Novel ballistic to diffusive crossover in the dynamics of a one dimensional Ising model with variable range of interaction

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Abstract.

The idea that the dynamics of a spin is determined by the size of its neighbouring domains was recently introduced (S. Biswas and P. Sen, Phys. Rev. E **80**, 027101 (2009)) in a Ising spin model (henceforth, referred to as model I). A parameter p is now defined to modify the dynamics such that a spin can sense domain sizes up to $R = pL/2$ in a one dimensional system of size L. For the cutoff factor $p \to 0$, the dynamics is Ising like and the domains grow with time t diffusively as $t^{1/z}$ with $z = 2$, while for $p = 1$, the original model I showed ballistic dynamics with $z \simeq 1$. For intermediate values of p, the domain growth, magnetisation and persistence show model I like behaviour up to a macroscopic crossover time $t_1 \sim pL/2$. Beyond t_1, characteristic power law variations of the dynamic quantities are no longer observed. The total time to reach equilibrium is found to be $t = apL + b(1-p)^3L^2$, from which we conclude that the later time behaviour is diffusive. We also consider the case when a random but quenched value of p is used for each spin for which ballistic behaviour is once again obtained.

1. Introduction

Dynamical phenomena is an important topic in statistical physics. Physical quantities in self organized and/or driven systems show rich time dependent behaviour in many cases. Some of the dynamical phenomena which have attracted a lot of attention are critical dynamics, quenching and coarsening, reaction diffusion systems, random walks etc.

In most of these phenomena, we find there is a single timescale leading to uniform time dependent behaviour which in many cases is a power law decay or growth [2]. However, in some complex systems, it has been observed that the dynamics is governed by a distinct short time behaviour followed by a different behavior at long times. For example, in spin systems, at criticality, the order parameter is observed to grow for a macroscopically short time [3] while at longer times it decays in an expected power law manner. For correlated random walks, e.g., the persistent random walk on the other hand, one finds a ballistic (i.e., when the root mean square (rms) displacement scales linearly with time) to diffusive (rms displacement varying as the square root of time) crossover in the dynamics [4]. Random walks on small world networks show a completely opposite behaviour, the number of distinct sites visited by the walker has an initial diffusive scaling followed by a ballistic variation with time [5]. This is also true for a biased random walker.

In this paper, we study a dynamical model of Ising spins in one dimension which is governed by a single parameter. The system is a generalised version of a recently proposed model in [1] (which we refer to as model I henceforth) where the state of the spins ($S = \pm 1$) may change in two situations: first when its two neighbouring domains have opposite polarity, and in this case the spin orients itself along the spins of the neighbouring domain with the larger size. This case may arise only when the spin is at the boundary of the two domains. The neighbouring domain sizes are calculated excluding the spin itself, however, even if it is included, there is no change in the results. A spin is also flipped when it is sandwiched between two domains of spins with same sign. When the two neighbouring domains of the spin are of the same size but have opposite polarity, the spin will change its orientation with fifty percent probability. Except for this rare event the dynamics in the above model is deterministic. This dynamics leads to a homogeneous state of either all spins up or all spins down. Such evolution to absorbing homogeneous states are known to occur in systems belonging to directed percolation (DP) processes, zero temperature Ising model, voter model etc. [6, 7].

Model I was introduced in the context of a social system where the binary opinions of individuals are represented by up and down spin states. In opinion dynamics models, such representation of opinions by Ising or Potts spins is quite common [8]. The key feature is the interaction of the individuals which may lead to phase transitions between a homogeneous state to a heterogeneous state in many cases [9].

Model I showed the existence of novel dynamical behaviour in a coarsening process when compared to the dynamical behaviour of DP processes, voter model, Ising models etc. [10, 11, 12, 13, 14]. The domain sizes were observed to grow as $t^{1/z}$ with the exponent z very close to unity. It may be noted that the dynamics of a domain wall can be visualised as the movement of a walker and therefore the value $z \simeq 1$ indicated that the effective walks are ballistic. When stochasticity is introduced in this model, such that spin flips are dictated by a so called "temperature" factor, it shows a robust behaviour in the sense that only for the temperature going to infinity there is conventional Ising model like behaviour with $z = 2$, i.e., the domain wall dynamics becomes diffusive in nature [15].

In this work, we have introduced the parameter p, which we call the cutoff factor, such that the maximum size of the neighbouring domains a spin can sense is given by $R = pL/2$ in a one dimensional system of L spins with periodic boundary condition. It may be noted that for $p = 1$, we recover the original model I where there is effectively no restriction on the size sensitivity of the spins. $R = 1$ corresponds to the nearest neighbour Ising model where $p \to 0$ in the thermodynamic limit.

By the introduction of the parameter p we have essentially defined a restricted neighbourhood of influence on a spin. Thus here we have a finite neighbourhood to be considered, which is like having a model with finite long range interaction. In addition, here we impose the condition that within this restricted neighbourhood, the domain structure is also important in the same way it was in model I. If one considers opinion dynamics systems (by which model I was originally inspired), the domain sizes represent some kind of social pressure. A finite cutoff (i.e., $p < 1$) puts a restriction on the domain sizes which may correspond to geographical, political, cultural boundaries etc. The case with uniform cutoff signifies that all the individuals have same kind of restriction; we have also considered the case with random cutoffs which is perhaps closer to reality.

