

# Neutrino parameters from matter effects in $P_{ee}$ at long baselines

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We show that the earth matter effects in the  $\nu_e \rightarrow \nu_e$  survival probability can be used to cleanly determine the third leptonic mixing angle  $\theta_{13}$  and the sign of the atmospheric neutrino mass squared difference,  $\Delta m_{31}^2$ , using a  $\beta$ -beam as a  $\nu_e$  source.

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Determination of the third neutrino mixing angle  $\theta_{13}$ , the sign of  $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$ , the three CP phases, and the absolute neutrino mass scale are necessary for reconstruction of the neutrino mass matrix, which will have important consequences for nuclear and particle physics, astrophysics and cosmology. The  $\nu_e \rightarrow \nu_\mu$  transition probability  $P_{e\mu}$ , has been identified as the “golden channel” [1] for measuring the Dirac phase  $\delta_{CP}$ ,  $sgn(\Delta m_{31}^2)$  and  $\theta_{13}$  in long baseline accelerator based experiments. However, this strength of the golden channel also brings in the well-known problem of parameter “degeneracies”, where one gets multiple fake solutions in addition to the true one [2]. Various ways to combat this vexing issue have been suggested in the literature, including combining the golden channel with the “silver” ( $P_{e\tau}$ ) [3] and “platinum” ( $P_{\mu e}$ ) channels. While each of them would have fake solutions, their combination helps in beating the degeneracies since each channel depends differently on  $\delta_{CP}$ ,  $sgn(\Delta m_{31}^2)$  and  $\theta_{13}$ . In this letter, we propose using the  $\nu_e \rightarrow \nu_e$  survival channel,  $P_{ee}$ , which is *independent* of  $\delta_{CP}$  and the mixing angle  $\theta_{23}$ . It is therefore *completely* absolved of degeneracies and hence provides a clean laboratory for the measurement of  $sgn(\Delta m_{31}^2)$  and  $\theta_{13}$ . This gives it an edge over the conversion channels, which are infested with degenerate solutions.

The  $P_{ee}$  survival channel has been extensively considered for measuring  $\theta_{13}$  with  $\bar{\nu}_e$  produced in nuclear reactors [4] and with detectors placed at a distance  $\simeq 1$  km. Reducing systematic uncertainties to the sub-percent level is a prerequisite for this program and enormous R&D is underway for this extremely challenging job. For accelerator based experiments, the survival channel,  $P_{ee}$ , has been discussed with sub-GeV neutrinos from a  $\beta$ -beam source at CERN and a megaton water detector in Fréjus at a baseline of 130 km [5, 13]. However, no significant improvement on the  $\theta_{13}$  limit was found in [5] for a systematic error of  $\gtrsim 5\%$ . This stems mainly from the fact that in these experiments one is trying to differentiate between two scenarios, both of which predict a large number of events, differing from each other by a small number due to the small value of  $\theta_{13}$ . Also, since  $sgn(\Delta m_{31}^2)$ , is ascertained using earth matter effects, there is no hierarchy sensitivity in these survival

channel experiments due to the the short baselines involved.

In this letter, we emphasize on the existence of large matter effects in the survival channel,  $P_{ee}$ , for an experiment with a very long baseline. Recalling that  $P_{ee} = 1 - P_{e\mu} - P_{e\tau}$  and since for a given  $sgn(\Delta m_{31}^2)$  both  $P_{e\mu}$  and  $P_{e\tau}$  will either increase or decrease in matter, the change in  $P_{ee}$  is almost twice that in either of these channels. Using the multi-GeV  $\nu_e$  flux from a  $\beta$ -beam source, we show that this large matter effect allows for significant, even maximal, deviation of  $P_{ee}$  from unity. This, can thus be a convenient tool to explore  $\theta_{13}$ . This is in contrast to the reactor option or the  $\beta$ -beam experimental set-up in [5], where increasing the neutrino flux and reducing the systematic uncertainties are the only ways of getting any improvement on the current  $\theta_{13}$  limit. We further show, for the first time, that very good sensitivity to the neutrino mass ordering can also be achieved in the  $P_{ee}$  survival channel owing to the large matter effects. We discuss plausible experimental set-ups with the survival channel and show how the large matter effect propels this channel, transforming it into a very useful tool to probe  $sgn(\Delta m_{31}^2)$  and  $\theta_{13}$  even with relatively large room for systematic uncertainties.

