

# Nonconservative kinetic exchange model of opinion dynamics with randomness and bounded confidence

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The concept of a bounded confidence level is incorporated in a nonconservative kinetic exchange model of opinion dynamics model where opinions have continuous values  $\in [-1, 1]$ . The characteristics of the unrestricted model, which has one parameter  $\lambda$  representing conviction, undergo drastic changes with the introduction of bounded confidence parametrised by  $\delta$ . Three distinct regions are identified in the phase diagram in the  $\delta - \lambda$  plane and the evidences of a first order phase transition for  $\delta \geq 0.3$  are presented. A neutral state with all opinions equal to zero occurs for  $\lambda \leq \lambda_{c_1} \simeq 2/3$ , independent of  $\delta$ , while for  $\lambda_{c_1} \leq \lambda \leq \lambda_{c_2}(\delta)$ , an ordered region is seen to exist where opinions of only one sign prevail. At  $\lambda_{c_2}(\delta)$ , a transition to a disordered state is observed, where individual opinions of both signs coexist and move closer to the extreme values ( $\pm 1$ ) as  $\lambda$  is increased. For confidence level  $\delta < 0.3$ , the ordered phase exists for a narrow range of  $\lambda$  only. The line  $\delta = 0$  is apparently a line of discontinuity and this limit is discussed in some detail.

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## I. INTRODUCTION

Many social phenomena can be modelled and analysed using the methods of statistical physics. Opinion dynamics is one such phenomenon where the microscopic interactions taking place between individuals can lead to cooperative behaviour of the society, like the emergence of a consensus or agreement. Recently, several opinion dynamics models with continuous opinion have been proposed in which the opinions are updated after a pair of individuals interact and exchange opinion. Here the interaction scheme is similar to a kinetic theory of gas, where one assumes that gas molecules can collide and consequently exchange energy and momentum. Two basic opinion dynamics models having such kinetic exchange scheme are due to Deffuant et al [1] and Lallouache et al [2], which involve completely different concepts. In this paper, we attempt to combine the concepts used in these two basic models in a single model. In section II, we briefly review these two models. In section III, the proposed model is described. The results are presented and discussed in section IV. Summary and conclusions are given in the last section.

## II. A BRIEF REVIEW OF TWO KINETIC EXCHANGE MODELS

Deffuant et al [1] introduced a simple model (DNAW model hereafter) in which opinion exchanges between two agents take place only when the difference in the original opinions is less than or equal to a preassigned quantity  $\delta$ . If  $o_i(t)$  is the opinion of the  $i$ th agent interacting with the  $j$ th agent at time  $t$  (with  $|o_i - o_j| \leq \delta$ ), then in this

model the opinions evolve according to:

$$\begin{aligned} o_i(t+1) &= o_i(t) + \gamma(o_j(t) - o_i(t)) \\ o_j(t+1) &= o_j(t) + \gamma(o_i(t) - o_j(t)). \end{aligned} \quad (1)$$

Here  $\gamma$  is a constant ( $0 \leq \gamma \leq 0.5$ ) called the convergence parameter and  $o_i$  lies in the interval  $[0, 1]$ . The dynamics is such that the opinions tend to come closer after interaction. Hence as the dynamics proceeds, convergence to a finite number of opinions happens; opinions cluster around a few values and individuals belonging to different clusters no longer interact. The initial distribution of the individual opinions is uniform and therefore symmetric. This symmetry is broken as the time evolved distribution has a multi peaked delta function form. When there is only one peak in the final distribution, it is said to be a case of *consensus*, two peaks imply *polarisation* and the existence of a larger (finite) number of peaks signifies *fragmentation* in the society.

The model is conservative as total opinion is conserved in each interaction. Obviously, in this conservative model, consensus would imply that opinions converge to the value  $1/2$ . Several models have been formulated incorporating the idea of bounded confidence later [3–5] and a general form of kinetic exchange type model proposed in [6].

More recently, a model in which kinetic exchanges take place with randomness, and where there is no conservation, has been introduced by Lallouache et al [2] (LCCC model hereafter). Any two agents can interact in this unrestricted model. The opinion evolution here follows the rule:

$$\begin{aligned} o_i(t+1) &= \lambda[o_i(t) + \epsilon o_j(t)] \\ o_j(t+1) &= \lambda[o_j(t) + \epsilon' o_i(t)]; \end{aligned} \quad (2)$$

where  $\epsilon, \epsilon'$  are drawn randomly from uniform distributions in  $[0, 1]$ . In this model  $\lambda$  is a parameter which is interpreted as ‘conviction’. The opinions are bounded,

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i.e.,  $-1 \leq o_i(t) \leq 1$ ; in case  $o_i$  exceeds 1 or becomes less than  $-1$ , it is set to 1 and  $-1$  in the respective cases.