In the next section, we describe the dynamical rule and quantities estimated. We present the results for the case when p is same for all spins in section III and IV and in section V we consider the case when the values of p for each spin is random, lying between zero and unity and constant over time for each spin. In the last section, we end with concluding remarks.

2. Dynamical rule and quantities calculated

As mentioned before, only the spins at the boundary of a domain wall can change its state. When sandwiched between two domains of same sign, it will be always flipped. On the other hand, for other boundary spins (termed the target spins henceforth), there will be two neighbouring domains of opposite signs. For such spins, we have the following dynamical scheme: let d_{up} and d_{down} be the sizes of the two neighbouring domains of type up and down of a target spin (excluding itself). In model I, the dynamical rule was like this: if d_{up} is greater (less) than d_{down}, the target spin will be up (down) and if $d_{up} = d_{down}$ the target spin is flipped with probability 0.5. Now, with the introduction of p, the definition of d_{up} and d_{down} are modified: $d_{up} = \min\{R, d_{up}\}$ and similarly $d_{down} = \min\{R, d_{down}\}$ while the same dynamical rule applies.

As far as dynamics is concerned, we investigate primarily the time dependent behaviour of the order parameter, fraction of domain walls and the persistence probability. The order parameter is given by $m = \frac{|L_{up}-L_{down}|}{L}$ where L_{up} (L_{down}) is the number of up (down) spins in the system and $L = L_{up} + L_{down}$, the total number of spins. This is identical to the (absolute value of) magnetisation in the Ising model.

The average fraction of domain walls D_w, which is the average number of domain walls divided by the system size L is identical to the inverse of average domain size. Hence the dynamical evolution of the order parameter and fraction of domain walls is expected to be governed by the dynamical exponent z; $m \propto t^{1/(2z)}$ and $D_w \simeq t^{-1/z}$ [2].

The persistence probability $P(t)$ of a spin is the probability that it remains in its original state up to time t [14] is also estimated. $P(t)$ has been shown to have a power law decay in many systems with an associated exponent θ. The persistence probability, in finite systems has been shown to obey the following scaling form [16, 17]

$$P(t, L) \propto L^{-\alpha} f(t/L^z). \quad (1)$$

The exponent $\alpha = \theta z$ is associated with the saturation value of the persistence probability at $t \to \infty$ when $P_{sat}(L) = P(t \to \infty, L) \propto L^{-\alpha}$ [16].

In the simulations, we have generated systems of size $L \leq 6000$ with a minimum of 2000 initial configurations for the maximum size in general. Depending on the system size and time to equilibriate, maximum iteration times have been set. Random updating process has been used to control the spin flips. In general, the error bars in the data are less than the size of the data points in the figures and therefore not shown.

3. Case with finite R ($p \to 0$)

In this section, we discuss the case when R is finite. Effectively this means that R does not scale with L and is kept a constant for all system sizes. Since R is kept finite, expressing $R = pL/2$ implies $p \to 0$ in the the thermodynamic limit. For $R = 1$, the model is same as the Ising model as the dynamical rule is identical to the zero temperature Glauber dynamics. But it may be noted that making $R > 1$ will make the dynamical rules different from the case of $R = 1$; as an example we show in Fig. 1 how making $R = 2$ or 3 changes the dynamical rule compared to $R = 1$.

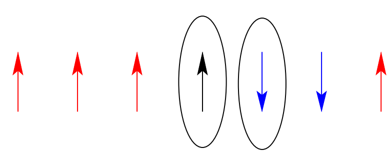

Figure 1. A schematic picture to show the dynamics in the present model for a finite value of R. Both the encircled spins will change their state with fifty percent probability for the nearest neighbour Ising model ($R = 1$). For $R = 2$, the encircled spin on the left will flip with probability 1/2 while the one on the right will flip with probability 1. For $R = 3$, the left one will not flip but the right one will.

We have simulated systems with $R = 2$ and $R = 3$ which show that the dynamics leads to the equilibrium configuration of all spins up/down. Not only that, the dynamic exponents also turn out to be identical to those corresponding to the nearest neighbour Ising values (i.e., $\theta = 0.375$ and $z = 2$). As R is increased, the finite size effects become stronger, however, it is indicated that the Ising exponents will prevail as the system size becomes larger. In an indirect way, we have shown later that $z = 2$ as $p \to 0$ using a general scaling argument. The behaviour of the different dynamic quantities for $R = 3$ are shown in Figs 2 and 3.