For simplicity, we start with one mass scale dominance (OMSD) and the constant density approximation. OMSD implies setting  $\Delta m_{21}^2 = 0$ . Under these conditions, for neutrinos of energy  $E$  and traveling through a distance  $L$ ,

$$P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 [1.27(\Delta m_{31}^2)^m L/E], \quad (1)$$

where the mass squared difference and mixing angle in matter are respectively

$$\begin{aligned} (\Delta m_{31}^2)^m &= \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2} \\ \sin 2\theta_{13}^m &= \sin 2\theta_{13} \Delta m_{31}^2 / (\Delta m_{31}^2)^m \end{aligned} \quad (2)$$

and  $A = 2\sqrt{2}G_F n_e E$  originates from the matter potential [6]. Here,  $n_e$  is the ambient electron density. From Eq. (1), the largest deviation of  $P_{ee}$  from unity is obtained when the conditions (i)  $\sin^2 2\theta_{13}^m = 1$  and (ii)  $\sin^2 [1.27(\Delta m_{31}^2)^m L/E] = 1$  are satisfied simultaneously.

The first condition is achieved at resonance which is obtained for  $A = \Delta m_{31}^2 \cos 2\theta_{13}$ . This defines the resonance energy as,  $E_{res} = \Delta m_{31}^2 \cos 2\theta_{13} / 2\sqrt{2}G_F n_e$ . The second condition gives the energy where the  $(\Delta m_{31}^2)^m$  driven oscillatory term is maximal,

$$E_{max}^m = \frac{1.27(\Delta m_{31}^2)^m L}{(2p+1)\pi/2}, p = 0, 1, 2.. \quad (3)$$

Maximum matter effect is obtained when  $E_{res} = E_{max}^m$  [7], which gives,

$$(\rho L)^{max} = \frac{(2p+1)\pi \cdot 5.18 \times 10^3}{\tan 2\theta_{13}} \text{ km gm/cc}. \quad (4)$$

$\rho$  is the matter density in gm/cc. This is the distance where  $P_{ee} \simeq 0$ . Although both  $E_{res}$  and  $E_{max}^m$  depend on the value of  $\Delta m_{31}^2$ , the distance at which we get the maximum matter effect is independent of  $\Delta m_{31}^2$ . However, it is controlled by  $\theta_{13}$  very sensitively. For average earth matter densities obtained using the Preliminary Reference Earth Model (PREM), one can find the typical distances at which the above conditions are satisfied for various values of  $\sin^2 2\theta_{13}$  [8]. For instance for  $p = 0$  and  $\sin^2 2\theta_{13} = 0.2$  and  $0.1$ , these distances are 7600 km and 10200 km respectively. For higher values of  $p$  the distance exceeds the earth's diameter for  $\theta_{13}$  in the current allowed range. Using  $(\rho L)^{max}$  corresponding to the PREM profile, from Eq. (4) one can estimate that the condition of maximal matter effects inside the earth's mantle is satisfied only for  $\sin^2 2\theta_{13} \gtrsim 0.09$ .