It is possible to rescale the opinions in the DNAW model so that they lie in the interval  $[-1, 1]$ . Continuous opinions are relevant in cases like supporting an issue, rating a movie etc. Thus setting the interval as  $[-1, 1]$  appears to be more meaningful since a positive (negative) value of the opinion will mean liking (disliking) the motion. The magnitude of the opinion would then simply correspond to the amount of liking or disliking. For the rest of the paper, we thus consider opinions  $\in [-1 : 1]$ .

As there is no conservation in the LCCC model, the average opinion given by  $m = |\sum_i o_i|/N$  for a population of  $N$  agents, evolves in time and  $m$  can play the role of an order parameter. It is, in fact, analogous to magnetisation in magnetic systems. One can say that there is order/disorder when  $m(t \rightarrow \infty)$  converges to a nonzero/zero value. The model shows a unique behaviour: below a critical value of  $\lambda \simeq 2/3$ , all opinions identically turn out to be zero while above it, there is a nonzero value of the average opinion. Thus for  $\lambda > 2/3$ , an ordered phase exists. Interestingly, the opinions in the ordered phase have either all positive or all negative values. Generalisation and variations of the LCCC model have been considered in some subsequent works [7–9].

Symmetry breaking has different connotations in the conserved DNAW model and the nonconserved LCCC model. In the former, if opinions are initially in the interval  $[-1, 1]$ , a consensus implies convergence of all opinions to zero value and this is regarded as symmetry breaking as mentioned earlier. In LCCC, the identical state of all zeros is also obtained below  $\lambda \simeq 2/3$  even without putting any restriction on the interactions. However, this state has been interpreted as a symmetric state [2]. This is following the idea that as  $m = 0$  here, it is like a paramagnetic state (which is a symmetric state in magnetic systems). But obviously, this is a very special paramagnetic state which also has zero fluctuation.

### III. THE MODEL

Having conservation in the opinions is rather unrealistic but the concept of having a bounded confidence level is relevant in many cases. On the other hand, it is true that people may have a conviction level in their and in others' opinions. We thus combine the LCCC model and the DNAW model by putting the restriction of the bounded confidence in the former. It has to be realised that the conviction and bounded confidence are independent concepts and the parameters associated with these may be regarded as independent. Putting a bounded confidence may be applicable to cases in which there are groups of people (e.g., members of a political party) who share similar opinion and interact within themselves primarily. On the other hand, conviction is associated with individual behaviour. It is known that in models with bounded confidence, for small confidence level, the soci-

ety is fragmented because of lack of communication. A large value of conviction is not expected to help reach consensus when the confidence level is low when we combine the two concepts, as the groups stick to their original opinions in this case. On the other hand, even when confidence level is large, very large values of conviction can make the society polarised as both positive and negative opinions can prevail simultaneously.

In the model proposed in the present paper, we follow eq (2) for the evolution of opinions but put the restriction that agents interact only when  $|o_i - o_j| \leq 2\delta$ .  $\delta$  is once again the parameter representing the confidence level and can vary from zero to 1.

We therefore have two parameters in the model,  $\lambda$  and  $\delta$ .  $\delta = 1$  is identical to the original LCCC model.  $\delta = 0$  is an interesting limit. Here, agents interact only when their opinions are exactly same. We will discuss this limit in greater detail later.

The order parameter in this model is the one defined for the LCCC model, i.e.,  $m = |\sum_i o_i|/N$ . To avoid confusion, we adopt the following terminology for the present nonconserved model: when  $m = 0$  and also the individual opinion distribution is a delta function peaked at zero, we call it a *neutral* state. If it is not a delta function at zero but  $m = 0$ , the state will be termed a *disordered* state. When  $m \neq 0$ , it is an *ordered* state; further if all individuals have identical opinion (which is perhaps only ideological as opinions are continuously distributed), it is a *consensus* state. Hence consensus is not merely an agreement in this terminology. Obviously, in the conserved system, the nomenclature of order and disorder is irrelevant. We intend to locate the regions of neutrality, order and disorder in the present model in the  $\delta - \lambda$  plane.

