

Neutrino Bremsstrahlung Process in highly degenerate magnetized electron gas

Indranath Bhattacharyya
 Department of Applied Mathematics
 University of Calcutta, Kolkata-700 009, INDIA
 E-mail : *i_bhattacharyya@hotmail.com*

Abstract

In this article the neutrino bremsstrahlung process is considered in presence of strong magnetic field, though the calculations for this process in absence of magnetic field are also carried out simultaneously. The electrons involved in this process are supposed to be highly degenerate and relativistic. The scattering cross sections and energy loss rates for both cases, in presence and absence of magnetic field, are calculated in the extreme-relativistic limit. Two results are compared in the range of temperature $5.9 \times 10^9 \text{ K} < T \leq 10^{11} \text{ K}$ and magnetic field $10^{14} - 10^{16} \text{ G}$ at a fixed density $\sim 10^{15} \text{ gm/cc}$, a typical environment during the cooling of magnetized neutron star. The interpretation of our result is briefly discussed and the importance of this process during the stellar evolution is speculated.

1 Introduction:

The neutrino emission process plays an important role in the late stages of the stellar evolution. It is known that the radiation of neutrino could be dominated over the ordinary electromagnetic radiation in case of highly dense and hot stellar structures, such as, white dwarves or neutron stars. Unlike the other mechanisms the neutrinos are produced directly from their point of origin and do not require the transport of energy to the surface of the stellar object before getting radiated. As a consequence the energy outflow is given directly by the rate at which the neutrinos are produced. Photo-neutrino process ($e^- + \gamma \rightarrow \nu + \bar{\nu}$), Pair-annihilation process ($e^- + e^+ \rightarrow \nu + \bar{\nu}$) and Plasma neutrino process ($\Gamma \rightarrow \nu + \bar{\nu}$) are the sole mechanisms that carry away the energy from star during its evolution period, although there are few more processes which might play more important role under some special environments. Chiu and his collaborators [1, 2, 3] calculated some important neutrino emission processes and pointed out their important role in astrophysics. In 1972 Dicus [4] reconsidered a few such processes in the framework of electro-weak interaction theory and calculated the energy-loss rate at the various stages of the stellar evolution. According

to the Standard Model the neutrino has the minimal properties such as zero mass, zero charge etc. Introduction of neutrino mass compatible with the experimental data is a bare minimum extension of the Standard Model. In the frame work of Standard Model with this little extension some calculations [5, 6] related to the neutrino process have already been carried out. Itoh et al. [7] considered some neutrino emission processes to calculate them numerically.

Pontecorvo [8] gave an idea that the neutrino may be emitted by the interaction of electron with the nucleus, called bremsstrahlung process. Gandel'man and Pinaev [9] carried out the detailed calculations for this process in the non-relativistic electron gas. After that Festa and Ruderman [10] extended this calculations for relativistic limit with considering the screening effect that becomes important in the high density. Dicus et al. [11] considered the process according to the Standard Model and compare the result with different screening effect. Saha [12] calculated the bremsstrahlung process according to photon-neutrino weak coupling theory which is very much satisfactory in explaining the neutrino-synchrotron process [13]. In this article we have considered the bremsstrahlung process in presence of strong magnetic field and this is the first attempt to do so. It has been shown by Festa and Ruderman [10] and then by Dicus et al. [11] that the bremsstrahlung process has maximum effect in the relativistic degenerate region; thus in the stellar object such as newly born neutron star the process is supposed to be very much effective. Here we are to verify whether the presence of strong magnetic field, which may be generated in the rotating neutron stars, will have any effect on the bremsstrahlung process. We have calculated the scattering cross section and then obtained the energy-loss rate for the bremsstrahlung both in presence and absence of strong magnetic field. We are going to study the influence of high magnetic field on the bremsstrahlung process and the region where its presence may take a crucial role in the neutrino emission. The ordinary neutrino bremsstrahlung process is very much significant for the neutrino energy generation process in the relativistic highly degenerate region; therefore a comparative study is required for this process in presence and also in absence of strong magnetic field.