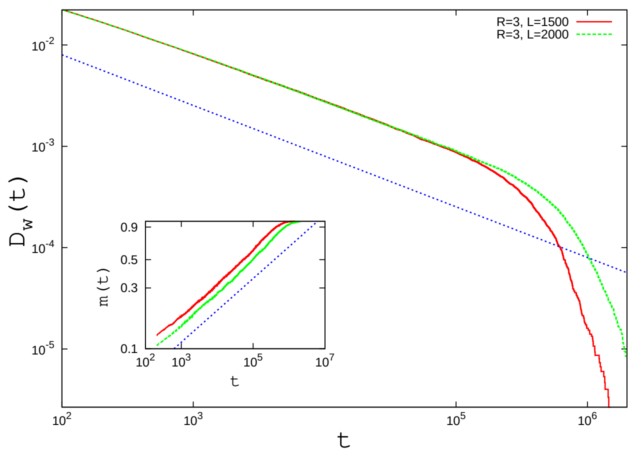

Figure 2. Decay of the fraction of domain walls $D_w(t)$ with time for $R = 3$ and two different system sizes shown in a log-log plot. The dashed line has slope equal to 0.5. Inset shows growth of magnetisation $m(t)$ with time for $R = 3$; the dashed line here has slope equal to 0.25.

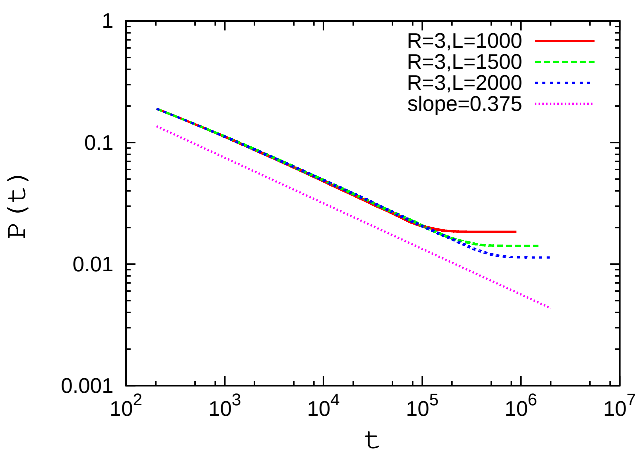

Figure 3. Decay of persistence probability $P(t)$ with time for three different sizes shown in a log-log plot. The straight line has slope 0.375.

4. Case with $p > 0$

In this section, we discuss the case when p is finite. We also assume that p is uniform, which means each spin experiences the same cutoff.

The equilibrium behavior is same for all p, i.e., starting from a random initial configuration, the dynamics again leads to a final state with $m = 1$, i.e., all spins up or all spins down. For $p = 1$, that is in model I, it was numerically obtained that $\theta \simeq 0.235$ and $z \simeq 1.0$ giving $\alpha \simeq 0.235$, while in the one dimensional Ising model $\theta = 0.375$ and $z = 2.0$ (exact results) giving $\alpha = 0.75$. It is clearly indicated that though model I and the Ising model have identical equilibrium behaviour, they belong to two different dynamical classes which correspond to $p = 1$ and $p \to 0$ limit respectively of the present model. It is therefore of interest to investigate the dynamics in the intermediate range of p.

4.1. Results for $0 < p < 1$

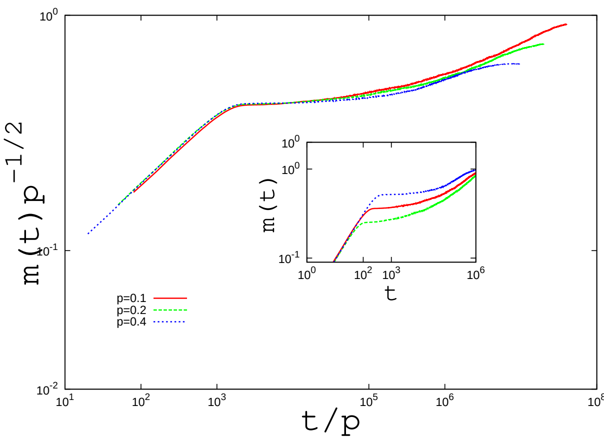

Figure 4. The collapse of scaled order parameter versus scaled time for different values of p, shows $z = 1$ for $t < t_1$. Inset shows unscaled data. System size $L = 3000$.

Drastic changes in the dynamics are noted for finite values of $p < 1$. The behaviour of all the three quantities, $m(t)$, D_w and $P(t)$ shows the common feature of a power law growth or decay with time up to an initial time t_1 which increases with p. The power law behaviour is followed by a very slow variation of the quantities over a much longer interval of time, before they attain the equilibrium values. The power law behaviour in the early time occur with exponents consistent with model I, i.e., $z \simeq 1$ and $\theta \simeq 0.235$. This early time behaviour accompanied by model I exponents is easy to explain: it occurs while the domain sizes are less than $pL/2$ such that the size sensitivity does not matter and the dynamics is identical to that in model I. As the domain size increase beyond this value, the sizes of the neighbouring domains as sensed by the boundary spin become equal making the dynamics stochastic rather than deterministic as a result of which the dynamics becomes much slower.

We thus argue that since domain size $\sim t^{1/z}$, the time up to which model I behaviour will be observed is $t_1 = (pL/2)^z$. Since z for model I is 1 we expect that $t_1 = pL/2$. For a fixed size L one can then consider the scaled time variable $t' = t/p$, and plot the relevant scaled quantities against t' for different values of p to get a data collapse up to $t'_1 = t_1/p$, independent of p. We indeed observe this, in Figures 4, 5 and 6, the scaling plots as well as the raw data are shown. From the raw data, t_1 is clearly seen to be different for different p.

