$$\begin{array}{cc} & \begin{array}{cc} e_L & E_L \end{array} \\ \begin{array}{c} e_R \\ E_R \end{array} & \left(\begin{array}{cc} m' & m \\ m & M \end{array}\right) \end{array} \tag{11}$$

Note that the charged lepton F gets an $SU(2)$-invariant mass M and is decoupled from the other fermions. The mixing angle ϕ is $\sim (m/M)$ and is common to both the charged and neutral sectors. To a good approximation the eigenstates in the neutral sector constitute one massless state and one heavy Dirac state. The charged sector consists of one light and one heavy fermion. All heavy states have a common mass M. The expression for $\widetilde{S}$ in this case is:

$$\begin{aligned} \widetilde{S} \;=\; & \frac{8\pi \sin^2\phi}{M_Z^2}\,[-\{\Pi_{LL}(M,M,0)-\Pi_{LL}(M,M,M_Z)\} \\ & -2\{\Pi_{LL}(0,0,0)-\Pi_{LL}(0,0,M_Z)\} \\ & +3\{\Pi_{LL}(0,M,0)-\Pi_{LL}(0,M,M_Z)\} \\ & -\{\Pi_{LR}(M,M,0)-\Pi_{LR}(M,M,M_Z)\}]\,. \end{aligned} \tag{12}$$

In this case we find that for $M = 1$ TeV (10 TeV), $\widetilde{S} = -3\times 10^{-4}$ (-2×10^{-2}), and it remains negative, increasing in magnitude as M increases. As in the vector doublet case $\widetilde{T}$ does not receive any contribution from the above mixing.

In summary, we have considered the mixing of sequential leptons with three different kinds of exotic multiplets, which are by themselves anomaly-free and in the unmixed situation make vanishing contributions to the oblique electroweak parameters S and T. In all cases the parameter T shows no impact. On the other hand, S is sensitive to the type of lepton multiplet, behaving differently in each case. For the mixing with a singlet neutrino the contribution is positive, but falls with increasing M. The other two cases exhibit a non-decoupling behaviour. In particular, while in both cases the magnitude grows with increasing M, for vector doublet leptons it is always positive whilst for vector triplets it is always negative. Hopefully, with a larger accumulation of data and a better control over systematics, LEP will be able to throw new light in this direction.

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Appendix

The Π functions used in the text (leaving aside the divergent pieces which cancel out in the final results) are given by :

$$\Pi_{LL}(M,M,M_Z) = M_Z^2\left[I_1 - I_2 - (x/2)I_0\right]/(4\pi^2) \tag{A.1}$$

$$\Pi_{LL}(M,M,0) = -\left[M^2 \ln M^2\right]/(8\pi^2) \tag{A.2}$$

$$\begin{aligned}\Pi_{LL}(0,M,M_Z) \;&=\; \Big[(12M_Z^2 - 18M^2)\ln M_Z^2 - 10M_Z^2 + 9M^2 \\ &\quad + 36(2M_Z^2 - M^2)I_3 - 72M_Z^2 I_4\Big]/(288\pi^2)\end{aligned} \tag{A.3}$$

$$\Pi_{LL}(0,M,0) = \left[-2M^2\ln M^2 + M^2\right]/(32\pi^2) \tag{A.4}$$

$$\Pi_{LL}(0,0,M_Z) = \left[3M_Z^2 \ln M_Z^2 - 5M_Z^2\right]/(72\pi^2) \tag{A.5}$$

$$\Pi_{LL}(0,0,0) = 0 \tag{A.6}$$

$$\Pi_{LR}(M,M,M_Z) = \left[M^2 I_0\right]/(8\pi^2) \tag{A.7}$$

$$\Pi_{LR}(M,M,0) = \left[M^2 \ln M^2\right]/(8\pi^2). \tag{A.8}$$

The integrals I_i, $i = 0,\cdots,4$ can be expressed in terms of $x = (M/M_Z)^2$, $y = 2\sqrt{x - 1/4}$, $z = x - 1$ and M_Z as:

$$I_0 = \ln M_Z^2 + \ln x - 2 + 2y\arctan(1/y) \quad \text{for} \;\; x > (1/4) \tag{A.9}$$

$$I_1 = (1/2)\ln M_Z^2 + (1/2)\ln x - 1 + y\arctan(1/y) \quad \text{for} \;\; x > (1/4) \tag{A.10}$$

$$\begin{aligned}I_2 \;&=\; (1/3)\ln M_Z^2 + (1/3)\ln x - 13/18 + 2x/3 + \frac{3y - y^3}{6}\arctan(1/y) \\ &\quad \text{for} \;\; x > (1/4)\end{aligned} \tag{A.11}$$

$$I_3 = [2\ln(1+z) + 2z^2\ln\{z/(1+z)\} - (1-2z)]/4 \;\; \text{for} \;\; x > 1 \tag{A.12}$$

$$I_4 = [6\ln(1+z) - 6z^3\ln\{z/(1+z)\} - 2 + 3z - 6z^2]/18 \;\; \text{for} \;\; x > 1. \tag{A.13}$$

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