

# Microscopic calculation of half lives of spherical proton emitters

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## Abstract

Half life values for proton radioactivity in nuclei have been calculated in the WKB approximation. The microscopic proton-nucleus potential has been obtained by folding the densities of daughter nuclei with two microscopic NN interactions, DDM3Y and JLM. The densities have been obtained in the Relativistic Mean Field approach in the spherical approximation using the force FSU Gold. No substantial modification of results has been observed if other common forces are employed. The calculated results for the decays from the ground state or the low-lying excited states in almost all the nuclei agree well with experimental measurements. Reasons for large deviations in a few cases have been discussed. Results in  $^{109}\text{I}$  and  $^{112,113}\text{Cs}$  show that the effect of deformation is small contrary to earlier calculations. Predictions for possible proton radioactivity have been made in two nuclei,  $^{93}\text{Ag}$  and  $^{97}\text{In}$ .

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Proton drip line nuclei play a very important role in nuclear astrophysics, namely in nova and supernova explosions, X-ray bursts associated with explosive hydrogen burning, rapid proton capture processes, etc. These processes involve the low excitation energy regime of the nucleus. The rapid proton capture process at low energy has its inverse in the proton radioactivity from the nuclear ground state or low energy states. In both the cases, the important feature is the Coulomb barrier which

has to be overcome by the proton through tunneling. It is thus very important to study the proton radioactivity process.

Although the first example of proton radioactivity from nuclei was observed in an isomeric state of  $^{53}\text{Co}$  in 1970, the next decay was observed only a decade later in  $^{151}\text{Lu}$ . Since then, with the improvement in experimental facilities, examples of proton radioactivity from ground states or low lying isomeric states has been found in a number of nuclei, all between  $Z = 51$  and  $Z = 83$ [1, 2].

Theoretical calculations have been employed to explain the observed lifetimes of proton radioactivity. Most of the investigations employ the picture of a point proton tunneling through a potential barrier and calculate the half life in the WKB approximation. The potential that has been predominantly used is the Woods-Saxon form added to the Coulomb potential [3, 4, 5]. Recently Basu *et al.* have constructed the proton-nucleus nuclear potential in the single folding model and used it to calculate the decay probability in observed spherical proton emitters[6]. However, in all these calculations, the density of the daughter nucleus is taken from phenomenological models. Replacement of this density by one obtained from some mean field calculation may be expected to reproduce the results more accurately.

Relativistic Mean Field (RMF) approach is now a standard tool in low energy nuclear structure. It has been able to explain different features of stable and exotic nuclei like ground state binding energy, deformation, radius, excited states, spin-orbit splitting, neutron halo, etc[7]. It is well known that in nuclei far away from the stability valley, the single particle level structure undergoes certain changes in which the spin-orbit splitting plays an important role. Being based on the Dirac Lagrangian density, RMF is particularly suited to investigate these nuclei because it naturally incorporates the spin degrees of freedom. Relativistic Hartree Bogoliubov (RHB) calculations have been used to predict the proton drip line in medium-heavy to superheavy nuclei with considerable success[8].

There exist different variations of the Lagrangian density as well as a number of different parameterizations. Recently, a new Lagrangian density has been proposed[9] which involves self-coupling of the vector-isoscalar meson as well as coupling between the vector-isoscalar meson and the vector-isovector meson. The corresponding parameter set is called FSU Gold[9]. In this work, we have employed mainly this force for our calculation, though we have also checked our calculations using the forces NL3[10] and NLSH[11] in some cases.

In the conventional RMF+BCS approach for even-even nuclei, the Euler-Lagrange equations are solved under the assumptions of classical meson fields, time reversal symmetry, no-sea contribution, etc. Pairing is introduced under the BCS approximation. Since accuracy of the nuclear density is very important in our calculation,

we have solved the equations in co-ordinate space. The strength of the zero range pairing force is taken as 300 MeV-fm for both protons and neutrons. These values have been chosen to represent a good fit for the binding energy values in the daughter nuclei. For odd number of nucleons, the tagging approximation has been used to specify the level occupied by the last odd nucleon of either type. We have observed that moderate variations of the pairing strength do not influence the life time to any great extent.