Although the model I behaviour is confirmed up to t_1 and explained easily, beyond t_1, the raw data do not give any information about the dynamical exponents z and θ as no straight forward power law fittings are possible. While an alternative method to calculate θ is not known, one may have an estimate of z using an indirect method. It

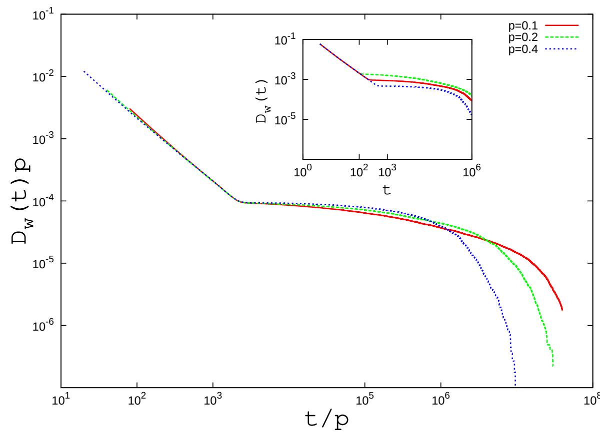

Figure 5. The collapse of scaled fraction of domain walls versus scaled time for different values of p; shows $z = 1$ for $t < t_1$. Inset shows unscaled data. System size $L = 3000$.

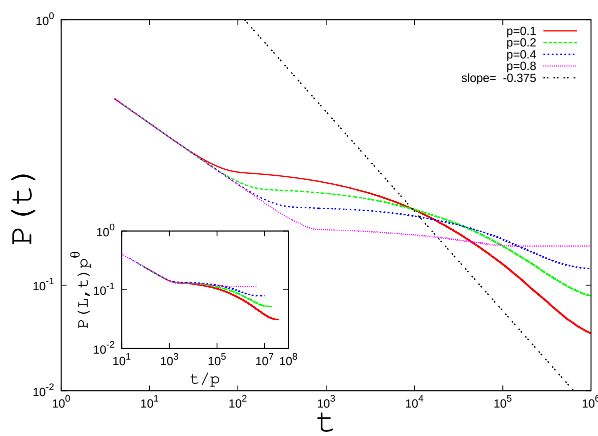

Figure 6. Persistence probability versus time for different values of p; the straight line with slope 0.375 shown for comparison. Power law behaviour can be observed only at the initial time. System size $L = 3000$. Inset shows the collapse of scaled persistence probability versus scaled time indicating $z = 1$ for $t < t_1$.

has been shown recently that for various dynamical Ising models, the time t_{sat} to reach saturation varies as L^x where x is identical to the dynamical exponent z [15, 18]. One may attempt to do the same here.

Actually it is possible to find out theoretically the form of t_{sat} from the qualitative behaviour of the dynamical quantities described above and the snapshot of the system (Fig. 7) at times beyond t_1. At $t > t_1$, the domain sizes of the neighbours of any spin at the boundary appear equal such that the domain walls perform random walks slowing down the annihilation process. Domain walls annihilate only after one of the neighbouring domains shrinks to a size $< pL/2$ again. In a small system, one can see that the slow process continues with only two domain walls separating two domains remaining in the system at later times (Fig. 7). Even in larger systems, there will be

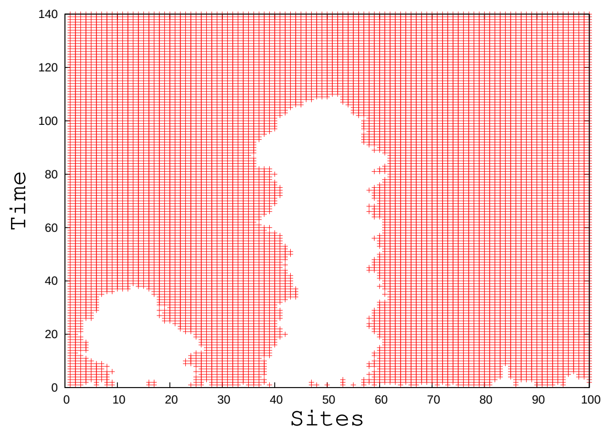

Figure 7. Snapshot for $p < 1.0$ ($p = 0.4$) for system size $L = 100$.

only a few domain walls remaining making $D_w \propto 1/N$ at $t > t_1$ as we note from the inset of Fig 5: D_w remains close to $O(1/N)$ for a long time before going to zero.