2 Calculation of scattering cross section:

In the bremsstrahlung process the electron interacts with the nucleus having the coulomb potential

$$\Phi(\vec{r}) = \frac{Ze}{|\vec{r}|} e^{-|\vec{r}|/\lambda_d} \quad (2.1)$$

i.e. a single charge with an exponential screening cloud (Yukawa like charge distribution), where λ_d is the Debye screening length given by

$$\frac{1}{\lambda_d^2} = \frac{4e^2}{\pi} E_F P_F \quad (2.2)$$

Here P_F and E_F represent the Fermi momentum and energy respectively. We consider the potential, given by (2.1), because the screening effect is important in the high density region. There will be four different Feynman Diagrams shown in the Figure-1 (Z-exchange diagram) and Figure-2 (W-exchange diagram). In our calculations the presence of magnetic field plays a crucial role, so it is to be handled with care. Without any loss of generality we can take the direction of magnetic field along z -axis. In presence of magnetic field the energy momentum relation of the electron becomes

$$(p_n^0)^2 = m_e^2 + p_z^2 + 2n \frac{H}{H_c} m_e^2 \quad (2.3)$$

where $H_c = 4 \cdot 414$ G stands for critical magnetic field. Here n represents the Landau level for the electron in the magnetic field. [The value of s is taken as ± 1 as per the spin of the electron is directed towards or opposite to the direction of magnetic field (along z -axis) respectively.]

The component of electron momentum along the direction of magnetic field remains unaffected. It is clear that the effect of magnetic field on the electron quantizes its energy to the direction perpendicular to H and thus transverse components would get replaced by $p_x^2 + p_y^2 \rightarrow 2nm_e^2 \frac{H}{H_c}$, whereas the longitudinal component p_z would be directed along the magnetic field. The Feynman diagrams for the bremsstrahlung process are same for both in presence and absence of magnetic field; only we have to keep in our mind that four momenta of the electronic lines, present in the diagrams, should be modified. First we shall calculate the ordinary bremsstrahlung process i.e. the process in absence of magnetic field. The matrix element can be constructed as

$$M_{fi} = -ie \frac{G_F}{\sqrt{2}} A_0(\vec{k}) J_\mu \mathcal{M}^\mu \quad (2.4)$$

where,

$$\mathcal{M}^\mu = \bar{u}(p') [\gamma^\mu (C_V - C_A \gamma_5) \frac{(p'^\tau \gamma_\tau + q^\tau \gamma_\tau + m_e)}{(p' + q)^2 - m_e^2 + i\epsilon} \gamma^0 + \gamma^0 \frac{(p^\tau \gamma_\tau - q^\tau \gamma_\tau + m_e)}{(p - q)^2 - m_e^2 + i\epsilon} \gamma^\mu (C_V - C_A \gamma_5)] u(p) \quad (2.5)$$

$$J_\mu = \bar{u}_\nu(q_1) \gamma_\mu (1 - \gamma_5) v_\nu(q_2) \quad (2.6)$$

$$A_0(\vec{k}) = \int \Phi(\vec{r}) e^{-i(\vec{k} \cdot \vec{r})} d^3r = -\frac{4\pi Z e}{|k^2 + q_{sc}^2|} \quad (2.7)$$

and

$$q = q_1 + q_2$$

The energy momentum conservation leads to

$$k + p = p' + q$$

where k is purely space like as in the case of photo-coulomb neutrino process [14, 15]. The term q_{sc} present in the equation (2.7) arises due to the screening effect and can be expressed as

$$q_{sc} = \frac{1}{\lambda_d} \quad (2.8)$$

Using some simplifications we can write the term $J_\mu \mathcal{M}^\mu$ as follows:

$$J_\mu \mathcal{M}^\mu = \left[\frac{(p'J)}{q^2/2 + (p'q)} + \frac{(pJ)}{q^2/2 - (pq)} \right] \bar{u}(p') \gamma^0 (C_V - C_A \gamma_5) u(p) \quad (2.9)$$

We can put this expression to the equation (2.4) to get the expression for the scattering matrix. Now the squared sum of the scattering matrix over the final spins is to be integrated over the final momenta. The squared sum of the expression $[\bar{u}(p') \gamma^0 (C_V - C_A \gamma_5) u(p)]$ gives

$$\sum |\bar{u}(p') \gamma^0 (C_V - C_A \gamma_5) u(p)|^2 = (C_V^2 - C_A^2) + (C_V^2 + C_A^2) \frac{(p'_0 p_0 - \vec{p}' \cdot \vec{p})}{m_e^2} \quad (2.10)$$