The effective NN interactions density dependent M3Y (DDM3Y)[12, 13] and that of Jeukenne, Lejeune and Mahaux (JLM)[15] have been used to construct the proton nucleus potential. Both these interactions have been derived from nuclear matter calculation and have been applied in finite nuclei with success.

The DDM3Y interaction[12, 13] is obtained from a finite range energy independent M3Y interaction by adding a zero range energy dependent pseudopotential and introducing a density dependent factor. The density dependence may be chosen as exponential[12] or be of the more physical form  $C(1 - \beta\rho^{2/3})$ [13]. The constants were obtained from nuclear matter calculation[14] as  $C = 2.07$  and  $\beta = 1.624 \text{ fm}^2$ . We have used this form in our calculation with the above parameters. This interaction has been employed widely in the study of nucleon nucleus as well as nucleus nucleus scattering, calculation of proton radioactivity, etc.

The JLM potential[15] has been applied to finite nuclei in the Local Density Approximation. Finite range of the interaction has been incorporated by including a Gaussian form factor[16]. We have already applied the JLM potential to obtain the semimicroscopic optical model potential and to study elastic scattering in lighter nuclei with success[17]. Here we have used global parameters for the interaction and the default normalizations from Bauge *et al.*[16]. We have not come across any earlier work which uses the JLM interaction to calculate decay probabilities.

The microscopic nuclear potential has been obtained by folding the DDM3Y or JLM interactions with the microscopic densities obtained in the RMF calculation. The Coulomb potential has been similarly obtained by folding the Coulomb interaction with the microscopic proton density. The total potential consists of the nuclear part, the Coulomb potential as well as the centrifugal potential. We have not included the contribution of isovector component of the folded potential *i.e.* the Lane potential. However, we expect its effect to be small. In the case of JLM interaction, we choose only the real part of the potential. The half life of the parent nucleus has been obtained from the probability of barrier penetration in the WKB approximation. The assault frequency is obtained from the zero-point vibration energy that, in turn, has been calculated from the Q-values following the prescription of Poenaru *et al.*[18] for cluster radioactivity extended for protons. The details of the

calculations can be obtained from other references, *eg.* [6] and are not detailed here. Unlike some other works, we have not multiplied the proton nucleus potential by any normalization factor and have used the values that were obtained from nuclear matter calculations.

The calculated binding energy values of the daughter nuclei for the decays studied are compared with experimental or estimated values in Table 1. One can see that the results are reasonably good. In the bottom part of the table, the binding energy values are presented for a few other nuclei which are not known to be associated with proton radioactivity experimentally. We will discuss their significance later.

The results for the proton radioactivity half life calculation, in nuclei where such decay has been observed from low energy states, are tabulated in Table 2 for odd mass parents and Table 3 for odd-odd ones along with the experimental values and their errors. For comparison, we have also presented the results for the DDM3Y interaction of Basu *et al.* [6], who have used a simple phenomenological distribution for the densities. The experimental values are obtained from the compilation[2] by Sonzogni. We have also presented the uncertainties in the calculated half life values for the errors in the measured Q-values within parentheses. In a few nuclei, the results for Ref. [6] were obtained using slightly different experimental Q-values as indicated in the tables. More recent Q-values from [2] are expected to modify their results slightly. We would like to emphasize that in most of these cases, our results compare favourably with that of [6].

We note that the result for  $^{105}\text{Sb}$  using the JLM interaction is substantially different from that using DDM3Y interaction. This is possibly due to the very low Q-value of the decay. In all other cases, where the Q-values are much larger, close to or greater than 1 MeV, the results for the two interactions are very close to each other as well as to those of [6]. It signifies that while in this energy range, the behaviours of DDM3Y and JLM are similar, at very low energy, they may be different. We also note that a recent work[20] failed to observe any proton radioactivity in  $^{105}\text{Sb}$ . Thus it is not possible to comment on the relative merits of the two interactions.