Thus t_{sat} will have two components, t_1, already defined and t_2, the time during which there is a slow variation of quantities over time and the last two domains remain. While $t_1 \propto pL$, one can argue that $t_2 \propto (1-p)^3 L^2$. The argument runs as follows: Let us for convenience consider the open boundary case. Here, the size sensitivity of the spins is $R^{open} = qL$ where $0 \leq q \leq 1$ with the system assuming the model I behaviour for $q \geq 0.5$. At very late times, there will remain only one domain boundary in the system separating two domains of size, say, γL and βL, such that $\gamma + \beta = 1$. With both $\gamma, \beta > q$ the domain wall will perform random walk until either of the domains shrinks to a size qL. (This picture is valid for $q < 0.5$ and otherwise the dynamics will be simple model I type). Let us suppose that the domain with initial size βL shrinks to qL in time t_2^{open} such that the domain wall performs a random walk over a distance s where $\beta L - s = qL$. This gives

$$t_2^{open}(\beta) \propto (\beta - q)^2 L^2.$$

Or, the average value of t_2^{open} is given by

$$t_2^{open} \propto \int_q^{1-q} (\beta - q)^2 L^2 d\beta = \frac{(1-2q)^3 L^2}{3}.$$

The result for the periodic boundary condition is obtained by putting $q = p/2$ such that

$$t_2 \propto (1-p)^3 L^2$$

and therefore

$$t_{sat} = apL + b(1-p)^3 L^2 \tag{2}$$

The above form is also consistent with the fact that $t_{sat} \propto L^2$ for $p = 0$ and $t_{sat} \propto L$ for $p = 1$.

For large L, the second term in the above equation will dominate making $t_{sat} \propto (1-p)^3 L^2$. In order to verify this, we have numerically obtained t_{sat} and plotted t_{sat}/L^2

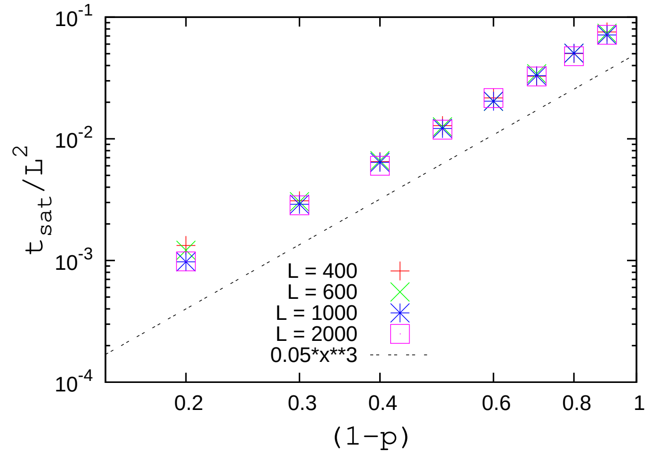

Figure 8. Scaled saturation time (t_{sat}/L^2) against $(1-p)$ for different L shows collapse with $t_{sat}/L^2 \propto (1-p)^3$.

against $(1-p)$ for different L and found a nice collapse and a fit compatible with eq (2) (Fig. 8) with $a \sim 1$ and $b \sim O(10^{-2})$. We conclude therefore that in the thermodynamic limit at later times, for any $p \neq 1$, $z = 2$, i.e., the dynamics is diffusive. This argument, in fact holds for $p \to 0$ as well showing that for R finite, $z = 2$, as discussed in the preceding section.

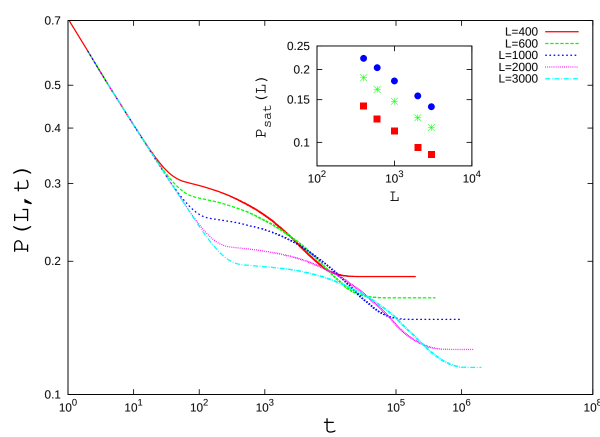

Figure 9. Persistence probability as a function of time for $p = 0.4$ for different sizes. Inset shows that the saturation values of the persistence probability shows a variation $L^{-\alpha}$ for values of $p = 0.8, 0.4, 0.2$ (from top to bottom) with $\alpha \simeq 0.230$.

We have discussed so far the time dependent behaviour and exponents only. But another exponent α which appears at $t \to \infty$ for the persistence probability can also be extracted here. The persistence probabilities show the conventional saturation at large times, with the saturation values depending on L. The log-log plot of $P(L, t \to \infty)$ against L shows that power law behaviour is obeyed here with the exponent α once again coinciding with the model I value, ~ 0.23 for any value of $p \neq 0$ (Fig. 9).

Having obtained α, we use eq (1) with trial values of z to obtain a collapse of the data PL^α versus t/L^z for any value of nonzero $p < 1$. As expected, an unique value of z does not exist for which the data will collapse over all t/L^z. However, we find that using $z = 1$, one has a nice collapse for initial times up to t_1 while with $z = 2$, the data collapses over later times (Fig. 10). The significance of the result is, an unique value of α is good for collapse for both time regimes. However, it is not possible to extract any value of θ for later times as θ is extracted from eq (1) in the limit $t/L^z < 1$ only.