## IV. RESULTS AND NUMERICAL ANALYSIS

We take a population of  $N$  agents and let it evolve according to the dynamical rules defined above (i.e., eq 2 with a bounded confidence level  $\delta$ ).

### A. Order parameter

The behaviour of the order parameter after the system reaches equilibrium is presented in Fig. 1. As a function of  $\lambda$ , we find that the order parameter first assumes non-zero values at a threshold value of  $\lambda = \lambda_{c_1}$  which is independent of  $\delta$ ;  $\lambda_{c_1} \simeq 2/3$  as in the original LCCC model. The order parameter increases with  $\lambda$  beyond  $\lambda_{c_1}$  up to a certain value of  $\lambda$  and decreases to zero as  $\lambda$  is increased further. The decrease becomes steeper with  $\delta$  and more so when the system size is increased. The results indicate that there are three distinct regions: one ordered region for intermediate values of  $\lambda$  and two regions at low and high values of  $\lambda$  where the order parameter vanishes. These two regions may be either disordered or neutral.

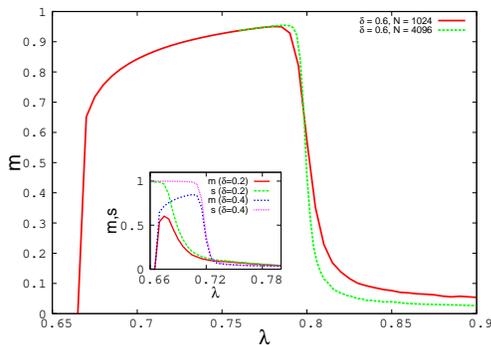


FIG. 1: (Color online) Variation of the order parameter  $m$  with  $\lambda$  for  $\delta = 0.6$  for  $N = 1048$  and  $4096$ . Inset shows  $m$  and  $s$  for  $\delta = 0.2$  and  $0.4$  against  $\lambda$  for  $N = 1048$ .

We also study the behaviour of another quantity  $s$  defined as

$$s = \langle |f_+ - f_-| \rangle, \quad (3)$$

where  $f_+$  denotes the fraction of population with opinion greater than zero and  $f_- = 1 - f_+$  in a particular configuration.  $\langle \dots \rangle$  denotes average over all configurations. It can be easily seen that  $s$  is equal to unity both in the neutral state and ordered state of LCCC (in the numerical study, we take an opinion value to be equal to zero when its absolute value is less than  $10^{-8}$  [10]). Deviation of  $s$  from unity will indicate that opinions with both signs are present in general. We notice that  $s$  remains close to unity as  $\lambda$  is increased from zero before showing a sharp fall close to a value of  $\lambda$  where the order parameter also starts to fall (Fig. 1 inset). Evidently, as the system enters the disordered state, individual opinions are  $> 0$  and  $\leq 0$  with equal probability.  $s$  has a monotonic behaviour and is useful to locate the transition to the disordered state. Moreover, comparison of  $s$  and  $m$  shows that these two measures become closer and tend to merge as  $\lambda$  is increased further. This indicates that as one moves deeper into the disordered region, opinions become more and more close to the extreme values  $\pm 1$ , leading to a polarisation tendency in the opinions.

## B. Individual opinion distribution

To understand the nature of the phases, the distribution of individual opinions may be studied. Such studies are known to lead to a correct speculation about phase transitions [11]. This study shows (Fig. 2) that the probability for zero opinion is nearly equal to 1 below  $\lambda_{c1} \simeq 2/3$  as in the LCCC model for all  $\delta$ . Hence a neutral state exists here as well and the confidence level is absolutely irrelevant as  $\lambda_{c1}$  is independent of  $\delta$ .