One can see that the results are also in very good agreement with experimental values in most of the cases. In a number of decays, the results do not match so well, *eg.* in  $^{147}\text{Tm}$ ,  $^{150}\text{Lu}$  or  $^{156}\text{Ta}$  and more prominently in  $^{185}\text{Bi}$  and in the decay from the excited state of  $^{177}\text{Tl}$ , the last two results being off by an order of magnitude.

In the case of  $^{177}\text{Tl}$ , the half life of the decay from the ground state is reproduced very well. We have assumed that all the parent states have one quasi-proton configuration. The excited state involved in the proton radioactivity in this nucleus is situated 0.8 MeV above the ground state. At this energy the state may have substantial contribution from three quasi-proton configurations thus hindering

the decay. Such contributions may also play a role in some other cases where the agreement is poorer.

The hindrance in  $^{185}\text{Bi}$  may also be explained easily. Davids *et al.* [21] observed proton radioactivity in  $^{185}\text{Bi}$  from  $1/2^+$  state of the parent to the ground state of the daughter. The parent state is expected to have the configuration  $\pi(h_{9/2})^2(s_{1/2})^{-1}$  configuration and yet it decays to the ground state of  $^{184}\text{Pb}$ . Thus this decay is more complicated and Davids *et al.* suggested that it takes place through the small  $2p-2h$  admixture in the ground state of the daughter[21]. The calculated admixture fraction of the  $2p-2h$  state to the ground state comes out to be  $\sim 0.08$ . This value is consistent with calculation in  $^{186,188}\text{Pb}$ [22] which gives values  $\leq 0.09$ . The  $2p-2h$   $0^+$  state in the daughter  $^{184}\text{Pb}$  is estimated at 0.6 MeV from systematics. Using this value, the half life for the decay to the ground state has been calculated. The branching ratio for the decay to this state is found out to be very small, of the order of  $10^{-5}$  only. So the decay to the ground state is expected to dominate.

Systematics of light Bi isotopes indicate that the lowest state is the one quasi-proton state  $9/2^-$ . Davids *et al.* also assumed that the observed  $1/2^+$  state involved in the proton radioactivity is an excited state. However as no proton or alpha radioactivity was observed which involves the  $9/2^-$  state, it has been concluded that the  $1/2^+$  state is the ground state[2]. It is possible to predict the half life of the decay from the above mentioned  $9/2^-$  state to the ground state of  $^{184}\text{Pb}$  assuming the Q-value of the decay to be the same as that for the decay from the  $1/2^+$  state. The half life of the decay (with  $l = 5$ ) comes out to be 26 ms. This value must be treated as the lower limit in case the  $9/2^-$  is the ground state.

We want to emphasize a very important result of our calculation. It has often been suggested that the half life values for proton radioactivity of  $^{109}\text{I}$  and  $^{112,113}\text{Cs}$  cannot be reproduced without the inclusion of deformation effects. It was pointed out that a deformation of the order of  $\beta \sim 0.05 - 0.15$  is essential to reproduce this data [23]. For example, Maglione *et al.*[5] have pointed out that none of the single particle states have reproduced the observed half life value within an order of magnitude. The authors used the Woods Saxon potential to obtain the single particle states. They observed that a deformation of  $\beta \sim 0.15$  is essential for reproduction of the experimental lifetime. However, our calculation reproduces the data for  $^{109}\text{I}$  with considerable accuracy. In the deformed calculations, it is usually assumed that the deformation of the parent and the daughter nuclei are identical. The daughter in this particular case is  $^{108}\text{Te}$ . Te nuclei are well known vibrational nuclei with very small deformation. One possibility may be that it is the deficiency of the Woods Saxon potential far away from the stability valley which is responsible for the failure of the calculations. We also stress that our results are nearly identical

for both the NN interactions. The results for  $^{113}\text{Cs}$  and to some extent  $^{112}\text{Cs}$  are not reproduced so well and this shortcoming may be an effect of deformation. However, in none of them do we have an order of magnitude disagreement between theory and experiment as obtained in [5].

