Neutron rich nuclei in a new binding energy formula and the astrophysical $r-$process

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May 11, 2019

Abstract

Neutron rich nuclei has been studied with a new phenomenological mass formula. Predictions of different mass formulas for the location of the neutron dripline are compared with those from the present calculation. The implications of the new mass formula for r-process nucleosynthesis are discussed. It is found that though the neutron drip line obtained from this formula differs substantially from other formulas, the r-process abundance upto mass 200 are unlikely to be significantly different. The errors inherent in the mass formula are found to play an insignificant role beyond mass $A = 80$.

1 Introduction

Rapid neutron capture or r-process nucleosynthesis is an important ingredient in the production of heavy elements. This process proceeds through very neutron rich regions of the nuclear landscape inaccessible in terrestrial laboratories. It is known not to depend strongly on the neutron absorption cross section. On the other hand, the ground state binding energy strongly influences the process. Experimental mass measurements are presently not possible in nuclei in the r-process path and one has to take recourse to theoretical predictions. In Gangopadhyay[1] (hereafter called Ref. I), a new phenomenological formula for ground state binding energies was introduced. In a subsequent work[2], we explored the proton rich side of the stability valley and studied the implication of the new formula for the rapid proton capture process.

All the phenomenological mass formulas have necessarily been devised by fitting the nuclei near the stability valley as the experimental values are available only for these nuclei. Neutron rich nuclei are experimentally very difficult to produce in terrestrial laboratories. It is not possible to test the performance of the mass formulas for such nuclei. An alternate test of the mass formula may be the reproduction of the r-process abundance.

In the present work we locate the neutron drip line for the mass formula of Ref I and compare our results with two standard mass formulas. Next, we study the astrophysical r-process abundance using the mass formula. We also investigate the effect that the errors in the predictions may have on the r-process.

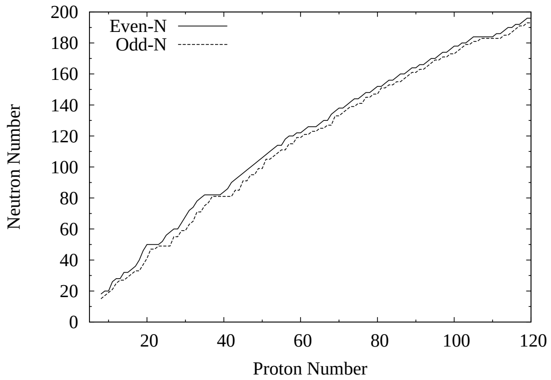

Figure 1: Neutron drip line according to the present calculation.

2 Results

2.1 Neutron dripline

The neutron dripline for a particular neutron number is defined to be the nucleus beyond which the neutron separation energy becomes zero or negative. Obviously, because of the pairing effect, the heaviest isotope of any element necessarily will have even neutron number. Studying the nucleon drip line is one of the major activities in nuclear physics. However, it has been possible to reach the neutron dripline in only very light nuclei which are beyond the ambit of mass formulas.

The calculated mass values of nuclei enable us to calculate the location of the dripline. We determine the location of the neutron dripline from the new binding energy formula of Ref. I and present it in Fig. 1 where we connect the neutron number of the last bound isotope, with odd and even number of neutrons separately for each element, starting from oxygen. As already noted, pairing effect ensures that the drip line for even neutron number lies beyond that for odd neutron number.

Gridnev *et al*[3] have studied extremely neutron rich Zr and Pb nuclei in deformed Hartree-Fock approach using various Skyrme forces. They predicted the existence of stability peninsula in very neutron rich isotopes. They suggested that it is an effect of the shell structure and may be a general feature in neutron rich nuclei. Though our predictions of neutron drip line are closer to the stability valley, we find that shell effects are manifested at the drip line also. As magic nuclei are more stable than their neighbouring isotopes due to the effect of shell closure, an element with neutron magic number is more likely to be the last bound nuclei for that isotopic chain. It is evident from Fig. 1 that at $N = 50$, 82 and and 184, the neutron drip line becomes parallel to the Z axis. At other shell or subshell closures also, this effect is observed though not as prominent as the neutron numbers mentioned above. We also see that at these three neutron numbers, the drip line for odd and even N come very close to each other, indicating the effect of the shell closure.