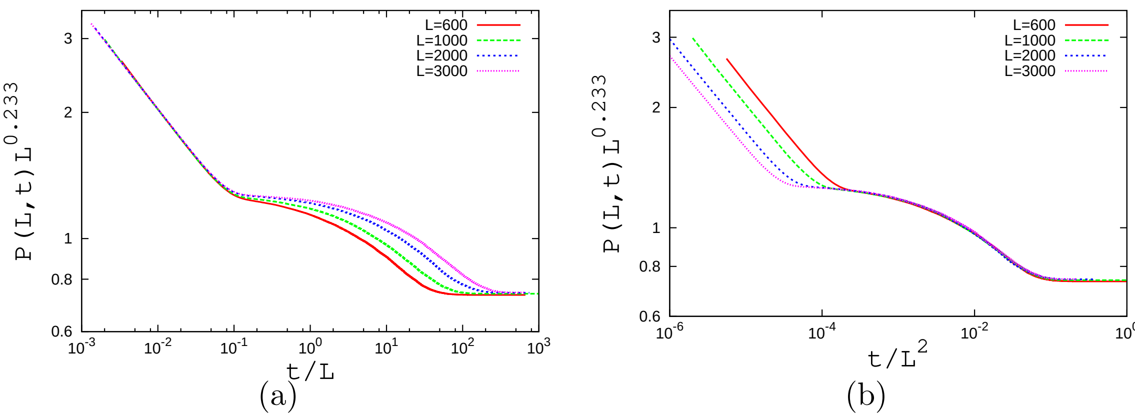

Figure 10. PL^α versus t/L^z for $p = 0.4$, shows a nice collapse for initial times up to t_1 using $z = 1$ and $\alpha = 0.233$ (a) while using $z = 2$ and the same value of α, the data collapses over later times (b).

4.2. Discussions on the results

At this juncture, several comments and discussions are necessary. We have obtained a crossover behaviour in this model where an initial ballistic behaviour for macroscopic time scales is followed by a diffusive late time behaviour. However, the diffusive behaviour at later times is not apparent in the simple log-log plots of the variables and can be extracted only from the study of the total time to equilibriate. This is due to the fact that the initial ballistic dynamics leaves the system into a non-typical configuration which is evidently far from those on diffusion paths. In fact in the diffusive regime, the coarsening process hardly continues in terms of domain growth as only few domain walls remain at $t > t_1$.

A consequence of this is evident in the behaviour of the persistence at later times. One may expect that the persistence exponent 3/8 may be obtained at very late times as here one has independent random walkers, few in number, which annihilate each other as they meet much like in a reaction diffusion process. However, such an exponent is not observed from the data (Fig. 6). Although with $z = 2$ we can obtain a collapse at later times, it is not possible to obtain a value of θ. Since persistence is a non-Markovian phenomena and it depends on the history, the exponent may not be apparent even if the phenomena is reaction diffusion like. Therefore to analyse the dynamical scenario further, we study the persistence in a different way. In order to study the persistence dynamics beyond $t = t_1$, we reset the zero of time at $t = t_1$. In case the number of domain walls left in the system at t_1 is of the order of the system size ($O(L)$), the behaviour of persistence should be as in the case of Ising model, i.e., a power law decay with exponent 3/8. On the other hand, if the number of independent random walkers is *finite* (i.e., vanishes in the $L \to \infty$ limit) which can not annihilate each other, the persistence probability is approximately

$$P_{rand}(t, L) = 1 - ct^{1/2}/L, \qquad (3)$$

where we have assumed that number of distinct sites visited by the walker is proportional to the distance travelled, which is $O(t^{1/2})$.

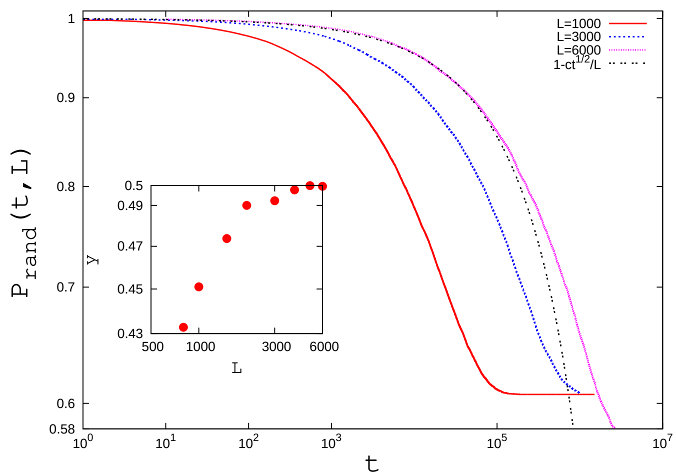

Figure 11. Persistence probability shows a decay as a function of time when t_1 is set as the initial time. The $L = 6000$ curve is fitted to the form $P_{rand}(t, L) = 1 - ct^y/L$ with $y = 0.5$ (shown with the broken line). Inset shows the variation of y with system size. $p = 0.4$ here.