To verify that our results are not peculiar to the chosen Lagrangian density, we have recalculated the results of three nuclei near the beginning, middle and the end of the mass region studied using the densities NL3[10] and NLSH [11]. All the calculations have used the DDM3Y interaction. We present the results in Table 4. We see that the results are very close to each other, *i.e.* they are independent of the force chosen. This is true even for  $^{113}\text{Cs}$ . The uncertainties in half life values due to the errors in measured Q-values are identical with those of our calculated values presented in Table 2 and are not shown.

We would also like to investigate the possibilities of observing proton radioactivity in nearby nuclei and chosen  $^{97}\text{In}$ ,  $^{93}\text{Ag}$ ,  $^{89}\text{Rh}$  and  $^{187}\text{Bi}$  for our purpose to be studied with FSU Gold. The binding energy results are already given in Table 1. For the first three decays, the Q-values are estimated and tabulated in [19] and presented in Table 5. The first is known to undergo  $\beta$ -decay while the latter two may possibly decay also via proton emission. The Q-value for  $^{187}\text{Bi}$  can be calculated from the binding energy difference between the parent and the daughter and is taken to be 1.130(19) MeV. Similar to the situation in  $^{185}\text{Bi}$ , the life time is calculated for the decay from the  $1/2^+$  state of the parent to the ground state of the daughter.

The results for the first three decays are tabulated in Table 5. The ground state of the nuclei  $^{89}\text{Rh}$ ,  $^{93}\text{Ag}$  and  $^{97}\text{In}$  are all assumed to be  $9/2^+$ . This follows from the systematics as well as the shell model picture. Only the lower limit of the half life values of the first two nuclei are known and both are  $1.5 \mu\text{s}$ . The half life of  $^{97}\text{In}$  is taken to be 5 ms from systematics. Assuming a possible proton radioactivity from the ground state of the parent to the ground state of the daughter, the half life values have been calculated using the DDM3Y interaction. As the Q-values in these nuclei are not known, we have calculated the upper and the lower limits of the half life values corresponding to the two limits of Q-values also. The picture is not clear due to the very large uncertainties in the Q-values as well as the experimental half life values. However, a few general remarks may be made. If we assume a half life value close to the lower limit in  $^{89}\text{Rh}$ , we see that the proton radioactivity has a very small branching ratio compared to  $\beta$ -decay. However, in  $^{93}\text{Ag}$ , it may be possible to observe proton radioactivity, indeed it may be the dominant form of decay. The situation in  $^{97}\text{In}$  is more complicated. Here, even assuming the longest half life, the dominant decay mode is expected to be proton radioactivity. The experimental life time value is from systematics and may not be correct. From the preceding

discussion, it is clear that possible  $^{93}\text{Ag}$  and  $^{97}\text{In}$  are potential candidates for proton radioactivity at ground state.

With respect to the  $1/2^+$  excited state in  $^{187}\text{Bi}$ , this level is seen to have a half life of 0.290 ms[24] and has a branching ratio  $> 50\%$  [25] for alpha decay. The calculated half life for proton radioactivity from this state, assuming a 9% admixture of  $2p - 2h$  state in the ground state of  $^{186}\text{Pb}$ , is 10.9 seconds. Thus the branching ratio for proton radioactivity is expected to be very small compared to the alpha decay branch.

To summarize, half life values for proton emitting spherical nuclei have been calculated in the WKB approximation. The microscopic proton-nucleus potential has been obtained by folding the densities of daughter nuclei with two microscopic NN interactions, DDM3Y and JLM. The densities have been obtained in the RMF approach using mainly the force FSU Gold. Results are very similar for other common forces. The calculated results for the decays from the ground state or the low-lying excited states in almost all the nuclei agree well with experimental measurements. Reasons for large deviations in a few cases have been discussed. Results in  $^{109}\text{I}$  and  $^{112,113}\text{Cs}$  show that the effect of deformation is small contrary to earlier calculations. Predictions for possible proton radioactivity have been made in two nuclei,  $^{93}\text{Ag}$  and  $^{97}\text{In}$ .