It may be noted that a plateau like region exists in the distribution where the individual opinions have more or less the same probability. The extent of the plateau depends on  $\lambda$  and shrinks with increasing  $\lambda$ . One can

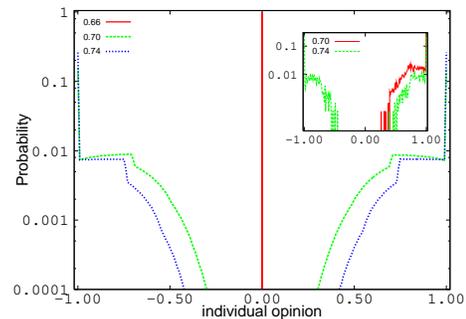


FIG. 2: (Color online) The distribution of individual opinions shown for  $N = 2048$  different values of  $\lambda$  for  $\delta = 0.4$  averaged over different configurations. Inset shows the same for a single realisation of the system. For  $\lambda < \lambda_{c1} \simeq 2/3$ , the average distribution is a delta function at zero. For  $\lambda_{c1} < \lambda < \lambda_{c2} \simeq 0.7183$  (for  $\delta = 0.4$ ), for a single configuration, opinions are all of one particular sign, while for averages over all configuration, the distribution is symmetric. For  $\lambda > \lambda_{c2}$ , the distribution is symmetric even for the single configuration.

investigate the reason for the existence of such a plateau region by some further analysis. Let us take specifically the example of  $\lambda = 0.74$  and  $\delta = 0.4$  where the plateau extends from about  $|o| = 0.75$  to nearly  $|o| = 1$ . It should be noted that even in equilibrium the individual opinions undergo dynamics. We tag individuals with absolute value of opinion greater than 0.75 at a time when the system has reached equilibrium and note their evolution. First, the probability  $P_t$  that these remain greater than 0.75 in subsequent times is calculated. For an ergodic case, i.e., if the opinions can span the entire opinion space with equal probability, this probability should be around 0.25. We find, however, that  $P_t$  is nearly 0.9 for  $\lambda = 0.74$  and  $\delta = 0.4$ . This means that the opinions remain within this interval with a high probability and one need not really bother about opinions with lesser values to analyse the situation in the plateau region.  $P_t$  may be close to unity if opinions reach a fixed point (or weakly fluctuate about one) and does not imply that a plateau like region will emerge. Hence we probe further by splitting the interval into further subdivisions. We estimate separately  $P_1$ ,  $P_2$  and  $P_3$  where,

- (i)  $P_1$  is the probability that opinions at later times are greater than 0.99
- (ii)  $P_2$  is the probability that opinions at later times lie between 0.89 and 0.99
- (iii)  $P_3$  is the probability that opinions at later times lie between 0.79 and 0.89.

One can define other probabilities but these three are sufficient for our purpose in the present case. We also take three mutually exclusive subsets of the tagged agents with initial values of  $|o|$  between 0.75 to 0.80 (subset A), between 0.85 to 0.9 (subset B) and between 0.95 to 1.0 (subset C). Here initial value implies the value at the time of tagging after the system has equilibrated. We separate out  $P_1$  as the probability of individual opinions

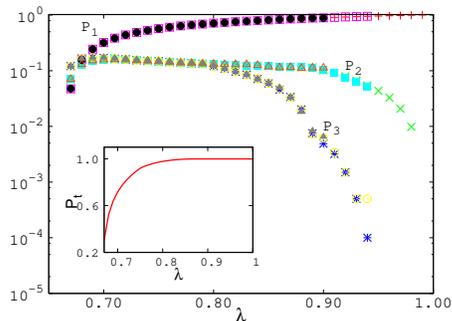


FIG. 3: (Color online) The probabilities  $P_1, P_2$  and  $P_3$  are shown against  $\lambda$  for subsets A, B and C. The data for the different subsets coincide for each of the three probabilities. Inset shows the total probability that starting with an opinion with absolute value greater than 0.75, it remains so at subsequent times. Data is for  $\delta = 0.4$  and  $N = 2048$ .

having values very close to unity is much greater than the plateau height (this is simply because of the imposed boundary condition on the opinions) as shown in Fig. 2. The idea is that if opinions do reach (nearly) fixed point values,  $P_2$  and  $P_3$  should be grossly different for all the subsets A, B and C (e.g., for A,  $P_2$  will be less than  $P_3$  etc.) On the other hand, if  $P_2$  and  $P_3$  turn out to be close to each other for any subset, one can conclude that the subspace  $0.79 < |o| < 1.0$  is uniformly accessible to the agents within that subset. Moreover if these probabilities are independent of subset A, B and C, one can argue that a plateau region will exist. We find all the three probabilities  $P_1, P_2$  and  $P_3$  to be independent of the subset considered (Fig. 3).  $P_1$  turns out to be much higher than the other probabilities as expected and indeed,  $P_2$  and  $P_3$  are very close to each other for  $\lambda = 0.74$ . Hence the plateau exists and the reason is the opinions here evolve in such a way that only part of the opinion space is spanned but spanned with equal probability. We have in fact obtained  $P_1, P_2, P_3$  and  $P_t$  for other values of  $\lambda$  for  $\delta = 0.4$  and shown them in Fig. 3.