One needs to remember that the formula has an average error of 376 keV. Thus, if the predicted binding energy of the last neutron in an odd N nucleus

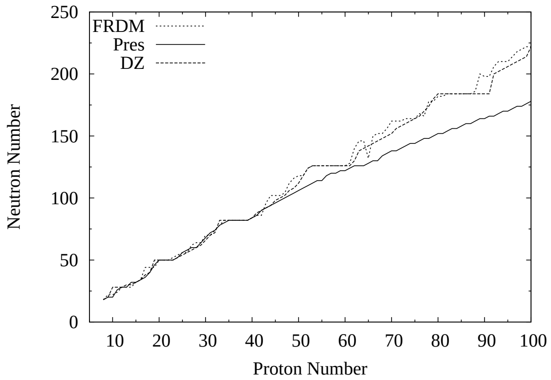

Figure 2: Neutron drip line for even N-nuclei predicted by the present formula (Pres), FRDM and Duflo-Zuker (DZ) formula.

is very small, it is possible that the nucleus is actually beyond the dripline. Conversely, prediction of a very small negative value of last neutron separation energy cannot guarantee that the dripline does not lie actually beyond. Thus, the driplines may conceivably shift by two nucleons either way.

As evident from Ref I, the neutron drip line predicted by the present formula differs substantially from that predicted by other common mass formulas, such as Finite Range Droplet Model (FRDM)[4] or Duflo-Zuker formula[5]. The results for drip line for even neutron nuclei for the present formula are compared with the above two calculations in Fig. 2. We note that beyond $Z \sim 64$, the prediction of the other two formulas differ significantly from the present calculation. However, in absence of experimental data, it is not possible to comment on their relative merits.

2.2 Astrophysical r-process

What about the effect of the new mass formula on nucleosynthesis? We have found the present formula to be successful in describing the astrophysical rp-process that occurs in $N \sim Z$ nuclei and involves proton rich nuclei[2]. Neutron rich nuclei take part in astrophysical r-process where neutrons are captured rapidly by nuclei leading to very neutron rich region. These then decay to end up as ordinary nuclei near the stability valley.

We choose to employ the canonical r-process framework within the waiting point approximation. In this approach, rapid neutron capture is in equilibrium with its inverse process, the (γ, n) process. Thus, the ratio of the number of nuclei with (Z,N+1) to that of (Z,N) is given by the Saha equation.

$$\frac{\mathcal{N}(Z,N+1)}{\mathcal{N}(Z,N)} = \frac{G^*(Z,N+1)}{2G^*(Z,N)} N_n T^{-3/2} 10^{-34.075} 10^{5.04 * S_n(Z,N+1)/T} \tag{1}$$

where T is the temperature in GK and $G^* = (2J_{gs}+1)G$ are the temperature dependent partition functions taken from Rauscher *et al*[6].

Neutrons are rapidly absorbed by the nuclei reaching very large neutron to proton ratio. Beta decay turns the neutrons into protons. Near the neutron