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## References

- [1] P.J. Woods and C.N. Davis, *Annu. Rev. Nucl. Part. Sci.* **47** (1997) 541.
- [2] A.A. Sonzogni, *Nucl. Data Sheets* **95** (2002) 1.
- [3] B. Buck, A. C. Merchant and S. M. Perez, *Phys. Rev. C* **45** (1992) 1688.
- [4] S. Aberg, P. B. Semmes and W. Nazarewicz, *Phys. Rev. C* **56** (1997) 1762.
- [5] E. Maglione, L.S. Ferreira and R.J. Liotta, *Phys. Rev. Lett.* **81** (1998) 538 ; *Phys. Rev. C* **59** (1999) R589.
- [6] D. N. Basu, P. Roy Chowdhury and C. Samanta, *Phys. Rev.* **72** (2005) 051601(R) ; P. Roy Chowdhury, C. Samanta and D. N. Basu, *Arxiv Nucl-Th* (2005) 0511090.

- [7] See *eg.* P. Ring, Prog. Part. Nucl. Phys. **37** (1996) 193.
- [8] G. A. Lalazissis, D. Vretenar and P. Ring, Nucl. Phys. **A650** (1999) 133 ; **A719** (2003) C209.
- [9] J. Piekarewicz J and B.G. Todd-Rutel, Phys. Rev. Lett. **95** (2005) 122501.
- [10] G.A. Lalazissis, J. König and P. Ring, Phys. Rev. **C55** (1997) 540.
- [11] M.M. Sharma, M.A. Nagarajan and P. Ring, Phys. Lett. **B312** (1993) 377.
- [12] A. M. Kobos, B. A. Brown, R. Lindsay and G. R. Satchler, Nucl. Phys. **A425** (1984) 205.
- [13] A. K. Chaudhuri, Nucl. Phys. **A449** (1986) 243 ; **A459** (1986) 41.
- [14] D.N. Basu, J. Phys. G: Nucl. Part. Phys. **30** (2004) B7.
- [15] J.P. Jeukenne, A. Lejeune and C.Mahaux, Phys. Rev. C **14** (1974) 1391.
- [16] E. Bauge, J.P. Delaroche and M. Girod, Phys. Rev. **C63** (2001) 024607.
- [17] G. Gangopadhyay and S. Roy, J. Phys. G: Nucl. Part. Phys. **31** (2005) 1111.
- [18] D. N. Poenaru, W. Greiner, M. Ivascu, D. Mazilu and I. H. Plonski, Z. Phys. A **325** (1986) 435.
- [19] G. Audi G, A.H. Wapstra and C. Thibault, Nucl. Phys. **A729** (2003) 337.
- [20] Z. Liu *et al.* , Phys. Rev. **C72** (2005) 047301.
- [21] C.N. Davids *et al.* , Phys. Rev. Lett. **76** (1996) 592.
- [22] J. Wauters *et al.* , Phys. Rev. C **50**, 2768 (1994).
- [23] V.P. Bugrov and S.G. Kadmskyi, Sov. J. Nucl. Phys. **49** (1989) 967.
- [24] J.C. Batchelder *et al.* , Eur. Phys. J. A **5** (1999) 49.
- [25] E. Coenen, K. Deneffe, M. Huyse, P. Van Duppen and J.L. Wood, Phys. Rev. Lett. **54** (1985) 1783.

Table 1: Binding energy values of daughter nuclei in the decays studied in the present work. Experimental values are from [19].