Note that in the equation (2.10) no neutrino momentum is present, so this part will not be taken into account during the integration over the final momenta of neutrinos. Let us now evaluate the squared spin sum of the expression $|\frac{(p'J)}{q^2/2+(p'q)} + \frac{(pJ)}{q^2/2-(pq)}|$ and then integrating over final momenta of the neutrinos we can obtain [See Appendix-A]

$$\begin{aligned} \int \sum \left| \frac{(p'J)}{q^2/2 + (p'q)} + \frac{(pJ)}{q^2/2 - (pq)} \right|^2 \frac{d^3 q_1 d^3 q_2}{(2\pi)^3 2q_1^0 (2\pi)^3 2q_2^0} (2\pi) \delta(q^0 - q_1^0 - q_2^0) \\ \approx \frac{1}{18(2\pi)^3 m_e^2} (p^0 - p'^0)^3 \left[\frac{|\vec{p} - \vec{p}'|}{p^0 + p'^0} \right]^2 \end{aligned} \quad (2.11)$$

Up to this step the calculations for the neutrino bremsstrahlung process would be same in both situations i.e. in presence as well as absence of magnetic field. It is assumed that in presence of magnetic field quantized transverse components of the momentum do not participate directly during the interaction between nucleus and electron. Only the z-component of the electron momentum takes part in this process. Thus in this case we can obtain an expression almost similar to the equation (2.11) with replacing $|\vec{p}'|$ and $|\vec{p}|$ by p'_z and p_z respectively. Now to integrate the squared sum of the matrix element over all final momenta we shall utilize the result obtained in the equation (2.10) and (2.11), but we

have to take care when the magnetic field is present. In this case the phase space factor d^3p' takes the form [See Appendix-B]

$$\int d^3p' = \pi \frac{H}{H_c} m_e^2 \int dp'_z \quad (2.12)$$

whereas in the ordinary bremsstrahlung process the integration over the final momentum of electron is done in the usual manner. In the center of mass frame and assuming the electron momentum is much high relative to its rest mass we can carry out the calculations. In this extreme relativistic limit we can evaluate the integral over the final momentum of electron and obtain the following expression.

$$\int \sum |M|^2 \frac{d^3q_1 d^3q_2 d^3p'}{(2\pi)^3 2q_1^0 (2\pi)^3 2q_2^0 (2\pi)^3 2p'^0} (2\pi) \delta(q^0 - q_1^0 - q_2^0) = \frac{8G_F^2 \alpha^2 Z^2}{9(2\pi)^3 (1+r^2)^2} (C_V^2 + C_A^2) \frac{p_0^3}{m_e^2 m_\nu^2} \quad (2.13)$$

This expression is obtained for the ordinary bremsstrahlung process when there will be no magnetic field. In presence of magnetic field this expression becomes

$$\frac{2G_F^2 \alpha^2 Z^2}{9(2\pi)^3} (C_V^2 + C_A^2) \frac{p_0}{m_\nu^2} \left(\frac{H}{H_c} \right) \quad (2.14)$$

Note that the term r arises due to the weak screening effect. It is given by

$$r \approx \frac{q_{sc}}{p^0}$$

The expression for C_V and C_A for the electron type of neutrino emission will differ from those in case of muon and tau neutrino emission, since W-boson exchange diagrams are present only when the electron neutrino anti-neutrino pair is emitted. Inserting the above terms into the expression of the scattering cross section for both of those cases and returning to the C.G.S. unit we can finally obtain

$$\sigma \approx 1.76 \times 10^{-50} \left(\frac{E}{m_e c^2} \right)^2 \frac{1}{(1+r^2)^2} \text{ cm}^2 \quad (2.15)$$

in absence of magnetic field, whereas

$$\sigma_{mag} \approx 4.41 \times 10^{-51} \left(\frac{H}{H_c} \right) \text{ cm}^2 \quad (2.16)$$

in presence of magnetic field.

It is worth noting that all three type of neutrinos are taken into account in our calculations. Our result (equation 2.16) shows that the scattering cross section for the bremsstrahlung process in presence of magnetic field will not depend on the energy of the incoming electron, but on the intensity of the magnetic field present in the surroundings.