As  $\lambda$  is increased beyond  $\lambda_{c1}$ , in a single configuration, only all positive or all negative values are obtained as  $N \rightarrow \infty$ , while the average over all configurations is symmetric about zero as expected. However, as  $\lambda$  is increased further, the opinions, even in a single configuration, assume both negative and positive values symmetrically (Fig. 2 inset). These results are consistent with the results for the quantity  $s$  presented earlier. Hence we infer that an order-disorder transition is taking place at a value  $\lambda_{c2} > \lambda_{c1}$  which is later confirmed from more detailed analysis. For  $\delta = 1$ , the LCCC model,  $\lambda_{c2}$  is equal to 1 as expected.

### C. Phase transitions and critical properties

The ordered, disordered and neutral regions may be identified in a phase diagram in the  $\delta - \lambda$  plane. To

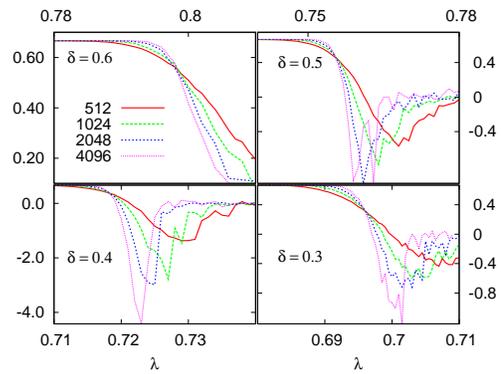


FIG. 4: (Color online) The Binder cumulant is shown for different values of  $\delta$  for  $N = 512, 1024, 2048$  and  $4096$ . For smaller values of  $\lambda$ , it shows clearly that a negative dip exists which approaches the intersection point as  $N$  increases. Other analysis suggests that the negative dip will be observed for all values of  $\lambda > 0.3$  if sufficiently larger system sizes are considered (see text). Colour code is same for all the figures.

obtain the phase boundaries in this plane, we estimate the phase transition points by traditional methods, i.e., attempt finite size scaling for the relevant physical variables, if possible. Among these variables is the fourth order Binder cumulant (BC) defined as

$$U = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}. \quad (4)$$

Here we discuss the case for  $\delta < 0.3$  and  $\delta \geq 0.3$  separately for reasons which will be clear later.

$\delta \geq 0.3$ : Plotting  $U$  against  $\lambda$ , we find that there is indeed a crossing point but interestingly, the BC shows a negative dip (Fig. 4) for  $\delta < 0.6$  for the system sizes considered. In fact it becomes more negative as the system size  $N$  is increased and the location of the negative dip approaches the crossing point as well. These are typical indications of a first order phase transition [12]. To confirm whether a first order transition is really taking place, we also plot the distribution of the order parameter very close to  $\lambda_{c2}$ . One expects the distribution to have peaks at nonzero values of  $m$  below the critical point (usually the distribution is a double gaussian). For a continuous phase transition, as the critical point is approached from below, the peaks occur at smaller and smaller values of  $m$ , finally merging at  $m = 0$  continuously at the critical point. For a first order phase transition, on the other hand, the peaks at nonzero values of the order parameter remain at more or less the same positions up to the transition point [12–16]. Here we find exactly this behaviour (Fig. 5); note that for finite systems, weak peaks will still show at nonzero values of  $m$  just above the transition point (instead of a perfect gaussian with mean zero).

We attempt to obtain scaling forms for the BC ( $U$ ), order parameter ( $m$ ) and a quantity analogous to susceptibility per spin (in magnetic systems) given by  $\chi =$

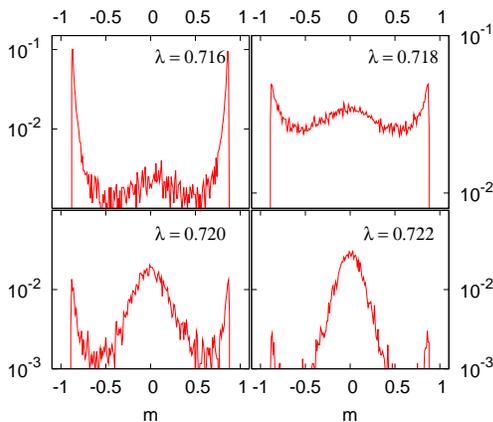


FIG. 5: (Color online) Distribution of the order parameter close to the transition point  $\lambda_{c_2} \simeq 0.7183$  for  $\delta = 0.4$  ( $N = 2048$ ).