Nucleus	BE/A(MeV)		Nucleus	BE/A(MeV)	
	Exp.	Calc.		Exp.	Calc.
<sup>104</sup> Sn	8.3836	8.3507	<sup>108</sup> Te	8.3032	8.2575
<sup>111</sup> Xe	8.181*	8.1386	<sup>112</sup> Xe	8.2295	8.1892
<sup>144</sup> Er	7.958*	7.8844	<sup>146</sup> Er	8.013*	7.9440
<sup>149</sup> Yb	7.929*	7.8905	<sup>150</sup> Yb	7.964*	7.9376
<sup>154</sup> Hf	7.918*	7.9126	<sup>155</sup> Hf	7.928*	7.9207
<sup>156</sup> Hf	7.9529	7.9365	<sup>159</sup> W	7.866*	7.8585
<sup>160</sup> W	7.8930	7.8780	<sup>163</sup> Os	7.805*	7.8079
<sup>164</sup> Os	7.8335	7.8313	<sup>165</sup> Os	8.842*	7.8467
<sup>166</sup> Os	7.8864	7.8696	<sup>170</sup> Pt	7.8083	7.8354
<sup>176</sup> Hg	7.7826	7.8031	<sup>184</sup> Pb	7.7827	7.7828
<sup>88</sup> Ru	8.313*	8.2009	<sup>94</sup> Pd	8.283*	8.1912
<sup>96</sup> Cd	8.265*	8.2040	<sup>186</sup> Pb	7.8053	7.7901

\*Estimated value

Table 2: Experimental and calculated proton decay half lives (T) for spherical proton emitters with even neutron number. The densities have been calculated using the force FSU Gold. Here DDM3Y and JLM indicate the calculated half life values obtained using the respective effective NN interactions. The last column presents the results of the calculation of [6]. The angular momentum of the proton involved is given by  $l$ . The experimental Q,  $l$  and half life values are from Ref. [2].

Nucleus	$l$	Q(MeV)	$\log_{10}T(s)$					
			Exp.			Present work		Ref[6]
			Errors			DDM3Y	JLM	DDM3Y
	( $\hbar$ )							
$^{105}\text{Sb}$	2	0.491(15)	2.049	-0.067	+0.058	2.27(46)	1.69(45)	1.97(46)
$^{109}\text{I}$	2	0.829(3)	-3.987	-0.022	+0.020	-4.03(4)	-4.01(4)	-4.25
$^{113}\text{Cs}$	2	0.978(3)	-4.777	-0.019	+0.018	-5.34(4)	-5.32(4)	-5.53 <sup>†</sup>
$^{145}\text{Tm}$	5	1.753(10)	-5.409	-0.146	+0.109	-5.20(6)	-5.10(6)	-5.14(6)
$^{147}\text{Tm}$	5	1.071(3)	0.591	-0.175	+0.125	0.98(4)	1.07(4)	0.98(4)
$^{147}\text{Tm}^*$	2	1.139(5)	-3.444	-0.051	+0.046	-3.26(6)	-3.27(6)	-3.39(5)
$^{151}\text{Lu}$	5	1.255(3)	-0.896	-0.012	+0.011	-0.65(3)	-0.55(3)	-0.67(3)
$^{151}\text{Lu}^*$	2	1.332(10)	-4.796	-0.027	+0.026	-4.72(10)	-4.73(10)	-4.88(9)
$^{155}\text{Ta}$	5	1.791(10)	-4.921	-0.125	+0.125	-4.67(6)	-4.57(6)	-4.65(6)
$^{157}\text{Ta}$	0	0.947(7)	-0.523	-0.198	+0.135	-0.21(11)	-0.23(11)	-0.43(11)
$^{161}\text{Re}$	0	1.214(6)	-3.432	-0.049	+0.045	-3.28(7)	-3.29(7)	-3.46(7)
$^{161}\text{Re}^*$	5	1.338(7)	-0.488	-0.065	+0.056	-0.57(7)	-0.49(7)	-0.60(7)
$^{165}\text{Ir}^*$	5	1.733(7)	-3.469	-0.100	+0.082	-3.52(5)	-3.44(5)	-3.51(5)
$^{167}\text{Ir}$	0	1.086(6)	-0.959	-0.025	+0.024	-1.05(8)	-1.07(8)	-1.27(8)
$^{167}\text{Ir}^*$	5	1.261(7)	0.875	-0.127	+0.098	0.74(8)	0.81(8)	0.69(8)
$^{171}\text{Au}$	0	1.469(17)	-4.770	-0.151	+0.185	-4.84(15)	-4.86(15)	-5.02(15)
$^{171}\text{Au}^*$	5	1.718(6)	-2.654	-0.060	+0.054	-3.03(4)	-2.96(4)	-3.03(4)
$^{177}\text{Tl}$	0	1.180(20)	-1.174	-0.349	+0.191	-1.17(25)	-1.20(25)	-1.36(25)
$^{177}\text{Tl}^*$	5	1.986(10)	-3.347	-0.122	+0.095	-4.52(5)	-4.46(5)	-4.49(6)
$^{185}\text{Bi}$	0	1.624(16)	-4.229	-0.081	+0.068	-5.33(13)	-5.36(13)	-5.44(13)