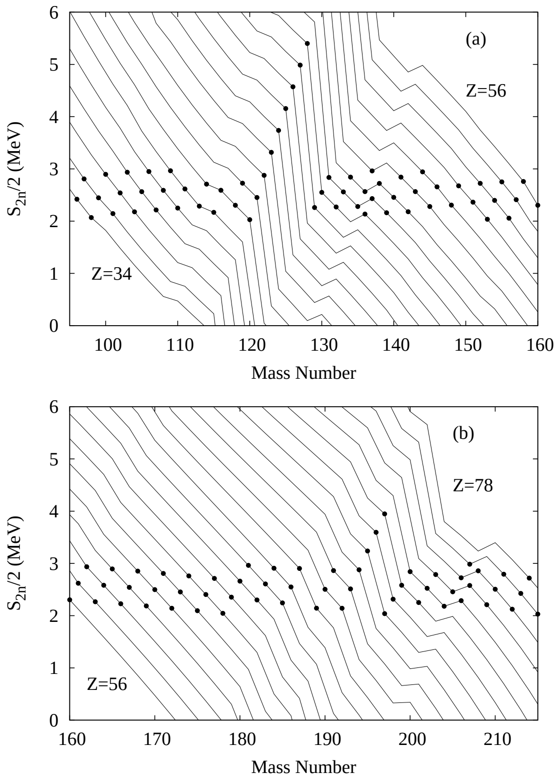

Figure 3: The two neutron separation energy S_{2n} and the r-process path.

shell closures, the r-process path approaches the stability valley resulting in peaks in the abundance distribution. After a certain time interval, r-process is assumed to stop abruptly. The frozen population then undergoes a beta decay cascade, including beta delayed neutron emission, to give the final abundance. The details of the method can be found in standard text books [see e.g. Iliadis[7]] and reviews[8].

The typical neutron flux densities densities range from $10^{21} - 10^{27}/\mathrm{cm}^3$ and the temperature varies from 1-2 GK. Putting these numbers in Saha equation, we find that away from the closed shell, the r-process proceeds typically along the paths characterised by $S_n \approx 2 - 3$ MeV. This signifies that the path lies among neutron rich nuclei. In Fig. 3, we have shown the path followed by the the r-process. To avoid the odd-even effect, it is a standard procedure to plot the path in terms of two neutron separation energy S_{2n}. The dark circles indicate the path taken by the r-process. When the path nears a closed shell, the neutron separation energy increases and the path shifts upwards towards more stable nuclei. Thus one expects a peak corresponding to the nuclei involved in the the r-process with neutron number close to a neutron closed shell. In Fig. 3, we see how the shell closures at $N = 82$ and $N = 126$ turn the r-process path towards higher proton number *i.e.* more stable nuclei.

In view of the difference between the neutron drip line predicted by different formula, it should be interesting to look at the differences between the r-process

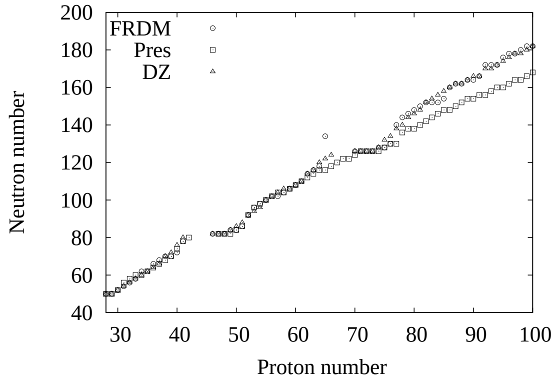

Figure 4: Lightest isotopes for which the two neutron separation energy is above 4 MeV. See text for details.

paths suggested by different mass formulas in Fig. 4. To keep the diagram simpler, we plot the neutron and proton numbers of the lightest isotopes of each element between $Z = 28$ and 100 for which the two neutron separation energy is above 4 MeV. As one can see, upto $A = 195$, the results of Duflo-Zuker formula and that of Ref I are nearly identical. FRDM predictions lie in slightly higher neutron numbers. However, all the three formulas are expected to produce r-process abundance peaks at nearly the same mass regions.

The abundance curve of r-process nuclei show prominent peaks around masses 80, 130 and 195. This corresponds to neutron closed shell $N=$ 50, 82 and 126. However, it is well known that a single r-process cannot explain the observed abundance of heavy nuclei including the above three peaks. Some recent calculations involve a large number of components under various conditions of density and temperature to look at the observed abundance. In the present work, rather than looking at a large number of possible processes, we choose certain standard scenarios and see whether the observed peaks are reproduced.