Under OMSD, the matter conversion probabilities are,  $P_{ex} = Y_{23} \sin^2 2\theta_{13}^m \sin^2 [1.27(\Delta m_{31}^2)^m L/E]$ , where  $Y_{23} = \sin^2 \theta_{23}$  for  $x = \mu$  and  $Y_{23} = \cos^2 \theta_{23}$  for  $x = \tau$ . The maximum matter effect condition in the conversion channels is also given by Eq. (4). However, there are suppression factors,  $\sin^2 \theta_{23}$  for  $P_{e\mu}$  and  $\cos^2 \theta_{23}$  for  $P_{e\tau}$ , not present in  $P_{ee}$ . Moreover, since  $P_{ee}$  does not contain  $\theta_{23}$ , the octant ambiguity as well as parameter correlations due to uncertainty in  $\theta_{23}$  are absent. In addition, as mentioned earlier, the  $P_{ee}$  channel does not contain the CP phase,  $\delta_{CP}$ . Both of these remain true in the presence of non-zero  $\Delta m_{21}^2$  [9]. In our numerical work, we solve the full three flavour neutrino propagation equation assuming the PREM [10] profile and keep  $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$  fixed at their present best-fit values of  $8.0 \times 10^{-5}$  eV<sup>2</sup> and 0.31 respectively [11]. We assume the true value of  $|\Delta m_{31}^2| = 2.5 \times 10^{-3}$  eV<sup>2</sup>.

In Fig. 1 we plot  $P_{ee}$  as a function of energy, at four different  $L$  and for three values of  $\sin^2 2\theta_{13}$ . The plots confirm that maximal matter effects come at  $L \simeq 10000$  km and  $L \simeq 7500$  km for  $\sin^2 2\theta_{13} = 0.1$  and  $0.17$  respectively for the normal hierarchy (NH). For the inverted hierarchy (IH) there is no significant matter effect for  $\nu_e$ . This large difference in the probabilities for NH and IH can be exploited for the determination of  $sgn(\Delta m_{31}^2)$ . Further, since the matter effect is a sensitive function of

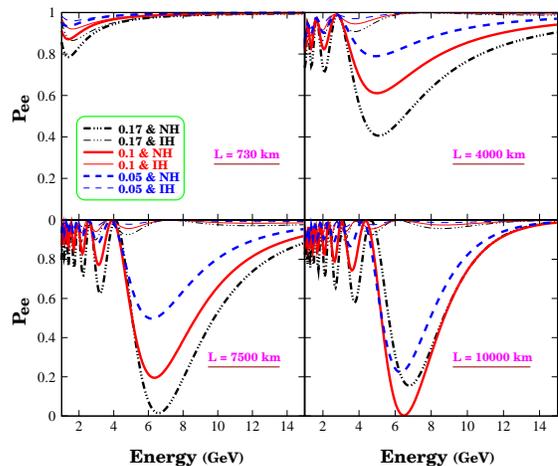


FIG. 1:  $P_{ee}$  in matter plotted versus neutrino energy. Thick (thin) lines are for normal (inverted) hierarchy.

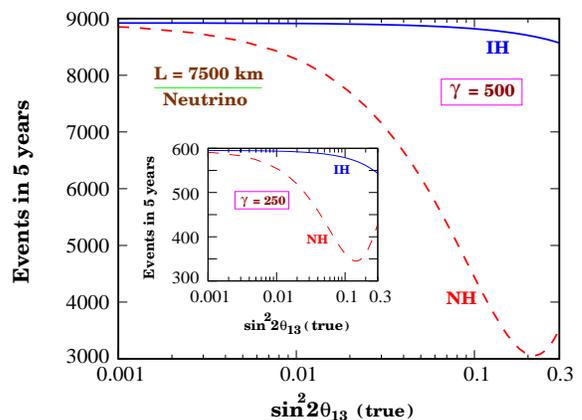


FIG. 2: Events in 5 years vs.  $\sin^2 2\theta_{13}$  for the normal (dashed line) and inverted (solid line) hierarchy for  $L = 7500$  km and  $\gamma = 500$ . The inset shows the same but for  $\gamma = 250$ .

$\theta_{13}$  it may also be possible to obtain information on this angle. We can also see that for a given value of  $\sin^2 2\theta_{13}$  ( $\gtrsim 0.09$ ) and  $E$ , the matter effect increases (almost linearly) with  $L$ , until the  $L$  for maximal matter effect is reached, beyond which matter effect falls. For values of  $\sin^2 2\theta_{13} < 0.09$  the condition for maximum matter effect is not met inside the earth's mantle and hence the matter effect and sensitivity to both hierarchy as well as  $\theta_{13}$  increase with  $L$ .