<sup>†</sup> Calculated for a Q -value of 0.977 MeV.

Table 3: Experimental and calculated proton decay half lives (T) for spherical proton emitters with odd neutron number. See caption of table 2 for details.

Nucleus	$l$ ( $\hbar$ )	Q(MeV)	$\log_{10}T(s)$					
			Exp.			Present work		Ref[6]
				Errors		DDM3Y	JLM	DDM3Y
$^{112}\text{Cs}$	2	0.824(7)	-3.301	-0.097	+0.079	-2.93(11)	-2.91(11)	-3.13 <sup>†</sup>
$^{150}\text{Lu}$	5	1.283(4)	-1.180	-0.064	+0.055	-0.59(4)	-0.49(4)	-0.58(4)
$^{150}\text{Lu}^*$	2	1.317(15)	-4.523	-0.301	+0.620	-4.24(15)	-4.24(15)	-4.38(15)
$^{156}\text{Ta}$	2	1.028(5)	-0.620	-0.101	+0.082	-0.22(7)	-0.23(7)	-0.38(7)
$^{156}\text{Ta}^*$	5	1.130(8)	0.949	-0.129	+0.100	1.66(10)	1.76(10)	1.66(10)
$^{160}\text{Re}$	2	1.284(6)	-3.046	-0.056	+0.075	-2.86(6)	-2.87(6)	-3.00(6)
$^{164}\text{Ir}$	5	1.844(9)	-3.959	-0.139	+0.190	-3.95(5)	-3.86(5)	-3.92(5)
$^{166}\text{Ir}$	2	1.168(8)	-0.824	-0.273	+0.166	-0.96(10)	-0.96(10)	-1.11(10)
$^{166}\text{Ir}^*$	5	1.340(8)	-0.076	-0.176	+0.125	0.22(8)	0.30(8)	0.21(8)

<sup>†</sup> Calculated for a Q-value of 0.823 MeV.

Table 4: Binding energy and proton decay half life values calculated using different RMF forces.

Nucleus	BE/A(MeV) of daughter			$\log_{10}T(s)$		
	FSU Gold	NL3	NLSH	FSU Gold	NL3	NLSH
$^{113}\text{Cs}$	8.1892	8.1868	8.2270	-5.34	-5.35	-5.32
$^{147}\text{Tm}$	7.9440	7.9657	7.9612	0.98	0.96	1.00
$^{147}\text{Tm}^*$				-3.26	-3.26	-3.24
$^{185}\text{Bi}$	7.7828	7.7817	7.7903	-5.33	-5.34	-5.31

Table 5: Calculated proton decay half lives (T) for  $^{89}\text{Rh}$ ,  $^{93}\text{Ag}$ ,  $^{97}\text{In}$ . The experimental half life in seconds including all the decays is denoted by  $\tau$ . Here I, II and III in the columns for calculated values refer respectively to the half lives for the mean Q-value, followed by the ones corresponding to its upper and lower limits. See caption of table 2 for more details.

Nucleus	$l$ ( $\hbar$ )	Q(MeV)	$\log_{10}\tau(s)$		$\log_{10}T(s)$		
			Exp.	I	II	III	
$^{89}\text{Rh}$	4	0.700(200)	> -5.824	-0.36	-3.07	3.76	
$^{93}\text{Ag}$	4	1.430(780)	> -5.824	-8.76	-12.15	-0.66	
$^{97}\text{In}$	4	1.812(780)	-2.301	-10.33	-12.97	-5.21	