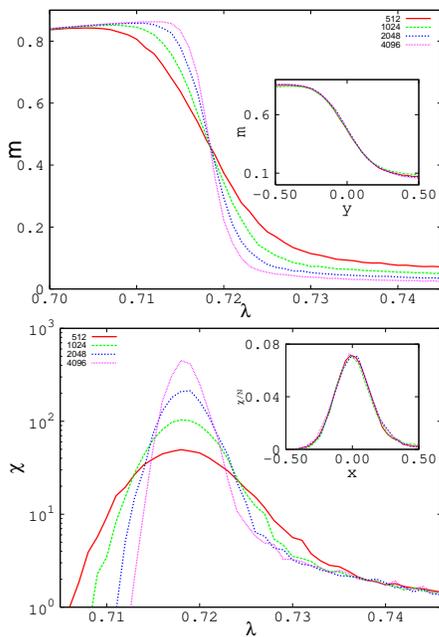


FIG. 6: (Color online) The variation of  $m$  and  $\chi$  are shown for different system sizes for  $\delta = 0.4$ . The insets show the scaling plots with  $m$  and  $\chi/N$  plotted against  $x = (\lambda - 0.7183)N^{0.53}$ .

$\frac{1}{N}[\langle M^2 \rangle - \langle M \rangle^2]$  where  $M$  is the total opinion,  $M = |\sum o_i|$ . The expected behaviour are given by

$$\begin{aligned} U &= f_1((\lambda - \lambda_{c_2})N^\mu) \\ m &= N^{-a} f_2((\lambda - \lambda_{c_2})N^\mu) \\ \chi &= N^b f_3((\lambda - \lambda_{c_2})N^\mu). \end{aligned} \quad (5)$$

For first order phase transitions in finite systems, one expects that instead of a delta function behaviour at the transition point, there will be a peak in the susceptibility which will diverge with the system size. The order parameter exponent  $a$  is expected to be close to zero. We find that the above scaling forms are indeed appropriate

TABLE I: The transition point and the values of the exponents for  $\lambda > 0.3$ . The typical errors in the data are  $\pm 1.0 \times 10^{-4}$  for  $\lambda_{c_2}$ ;  $\pm 0.01$  for  $\mu$ ,  $O(10^{-3})$  for  $a$  and  $O(10^{-2})$  for  $b$ .

$\delta$	$\lambda_{c_2}$	$\mu$	$a$	$b$
0.30	0.6958	0.56	0.00	1.04
0.40	0.7183	0.53	0.00	1.05
0.50	0.7555	0.50	0.00	1.02
0.60	0.7980	0.43	0.00	0.90
0.70	0.8415	0.34	0.035	0.90
0.80	0.8850	0.26	0.00	1.00
0.90	0.9530	0.20	0.025	0.90

in the present case, the data collapse to a single curve for specific values of  $a$ ,  $b$  and  $\mu$  (shown for  $m$  and  $\chi$  in Fig. 6). The value of  $a$  is indeed very close to zero and  $b \simeq 1$  for all values of  $\delta \geq 0.3$ . However, the value of  $\mu$  appears to have a systematic variation with  $\delta$ . Since  $\delta$  effectively puts a restriction on the number of compatible neighbours, it is not surprising that  $\mu$ , which is associated with  $N$ , shows a dependence on  $\delta$ . The values of the transition point  $\lambda_{c_2}$  and the exponents are presented in Table I.

All the above discussions are however, valid for  $\delta > 0.3$  only. The first order phase transition is most strongly observed close to  $\delta = 0.4$ . As for the negative dip, it is not observed for  $\delta \geq 0.6$  with  $N \leq 4096$ , but the values of the exponents indicate that the transition is first order-like. The negative dip for  $\delta \geq 0.6$  is thus expected to be observed for even higher values of  $N$  [12].