We have calculated the r-process abundance in three standard scenarios. In all cases, we assume a r-process time of two seconds. The beta decay life times and the beta delayed neutron emission probabilities have been taken from Möller *et al*[9]. We finally add the three results with relative weights to compare with the experimental abundance measurements. The newly developed formula have been used for masses. The three processes have been characterized by the following neutron density and temperature values; Component I : $N_n = 10^{21}/\text{cm}^3, \quad T = 1.9\ GK$, Component II : $N_n = 10^{25}/\text{cm}^3, \quad T = 1.5\ GK$ and Component III : $N_n = 10^{27}/\text{cm}^3, \quad T = 1.3\ GK$. The relative weights are taken as 30, 10 and 1, respectively. The total abundance has been normalized to reproduce the abundance at $A = 130$. No other attempt has been made to fit the experimental data.

Our results are presented in Fig. 5. One can see that the peaks seen around mass 80, 130 and 195 have been reproduced to some extent. Usually, one uses a large number of components, with neutron density, temperature, time and weights fitted to reproduce observed data. However, this may actually lead to spurious components which will be indistinguishable from an actual event. Rather than fitting the r-process abundance, the aim of the present work is to

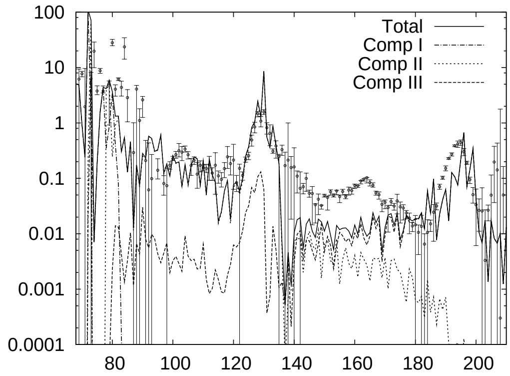

Figure 5: The abundance pattern from a weighted sum of three r-process components. See text for details.

see whether the peaks in the abundance distribution can be reproduced using the mass formula of Ref I.

Finally, a comment on the significance of the errors in calculating binding energy using the present formula needs to be made. Canonical r-process calculations indicate that the nuclei involved in the r-process have neutron separation energies lying between 2 to 3 MeV. As pointed out in Ref I, the root men square (r.m.s.) error in the ground state binding energies is 0.376 MeV for 2140 nuclei. No global mass formula, be it microscopic or phenomenological, shows a significantly better agreement. Near the stability valley, single nucleon separation energy (proton or neutron) is of the order of 8 MeV. Thus, a small error in binding energy does not influence the results of processes which involve nuclei near this valley. It is important to investigate the effect of the error, that is inherent in all global mass formulas, in the determination of the r-process path.

To see what effect this may have on the final abundance distribution, we have varied the mass values randomly following a Gaussian distribution with the calculated value as mean and standard deviation equal to the rms error quoted above. From the distribution of final abundance values for different masses, we have extracted the standard deviations. We find that the error plays a significant role in light masses. In nuclei beyond $A = 80$, inclusion of the error does not affect the results significantly.

3 Summary

The mass formula developed in Ref. I[1] has been employed to study the nuclei near the neutron drip line. The location of the neutron drip line has been calculated using the formula. The drip line predicted by the present formula differs substantially in heavy nuclei from predictions for two standard formulas. The abundance pattern resulting from the astrophysical r-process has been studied in the canonical approach under the waiting point approximation. Three different r-process scenarios have been considered which are seen to give rise to the observed peaks in mass 80, 130 and 195 region. The differences between the present and the other two formulas are unlikely to affect the abundance pattern significantly below $A = 200$. The error in the mass formula does not significantly affect the abundance pattern beyond mass 80.

Acknowledgments

This work has been carried out with financial assistance of the UGC sponsored DRS Programme of the Department of Physics of the University of Calcutta. Computer facilities created under DST-FIST Programme of the University of Calcutta have been utilized. Chirashree Lahiri acknowledges the grant of a fellowship awarded by the UGC.

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