We find that in the present case, resetting the zero of time at t_1, the persistence probability shows a decay before attaining a constant value. The decay for a large initial time interval can be fitted to a form $\tilde{P}(t) = 1 - ct^y$ where the exponent y increases with L and clearly tends to saturate at 0.5 as the system size is increased. This shows that the persistence probability is identical to (3) in form (Fig. 11). This signifies that at $t > t_1$, the dynamics only involves the motions of random walkers which do *not* meet and annihilate each other for a long time and explains the fact that domain walls remain a constant over this interval. Only at very large times close to equilibriation the domain walls meet and the persistence probability starts deviating from the behaviour given by (3). Actually once one of the neighbouring domains becomes less than $pL/2$ in size, the random walk will cease to take place and will become ballistic, which finally leads to annihilation within a very short time. Therefore although we have at later times independent walkers performing random walk, the power law behaviour with exponent 3/8 will never be observed (even when the origin of the time is shifted) as the annihilation here is not taking place as in a usual reaction diffusion system but determined by the model I like dynamics. It may also be noted that beyond $t = t_1$, annihilations occur only when the system is very close to equilibriation unlike in a reaction diffusion system where annihilations occur over all time scales.

The reason why a single value of α is valid for both $t > t_1$ and $t < t_1$ is also clear from the above study. We expect that at $t = t_1$, the number of persistent sites $\propto L^{-\alpha}$ with the value of $\alpha \simeq 0.235$ as in model I. The additional number of sites which become non-persistent beyond t_1 is proportional to $(t - t_1)^y/L$ and therefore at $t = t_{sat}$ expected number of persistent site is

$$c_1 L^{-\alpha} - c_2(t_{sat} - t_1)^y/L = c_1 L^{-\alpha} - c_2 t_2^y/L \ ,$$

where c_1, c_2 are proportionality constants. Since in the thermodynamic limit $y \to 1/2$ and $t_2 \propto L^2$, the number of persistence sites remains $\propto L^{-\alpha}$. Here we have assumed c_2 to be independent of L, the assumption is justified by the result.

5. The case with quenched randomness

In this section, we briefly report the behaviour of the system when each spin is assigned a value of p $(0 < p \leq 1)$ randomly from a uniform distribution. The randomness is quenched as the value of p assumed by a spin is fixed for all times.

Here we note that the equilibrium behaviour, all spins up or down is once again achieved in the system. However the time to reach equilibrium values are larger than the $p = 1$ case.

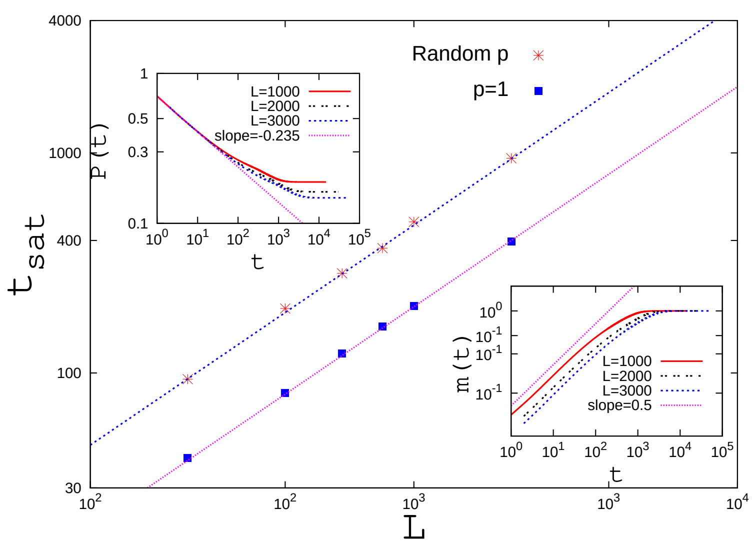

Figure 12. Saturation time (t_{sat}) against system size L shows $z = 1$. Inset on the top left shows the persistence probability $P(t)$ with time which follows a power law decay with exponent ~ 0.235 initially. The other inset on the bottom right shows the growth of magnetisation $m(t)$ with time where the initial variation is like $m(t) \sim t^{1/2}$.

The entire dynamics of the system, once again, can be regarded as walks performed by the domain walls. For $p = 1$ for all sites, the walks are ballistic with the tendency of a domain wall being to move towards its nearest one. For $0 < p \leq 1$ but same for all sites, as discussed in the previous section, the walk is either ballistic (at initial times) or diffusive (at later times) but identical for all the walkers. When p is different for each site, one expects that when a site with a relatively large p is hit, the corresponding domain wall will move towards its nearest domain wall while when a site with relatively small p is hit, the dynamics of the domain wall will be diffusive.

It has been previously noted that model I with noise (of a different kind) which induces similar mixture of diffusive and ballistic motions shows an overall ballistic behaviour (for finite noise) with the value of the dynamic exponent equal to unity [15]. In the present model with quenched randomness also, we find, by analyzing the saturation times that $z = 1$. However, the variation of the magnetisation, domain walls and persistence show power law scalings with exponents corresponding to model I only for an initial range of time (Fig 12).