$\delta < 0.3$ : When  $\delta$  is decreased below 0.3, the results do not give any clear indication about the nature of the phase transition and shows some anomalous behaviour. A rather uncharacteristic behaviour of the order parameter and the Binder cumulant is observed. The order parameter  $m$  shows a nonmonotonic behaviour when plotted as a function of  $\delta$  with fixed  $\lambda$  or vice versa (Fig. 7). A hump appears in the  $m$  versus  $\lambda$  plots for  $\delta < 0.3$  and  $\lambda \geq 0.7$  showing the existence of a local maximum value. A closer examination reveals that this hump shrinks in size, albeit very slowly, when  $N$  is increased for  $\lambda > 0.71$  (see inset (b) of Fig. 7) and is expected to disappear in the thermodynamic limit. The same is true for the hump appearing in the plot of  $m$  versus  $\lambda$  for  $\delta = 0.25$ . For  $\delta < 0.3$ , there are large fluctuations and irregularities in the BC as well which does not allow one to do a finite size scaling analysis and get the exponents. The irregular behaviour of the BC and the order parameter may be because of the fact that the interactions become less likely to occur as the confidence level is decreased. For this reason we restrict our study to  $\delta \geq 0.1$ .

We have estimated, somewhat approximately,  $\lambda_{c_2}$  for  $\delta < 0.3$  from the crossing point of the order parameter curves for different sizes. For example, from the data

of  $m$  against  $\lambda$  for  $\delta = 0.25$  shown in Fig. 7, we get  $\lambda_{c_2} \approx 0.7$ . The complete phase diagram in the  $\delta - \lambda$  plane is shown in Fig. 8.

#### D. Discussions

Let us try to understand the results by analysing the role of the confidence level  $\delta$ . Let the  $i$ th agent with opinion  $o_i$  interact with another agent with opinion  $o_i + x$  where  $|x| \leq 2\delta$ . Then,

$$o_i(t+1) = \lambda(1 + \epsilon)o_i(t) + \lambda\epsilon x. \quad (6)$$

Consider the case when  $\delta$  is small. On an average, when  $\lambda$  is smaller than  $2/3$ , the first term will make  $o_i$  smaller in magnitude while the contribution of the second term can be neglected. Then we find that  $o_i(t) \rightarrow 0$  as  $t \rightarrow \infty$  implying the convergence to a neutral state. Since for  $\delta = 1$ , it is already known that there is a transition to the neutral state at  $\lambda \simeq 2/3$ , we conclude that for any  $\delta$  this is the case as is confirmed by the numerical results. Actually, when individual opinions decrease towards zero because of the effect of the first term, the difference in opinions automatically becomes less ( $x \rightarrow 0$ ) so that large and small  $\delta$  values have the same effect; the second term does not contribute eventually. Thus  $\lambda_{c_1}$  is independent of  $\delta$ .

What happens at higher values of  $\lambda$ ? First consider small values of  $\delta$  again which means  $x$  is small too. Hence the second term still contributes less compared to the first and  $o_i$  will retain its original sign in most cases if  $\lambda$  is sufficiently large. So it is expected that there will be a region where opinions of both signs are present and  $m = 0$ , as originally opinions are uniformly distributed with positive and negative signs.

However, if  $\delta$  is large, there is no guarantee that the second term is small and opinions will retain their original signs, unless  $\lambda$  is also very high. This explains why we observe the transition to the disordered state at a higher value of  $\lambda$  as  $\delta$  is increased. Also, it is not surprising that there will be an ordered region between  $\lambda_{c_1}$  and  $\lambda_{c_2}$  (as already it is known to be present for  $\delta = 1.0$ ) where the LCCC property of all opinions assuming the same sign is still valid.

Although we have restricted to  $\delta \geq 0.1$  in the numerical simulations, the case when  $\delta$  is exactly equal to zero can be discussed theoretically to some extent. If an interaction takes place, the opinion for the  $i$ th agent follows the evolution equation

$$o_i(t+1) = \lambda(1 + \epsilon)o_i(t). \quad (7)$$

This equation is nothing but the dynamical equation obtained for the LCCC model in the limit of a single parameter map [2, 17, 18], where the transition to an ordered state occurs at a value of  $\lambda = e/4$ . However, in the present model the above is valid only when there is a second agent with opinion equal to  $o_i$  as well. Since this