3 Calculation of energy loss rate:

In the extreme relativistic case the energy loss rate in erg per nucleus per second for the neutrino bremsstrahlung process is calculated by the formula

$$\mathcal{E}_\nu^Z = \frac{2}{(2\pi)^3 \hbar^3} \int \frac{d^3p}{[e^{\frac{E-E_F}{\kappa T}} + 1]} c\sigma E e^{\frac{E-E_F}{\kappa T}} \quad (3.1)$$

where E_F stands for Fermi energy of the electron. We are considering the case in which the electrons are highly degenerate. It is well known that in the degenerate region the energy of the electron remains below the Fermi energy level. To obtain the energy-loss rate in erg per gram per second \mathcal{E}_ν^Z is divided by Am_p and it is obtained as

$$\mathcal{E}_\nu = \frac{Z^2}{A} \times 5 \cdot 26 \times 10^{-3} \times \frac{x_F^6 e^{1-x_F}}{(1+r^2)^2} T_{10}^6 \quad \text{erg/gm} - \text{sec} \quad (3.2)$$

where, $T_{10} = T \times 10^{-10}$

The x_F represents the ratio of the Fermi temperature to the maximum temperature of the degenerate electron gas. The degeneracy will be attained only when the following condition will be satisfied [16].

$$x_F^2 > 2\pi^2 \quad (3.3)$$

It can be obtained $x_F \approx 6$, considering the fact that temperature and density of the electron gas present in the core of a newly born neutron star would be approximately 10^{12} K and 10^{15} gm/cc respectively. The term r arises due to the weak screening, related to x_F by $r \approx 0 \cdot 096 \times x_F$. Thus we can calculate the term r , present in the equation (3.2). Finally the expression for the energy-loss rate becomes

$$\mathcal{E}_\nu = \frac{Z^2}{A} \times 0 \cdot 93 \times 10^{12} \times T_{10}^6 \quad \text{erg/gm} - \text{sec} \quad (3.4)$$

In the same manner we can obtain the energy loss rate in presence of magnetic field. In that case the the phase space factor will be replaced according to the rule defined in (B6). In the same manner energy-loss rate can be calculated as

$$\mathcal{E}_\nu^{mag} = \frac{Z^2}{A} \times 0 \cdot 51 \times 10^6 \times H_{13}^2 \times T_{10}^2 \quad \text{erg/gm} - \text{sec} \quad (3.5)$$

where, $H_{13} = H \times 10^{-13}$

We have computed (Table-1) the logarithmic value of the energy loss rate in the temperature range $0 \cdot 8 \times 10^{10} - 10^{11}$ K and the magnetic field $10^{14} - 10^{16}$ G at a fixed density $\rho = 10^{15}$ gm/cc.

4 discussion:

The neutrino bremsstrahlung process is an important energy generation mechanism during the stellar evolution and very much effective in the highly degenerate region, for examples, in the cores of low mass red giants, white dwarves etc. In addition to this degenerate nature if the electron gas is highly relativistic the energy-loss rate through the bremsstrahlung process would be significantly high. It has already been calculated that the neutrino luminosity in the crust of neutron star is high enough [17], but it is yet to be verified what would be the effect of neutrino emission by the bremsstrahlung process in the core region, particularly when the core is strongly magnetized. The discovery of radio pulsars showed that the collapse of normal stars results not only in supernova explosions, but may also generate strong magnetic field. In some stellar objects like neutron stars and magnetars the magnetic field may reach to 10^{16} G and influences the neutrino emission process. It is known that the neutron star is born as a result of type II supernova explosion. In the newly born neutron star the core temperature becomes 10^{12} K which drops down to 10^{11} K within few seconds of its birth and then slowly cools down until the temperature reaches to 2×10^8 K, after which the electromagnetic emission dominates over the neutrino emission [18]. It is worth noting that the electron gas, still left in the stellar core, is highly degenerate and relativistic.