6. Summary and concluding remarks

In summary, we have proposed a model in which a cutoff is introduced in the size of the neighbouring domains as sensed by the spins. The cutoff R is expressed in terms of a parameter p. At $p \to 0$ (finite R) and $p = 1$ it shows pure diffusive and ballistic behavior respectively. In the uniform case where p is same for all spins, a ballistic to diffusive crossover occurs in time for any nonzero $p \neq 1$. Usually in a crossover phenomenon, where a power law behaviour occurs with two different exponents, the crossover is evident from a simple log-log plot. In this case, however, the crossover phenomena is not apparent as a change in exponents in simple log-log plots does not appear. The crossover occurs between two different types of phenomena, the first is pure coarsening in which domain walls prefer to move towards their nearest neighbours as in model I and one gets the expected power law behaviour. At t_1, as mentioned before, some special configurations are generated and therefore the second phenomena involves pure diffusion of a few domain walls (density of domain walls going to zero in the thermodynamic limit) which remain non-interacting up to large times. Naturally, the only dynamic exponent in the diffusive regime is the diffusion exponent $z = 2$ which is *distinct* from the growth exponent $z = 1$. So the two dynamic exponents not only differ in magnitude, they are connected to distinct phenomena as well. This crossover behaviour is therefore a striking feature for the model. For R finite ($p \to 0$), there is no crossover effect, as the time t_1 is too small to generate these special configurations and usual reaction diffusion type of behaviour prevails.

Persistence probability, in whichever way one sets the zero of time, does not show any power law behaviour in the second time regime. At the same time, a single value of α is required for the collapse in the two regimes.

Another point of interest is that while $z = 2$ is expected for nonzero $p \neq 1$ values at later times, the behaviour of the total time to equilibriate as a function of p is not obvious. Our calculation shows that it is proportional to $(1-p)^3$, which is another important result of the present work.

We also found that making p a quenched random variable taken from an uniform distribution, one gets back model I like behaviour to a large extent. However, choosing a different distribution might lead to different results. The fact that the model has different behaviour with uniform p and with quenched random value of p is reminiscent of the different behaviour observed in agent based models with savings in econophysics [19].

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References

[1] S. Biswas and P. Sen, Phys. Rev. E **80**, 027101 (2009).
[2] A. J. Bray, Adv. Phys. **43** 357 (1994) and the references therein.
[3] H. K. Janssen, B. Schaub and B. Schmittmann, Z.Phys. **B73** 539 (1989)
[4] E. Renshaw and R. Henderson, Journal of Applied Probability **18**, 403 (1981), D. Szasz and B. Toth, Journal of Statistical Physics **37**,(1984); I. M. Sokolov, J. Klafter, and A. Blumen, Phys. Rev. E **61**, 2717(2000).
[5] E. Almaas, R. V. Kulkarni and D. Stroud, Phys. Rev. E **68**, 056105(2003) A. Procacci, R. Sanchis and B. Scoppola, Lett. Math. Phys. **83**, 181 (2008).
[6] H. Hinrichsen, Adv. Phys. **49** 815 (2000); J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press, Cambridge), 1999; G. Odor, Rev. Mod. Phys. **76** 663 (2004).
[7] T. M. Liggett *Interacting Particle Systems: Contact, Voter and Exclusion Processes* (Springer-Verlag Berlin 1999).
[8] D. Stauffer, in *Encyclopedia of Complexity and Systems Science* edited by R. A. Meyers (Springer, New York, 2009); K. Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys C **11** 1157 (2000); S. Galam, Int. J. Mod. Phys. C **19** 409 (2008).
[9] A. Baronchelli, L. Dall'Asta, A. Barrat, and V. Loreto, Phys. Rev. E **76**, 051102 (2007); C. Castellano, M. Marsili and A. Vespignani, Phys. Rev. Lett. **85** 3536 (2000).
[10] J. Fuchs, J. Schelter, F. Ginelli and H. Hinrichsen, J. Stat. Mech. P04015 (2008).
[11] D. Stauffer and P. M. C. de Oliveira, Eur. Phys. J B **30** 587 (2002.)
[12] J. R. Sanchez, arXiv:cond-mat/0408518v1
[13] P. Shukla, J. Phys. A Math. Gen. **38** 5441 (2005).
[14] B. Derrida, A. J. Bray and C. Godreche, J.Phys. A **27** L357 (1994). For a review on persistence, see S. N. Majumdar, Curr. Sci. **77** 370 (1999).
[15] P. Sen, Phys. Rev. E **81**, 032103(2010)
[16] G. Manoj and P. Ray, Phys. Rev. E **62** 7755 (2000); G. Manoj and P. Ray, J. Phys A **33** 5489 (2000).
[17] S. Biswas, A. K. Chandra and P. Sen, Phys. Rev. E **78**, 041119 (2008)
[18] P. Sen and S. Dasgupta, J. Phys. A Math. Gen. **37** 11949 (2004).
[19] A. Chatterjee, B. K. Chakrabarti, S. S. Manna, Physica A **335** (2004) 155; Phys. Scripta T **106** (2003) 36.