In what follows, we will show how, in a plausible experiment, one can use this near-resonant matter effect in the survival channel,  $P_{ee}$ , to constrain  $\theta_{13}$  and  $sgn(\Delta m_{31}^2)$ . Fig. 1 shows that the requirements for such a program include a  $\nu_e$  beam, baselines of at least a few thousand km and average energies around 6 GeV. The detector should be able to observe  $e^-$  unambiguously at these energies.

Pure  $\nu_e/\bar{\nu}_e$  beams can be produced from completely ionized radioactive ions accelerated to high energy decaying through the beta process in a storage ring, popularly

known as  $\beta$ -beams [12, 13, 14]. The ions considered as possible sources for beta beams are  $^{18}\text{Ne}$  and  $^8\text{B}$  for  $\nu_e$  and  $^6\text{He}$  and  $^8\text{Li}$  for  $\bar{\nu}_e$ . The end point energies of  $^6\text{He}$  and  $^{18}\text{Ne}$  are  $\sim 3.5$  MeV while for  $^8\text{B}$  and  $^8\text{Li}$  this can be larger  $\sim 13$ -14 MeV [15]. For the Lorentz boost factor  $\gamma = 250(500)$  the  $^6\text{He}$  and  $^{18}\text{Ne}$  sources have peak energy around  $\sim 1(2)$  GeV whereas for the  $^8\text{B}$  and  $^8\text{Li}$  sources the peak occurs at a higher value around  $\sim 4(7)$  GeV. Since the latter is in the ball-park of the energy necessary for near-resonant matter effects as discussed above, we will work with  $^8\text{B}$  ( $^8\text{Li}$ ) as the source ion for the  $\nu_e$  ( $\bar{\nu}_e$ )  $\beta$ -beam and  $\gamma = 250$  and 500. A CERN  $\beta$ -beam facility with  $\gamma \simeq 250$  should be possible with the existing SPS, while  $\gamma \leq 500$  could be achieved with upgrades of the existing accelerators. The Tevatron is also being projected as a plausible accelerator for the  $\beta$ -beam.

Water Čerenkov detectors have excellent capability of separating electron from muon events. Since this technology is very well known, megaton water detectors are considered to be ideal for observing  $\beta$ -beams. Such detectors do not have any charge identification capacity. But in a  $\beta$ -beam, the  $\beta^-$  and  $\beta^+$  emitters can be stacked in different bunches and the timing information at the detector can help to identify the  $e^-$  and  $e^+$  events [16].

It is well known that there are no beam induced backgrounds for  $\beta$ -beams. In this experimental set-up, the process  $\nu_e \rightarrow \nu_\tau \rightarrow \tau^- \rightarrow e^-$  could mimic the signal. We have checked that the background to signal ratio for these events in the relevant energy range is  $\sim 10^{-2}$  and can be neglected for the disappearance mode.  $e^-$  events from  $K$  and  $\pi^-$  decays are also negligible. The atmospheric background can be estimated in the beam off mode and reduced through directional, timing, and energy cuts.

Proposals for megaton water detectors include UNO [17] in USA, HyperKamiokande [18] in Japan and MEMPHYS [19] in Europe. If the  $\beta$ -beam is produced at CERN, then baselines in the range 7000-8600 km would be possible at any of the proposed locations for the UNO detector. Likewise, if the  $\beta$ -beam source be at FNAL, then the far detector MEMPHYS would allow for  $L = 7313$  km. HyperKamiokande could also be considered as the far detector and in that case  $L = 10184$  (9647) km if the source be at FNAL (CERN).