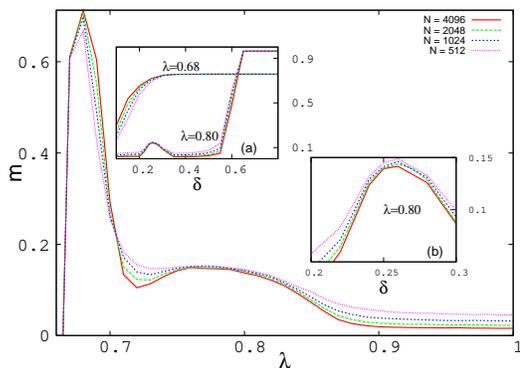


FIG. 7: (Color online) Variation of the order parameter  $m$  with  $\lambda$  for  $\delta = 0.25$ s. Inset (a) shows the variation of  $m$  with  $\delta$  for two fixed values of  $\lambda$ . Inset (b) shows a magnified portion of inset (a) to show the finite size effects. All data are shown for four different values of  $N$  following the same colour code.

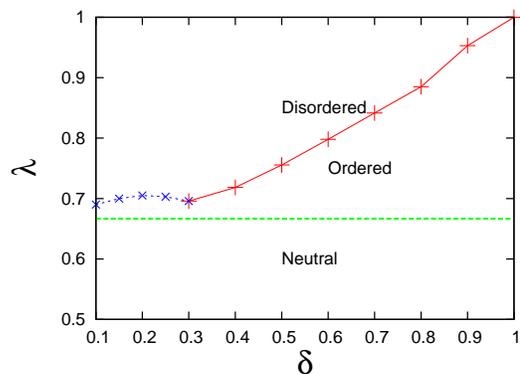


FIG. 8: (Color online) The phase diagram in the  $\delta - \lambda$  plane shows the existence of the neutral region (for  $\lambda \leq \lambda_{c1} \simeq 2/3$ ), the ordered region and the disordered region. The ordered and disordered regions are separated by a first order boundary (continuous line in red) for  $\delta \geq 0.3$  obtained using a finite size scaling analysis. For  $\delta < 0.3$ , the phase boundary (broken line in blue) has been obtained approximately only from the behaviour of the order parameter (see text).

will be extremely rare, effectively most of the opinions remain frozen and the single parameter map is not representative of all the agents' opinion evolution. In fact,  $\delta = 0$  may be regarded as a line of discontinuity in the phase diagram of the model in the  $\delta - \lambda$  plane as it will neither have the neutral state nor the ordered state anywhere. The disordered state is also different in nature for  $\delta = 0$ ; here the individual opinion distribution will be flat while for  $\delta \neq 0$ , however small, it is not so.

#### V. SUMMARY AND CONCLUSIONS

In summary, we have studied a model of continuous opinion dynamics with an attempt to merge the concepts of confidence level and conviction. We find the interesting result that with a large value of conviction and with

any finite bound on the confidence level (i.e.,  $\delta < 1$ ), a disordered state exists with tendency to polarisation. This is indeed justified, if agents are convinced to a large extent in their opinions and interact with like minded people only, the sign of the opinion, (representing liking/disliking) is likely to be maintained, giving rise to a polarised society. The neutral state with all opinions equal to zero remains unperturbed with the introduction of the bounded confidence. It is found that at least for  $\delta > 0.3$ , the order-disorder transition is first order in nature. For smaller confidence level, when  $\delta < 0.3$ , the ordered phase shrinks to a narrow region of the phase diagram.

In conclusion, we obtain a phase diagram with many features when the concept of bounded confidence is incor-

porated in the LCCC model of opinion dynamics. The overall result is that when bounded confidence level is large, there will be order in the society provided people are not too rigid. In the original DNAW model also, it had been shown that above a certain confidence level, society behaves more homogeneously. We show that this tendency remains true but only up to a certain level of conviction. This seems to be a realistic scenario and thus the combination of concepts from two different models in the present model of opinion dynamics is successful in reproducing this desired feature of a society.

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- [10] Numerically, the opinions should assume infinitesimally small values with both plus and minus signs in the neutral state giving  $s = 0$ . As these values tend to zero in the thermodynamic limit  $s$  should actually be unity below  $\lambda_{c_1}$  and therefore opinion values with absolute value less than a small quantity (here  $10^{-8}$ ) are taken as zero. For finite systems, it is also true that opinions have both plus and minus signs only up to a finite time after which a slight imbalance in the opinion signs drives the system to a state where opinions assume only one sign. This occurs at longer and longer times as the system size is increased indicating that in the thermodynamic limit, the opinions will remain of both signs as expected.
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