We are going to verify the role of the magnetic field on the neutrino bremsstrahlung process. In the neutrino synchrotron radiation [19, 13, 20, 21] neutrino anti-neutrino pair emission takes place since the electron changes its Landau levels, but in the bremsstrahlung the Landau levels are assumed to be unchanged throughout the process. The process occurs through the change of magnitude of the component of electron linear momentum directed along the magnetic field. It is found from Table-1 that in the temperature range 10^{10} K $\leq T \leq 10^{11}$ K and at the density 10^{15} gm/cc the energy loss rate for the ordinary bremsstrahlung process is greater than that in presence of strong magnetic field ($10^{14} - 10^{16}$ G). We can interpret that in the early stage of neutron star cooling, when the temperature remains above 10^{10} K, the effect of the bremsstrahlung process get lowered due to the presence of magnetic field, although the energy loss rate is still very high. If the strength of the magnetic field goes below the critical value, the process would become free from the influence of magnetic field, and a greater amount of neutrino energy is produced. It is evident from our work that though, in general, the magnetic field makes the bremsstrahlung process a bit less effective, but there exists a particular region ($5 \cdot 9 \times 10^9$ K $< T < 10^{10}$ K, $H \sim 10^{16}$ G and $\rho \sim 10^{15}$ gm/cc) during neutron star cooling, where the bremsstrahlung process contributes a greater

amount of neutrino energy loss by the influence of magnetic field compared to the situation when there would be no magnetic field at all. Therefore, it can be predicted that if the temperature falls below 10^{10} K, the process would give maximum effect due to the presence of super strong magnetic field having intensity 10^{16} G. Our study reveals that the neutrino bremsstrahlung process is an important energy generation mechanism in the late stages of the stellar evolution, even in presence of magnetic field.

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6 Appendix-A:

We have chosen a frame in which

$$\vec{q} = \vec{q}_1 + \vec{q}_2 \quad (A1)$$

In this frame we obtain

$$\frac{q^2}{2} + (p'q) = \frac{(p'^0 - p^0)(p'^0 + p^0)}{2} \quad (A2)$$

$$\frac{q^2}{2} - (pq) = -\frac{(p'^0 - p^0)(p'^0 + p^0)}{2} \quad (A3)$$

Thus,

$$\sum \left| \frac{(p'J)}{q^2/2 + (p'q)} + \frac{(pJ)}{q^2/2 - (pq)} \right|^2 = \frac{4}{(p'^0 - p^0)^2(p'^0 + p^0)^2} \sum |(p' - p)J|^2 \quad (A4)$$

Let us assume

$$P = p - p' = q - k \quad (A5)$$

we have,

$$\begin{aligned} \sum |(PJ)|^2 &= \sum |\bar{u}_\nu(q_1)P^\mu \gamma_\mu (1 - \gamma_5)v(q_2)|^2 \\ &= \frac{2}{m_\nu^2} [2(q_1P)(q_2P) - (q_1q_2)P^2] \\ &= \frac{2}{m_\nu^2} [(m_\nu P^0)^2 + \{(1 - 2\cos^2\alpha) |\vec{q}_1|^2 + (q_1^0)^2\} |\vec{P}|^2] \end{aligned} \quad (A6)$$

Now using

$$\int d^3q_2 = \frac{4\pi}{3} |\vec{q}_2|^3 = \frac{4\pi}{3} |\vec{q}_1|^3 \quad (A7)$$

and

$$d^3q_1 = |\vec{q}_1|^2 d|\vec{q}_1| \sin\alpha d\alpha d\phi \quad (A8)$$

we can obtain

$$\begin{aligned} & \int \sum |(PJ)|^2 \frac{d^3q_1}{2q_1^0} \frac{d^3q_2}{2q_2^0} \delta(2q_1^0 - q^0) \\ &= \frac{2\pi^2}{3m_\nu^2} \int \int_{\alpha=0}^{\pi} \frac{|\vec{q}_2|^4}{q_1^0} [(m_\nu P^0)^2 + \{(1-2\cos^2\alpha)|\vec{q}_1|^2 + (q_1^0)^2\} |\vec{P}|^2] \delta(q_1^0 - \frac{q_0}{2}) dq_1^0 \sin\alpha d\alpha d\phi \\ &= \frac{4\pi^2}{3m_\nu^2} \int \frac{|\vec{q}_2|^4}{q_1^0} [(m_\nu P^0)^2 + \{\frac{|\vec{q}_1|^2}{3} + (q_1^0)^2\} |\vec{P}|^2] \delta(q_1^0 - \frac{q_0}{2}) dq_1^0 \\ &\approx \frac{\pi^2}{18m_\nu^2} (p^0 - p'^0)^5 |\vec{p} - \vec{p}'|^2 \end{aligned} \quad (A9)$$

We have assumed $m_\nu \ll q_1^0 < q_0$ and used the following criteria

$$m_\nu \rightarrow 0$$

$$P^0 = q^0 = p^0 - p'^0$$

$$\vec{P} = \vec{k} = \vec{p} - \vec{p}'$$

since k is space like, whereas q is time like in our chosen frame.