For our numerical analysis we use the standard  $\chi^2$  technique with  $\chi_{total}^2 = \chi_{pull}^2 + \chi_{prior}^2$ , where  $\chi_{prior}^2 = [(|\Delta m_{31}^2| - |\Delta m_{31}^2(\text{true})|)/\sigma(\Delta m_{31}^2)]^2$ . In  $\chi_{pull}^2$  we consider 2%  $\beta$ -beam flux normalisation error and 2% error for detector systematics which is more conservative than the value of 2% usually considered in current literature [5, 13]. For the full definition of  $\chi_{pull}^2$  and details of our statistical analysis, we refer to [20]. The prospective ‘‘data’’ is generated at the ‘‘true’’ values of oscillation parameters, assuming 440 kton of fiducial volume for the detector with 90% detector efficiency, threshold of 4 GeV and energy smearing of width 15%. For the  $\nu_e$   $\beta$ -beam we

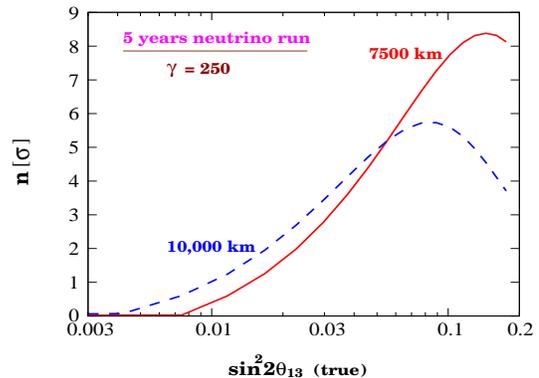


FIG. 3: Sensitivity to hierarchy for  $L = 7500$  (solid line) and 10000 km (dashed line) and  $\gamma = 250$ , as a function of true value of  $\sin^2 2\theta_{13}$ .

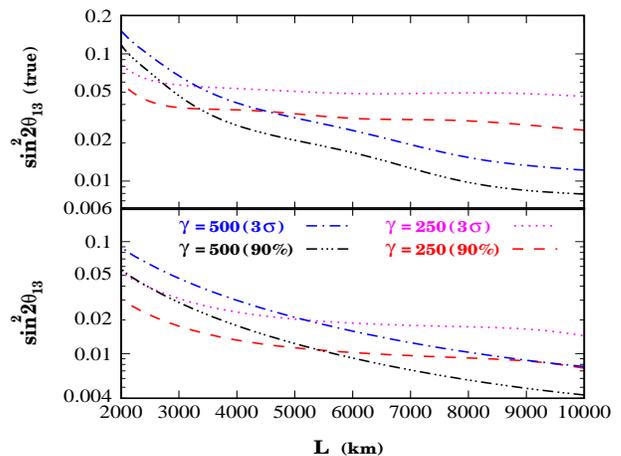


FIG. 4: The upper panel shows the smallest value of true  $\sin^2 2\theta_{13}$  at which normal and inverted hierarchy can be distinguished while the lower panel gives the sensitivity to  $\sin^2 2\theta_{13}$  at various baselines at 90% and  $3\sigma$  C.L., for two values of  $\gamma$ .

have assumed  $1.1 \times 10^{18}$  useful  $^8\text{B}$  decays per year and show results for 5 years of running of this beam. The number of events as a function of  $\sin^2 2\theta_{13}$  at  $L = 7500$  km with a  $\gamma = 500$   $\nu_e$   $\beta$ -beam is shown in Fig. 2 for NH and IH. The inset in Fig. 2 shows the number of events in 5 years expected from a lower  $\gamma = 250$ . We have used the neutrino-nucleon interaction cross-sections from [21].

In Fig. 3 we show the sensitivity ( $n\sigma, n = \sqrt{\chi^2}$ ) of the survival channel to the neutrino mass ordering for  $L = 7500$  and 10000 km and  $\gamma = 250$ . If the true value of  $\sin^2 2\theta_{13} = 0.05$ , then one can rule out the inverted hierarchy at the  $4.8\sigma$  ( $5.0\sigma$ ) level with  $L = 7500$  (10000) km. For  $L = 7500$  (10000) km, the wrong inverted hierarchy can be disfavored at the 90% C.L. if the true value of  $\sin^2 2\theta_{13} > 0.03$  (0.025). The sensitivity improves significantly if we use  $\gamma = 500$  instead of 250, since (i) the flux at the detector increases, and (ii) the flux peaks at  $E$  closer to 6 GeV, where we expect largest matter effects

[20]. For  $\gamma = 500$ , the inverted hierarchy can be disfavored at  $2.6\sigma(3.8\sigma)$  for a lower value of  $\sin^2 2\theta_{13} = 0.015$ . Minimum values of  $\sin^2 2\theta_{13}$  at which the inverted hierarchy can be ruled out at 90% and  $3\sigma$  C.L. for different values of  $L$  are shown in the upper panel of Fig. 4 for  $\gamma = 250$  and 500. From the figure one can see that for  $\gamma = 500$  and  $L = 7500$  (10000) km the wrong inverted hierarchy can be disfavored at the 90% C.L. if the true value of  $\sin^2 2\theta_{13} > 1.0 \times 10^{-2}$  ( $8.0 \times 10^{-3}$ ). If instead we use a total systematic error of 5% then we get the above sensitivity limits as  $\sin^2 2\theta_{13} > 1.6 \times 10^{-2}$  ( $1.2 \times 10^{-2}$ ) at 90% C.L.

If the true value of  $\sin^2 2\theta_{13}$  turns out to be smaller than the sensitivity reach shown in the upper panel of Fig. 4 for a given  $L$ , then it would not be possible to determine the hierarchy at the given C.L. However, we would still be able to put better constraints on  $\sin^2 2\theta_{13}$  itself. The lower panel of Fig. 4 demonstrates as a function of  $L$  the sensitivity to  $\theta_{13}$ , i.e., the minimum value of  $\sin^2 2\theta_{13}$  which can be statistically distinguished from  $\sin^2 2\theta_{13} = 0$  at 90% and  $3\sigma$  C.L. Both Figs. 3 and 4 show that the sensitivity improves with  $L$ , even though the flux falls as  $1/L^2$ . This results from matter effects increasing with  $L$ , as noted before. For  $L = 7500$  (10000) km, we can constrain  $\sin^2 2\theta_{13} < 6.3 \times 10^{-3}$  ( $4.3 \times 10^{-3}$ ) at the 90% C.L. for  $\gamma = 500$ . For a 5% systematic error the above numbers are changed to  $\sin^2 2\theta_{13} < 1.0 \times 10^{-2}$  ( $7.3 \times 10^{-3}$ ).

How does this compare with alternate possibilities? If the energy can be reconstructed accurately, then the result can be improved further. For instance, for  $L=7500$  km, if one could preferentially select the energy in the range 5.0-7.5 GeV, then the normal and inverted hierarchy would be differentiated for  $\sin^2 2\theta_{13} = 7.47 \times 10^{-3}$  at 90% C.L. for  $\gamma = 500$ .

Using the  $P_{e\mu}$  channel and a  $\beta$ -beam source at a distance of 7152 km (CERN-INO) from the detector, the NH can be distinguished from IH at 90% C.L. for  $\gamma = 500$  if  $\sin^2 2\theta_{13} > 7.7 \times 10^{-3}$  [20]. For a neutrino factory at 7500 km, the wrong IH can be discarded at the  $3\sigma$  C.L. if  $\sin^2 2\theta_{13} > 3 \times 10^{-4}$  [22].

We have presented our results using a  $\nu_e$  beam and assuming NH to be the true hierarchy. Similar results can also be obtained with a  $\bar{\nu}_e$  beam for IH. It is also possible to run both beams simultaneously.

In conclusion, we propound the possibility of using large matter effects in the survival channel,  $P_{ee}$ , at long baselines for determination of the neutrino mass ordering ( $sgn(\Delta m_{31}^2)$ ) and the yet unknown leptonic mixing angle  $\theta_{13}$ . Matter effects in the transition probabilities  $P_{e\mu}$  and  $P_{e\tau}$  act in consonance to give an almost two-fold effect in the survival channel. In addition, the problem of spurious solutions due to the leptonic CP phase and

the atmospheric mixing angle  $\theta_{23}$  does not crop up. The development of  $\beta$ -beams as sources of pure  $\nu_e/\bar{\nu}_e$  beams enables one to exclusively study the  $P_{ee}$  survival probability and adds a new direction to the prospects of a future  $\beta$ -beam.

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