Now introducing normalized factors and also using (A4) we obtain

$$\begin{aligned} & \int \sum \left| \frac{(p'J)}{q^2/2 + (p'q)} + \frac{(pJ)}{q^2/2 - (pq)} \right|^2 \frac{d^3q_1 d^3q_2}{(2\pi)^3 2q_1^0 (2\pi)^3 2q_2^0} (2\pi) \delta(q^0 - q_1^0 - q_2^0) \\ &\approx \frac{1}{18(2\pi)^3 m_\nu^2} (p^0 - p'^0)^3 \left[\frac{|\vec{p} - \vec{p}'|}{p^0 + p'^0} \right]^2 \end{aligned} \quad (A10)$$

This is same as the equation (2.11).

7 Appendix-B

In presence of magnetic field the phase space factor is replaced by the following relation [22]

$$\frac{2}{(2\pi)^3} \int d^3p = \frac{1}{(2\pi)^2} \sum_{n=0}^{n_{max}} g_n \int dp_z \quad (B1)$$

where g_n represents degeneracy factor of the Landau levels i.e.

$$g_0 = 1, \quad g_n = 2 \quad (n \geq 1) \quad (B2)$$

The maximum Landau level n_{max} can be obtained from the following relation

$$n_{max} = \frac{1}{2m_e^2} \left(\frac{H}{H_c} \right) [(p_{n_{max}}^0)^2 - (p^0)^2] \quad (B3)$$

where,

$$(p^0)^2 = p_z^2 + m_e^2 \quad (B4)$$

For $n_{max} < 1$ we have,

$$H > \frac{1}{2m_e^2} [(p_{n_{max}}^0)^2 - (p^0)^2] H_c \quad (B5)$$

It shows that for a very high magnetic field only $n = 0$ Landau level would contribute in the phase space. In this article we consider the environment is highly magnetized, which gives

$$(p_{n_{max}}^0)^2 - (p^0)^2 > 2m_e^2$$

and therefore

$$H > H_c$$

In that case the phase space factor will take the form

$$\int d^3p = \pi \frac{H}{H_c} m_e^2 \int dp_z \quad (B6)$$

It is same as the equation (2.12).

If the magnetic field is comparatively lower the higher Landau levels contribute in the phase space as per the condition (B3).

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T_{10}	$log(\frac{A}{Z^2}\mathcal{E}_\nu)$			Absence of magnetic field
	Presence of magnetic field			
	10^{14}	10^{15}	10^{16}	
0.8	7.51	9.51	11.51	11.39
0.9	7.62	9.61	11.62	11.69
1	7.71	9.71	11.71	11.97
2	8.31	10.31	12.31	13.77
3	8.66	10.67	12.67	14.83
4	8.91	10.91	12.91	15.58
5	9.10	11.10	13.10	16.16
6	9.26	11.26	13.26	16.64
7	9.40	11.40	13.40	17.04
8	9.51	11.51	13.51	17.39
9	9.62	11.62	13.62	17.69
10	9.71	11.71	13.71	17.97

Table 1: Logarithmic expression for energy loss rate at $\rho = 10^{15} \text{ gm/cm}^3$, and magnetic field $H = 10^{16}, 10^{15}, 10^{14} \text{ G}$ due to the neutrino bremsstrahlung process in presence and absence of magnetic field respectively in the temperature range $0.8 \times 10^{10} - 10^{11} \text{ K}$. The bold number indicates that the former process dominates over the later.

Figure Caption :

Figure-1: Feynman diagram for the neutrino bremsstrahlung process in presence of magnetic field with Z boson exchange.

Figure-2: Feynman diagram for the neutrino bremsstrahlung process in presence of magnetic field with W boson exchange.