Magnetic Moment of the Ω^- in QCD sumrule (QCDSR)

Jishnu Dey [1,3], Mira Dey [2,3],
and
Ashik Iqubal [4]

November 15, 2018

Abstract : The Ω^- magnetic moment was measured very accurately and experimentalists remarked that it differs from the theoretical estimates, thus posing a challenge to the latter. One such estimation uses QCDSR. We revisit this sumrule method, using condensate parameters which were obtained from fitting the differences $(\mu_p - \mu_n)$, $(\mu_{\Sigma^+} - \mu_{\Sigma^-})$ and $(\mu_{\Xi^0} - \mu_{\Xi^-})$ [1] and confirm the experimental number. The $\mu_{\Delta^{++}}$ is also found to agree with the experimental estimate.

Keywords : QCD sumrules, magnetic moments of baryons.

(1) Azad Physics Centre, Dept. of Physics, Maulana Azad College, Calcutta 700 013, India and ICTP, Trieste, Italy
(2) Dept. of Physics, Presidency College, Calcutta 700 073, India and and ICTP, Trieste, Italy
(3) *Work supported in part by DST grant no. SP/S2/K18/96, Govt. of India,*
permanent address : 1/10 Prince Golam Md. Road, Calcutta 700 026, India, email : deyjm@giascl01.vsnl.net.in.
(4) University College of Science, University of Calcutta, Acharya Prafulla Chandra Road, Calcutta, India.

The Ω^- magnetic moment, μ_{Ω^-}, has been the subject of many studies [2, 3, 4, 5, 6]. The magnetic moment was unknown when [2] was published but on hindsight the value predicted there, within the acceptable parameter range, agrees with the present accurately determined experimental result [7]. The results of Lee [3] using QCD sumrules and those from the lattice calculation [4] underestimate it whereas the light- cone relativistic quark model [5] and the chiral quark soliton model [6] overestimate it. We re-investigate this intriguing situation by looking at the calculations of Lee using a slightly different point of view advocated in [1] and find that one indeed gets good agreement with experiment. Further, as pointed out by Lee, the μ_{Ω^-} depend sensitively on the magnetic susceptibility so that we can pinpoint this parameter more effectively.

The QCD sumrule method is a very powerful tool in revealing a deep connection between hadron phenomenology and vacuum structure [8] via a few condensates like

$$a = -2\pi^2 < \bar{q}q >,\ b =< g^2G^2 > , \tag{1}$$

related to the quark (q) and gluon (G) vacuum expectation values. This can be used for evaluating magnetic moments of hadrons [9] where some new parameters enter, for example, χ, κ and ξ, defined through the following equations :

$$< \bar{q}\sigma_{\mu\nu}q >_F = e_q\chi < \bar{q}q > F_{\mu\nu}, \tag{2}$$

$$< \bar{q}gG_{\mu\nu}q >_F = e_q\kappa < \bar{q}q > F_{\mu\nu}, \tag{3}$$

$$< \bar{q}\epsilon_{\mu\nu\rho\gamma}G^{\rho\gamma}\gamma_5 q >= e_q\xi < \bar{q}q > F_{\mu\nu}. \tag{4}$$

where the F denotes the usual external electromagnetic field tensor. Lee [3] very carefully evaluated the contributions of these operators to the magnetic moments of the Ω^- and Δ^{++}, the latter emerging from the former when the quark mass m_s, is put equal to zero, the parameter f and ϕ are put equal to 1 and the quark charge $e_s = -1/3$ is replaced by $e_u = 2/3$. The parameter f and ϕ measure the values of quark condensates and quark spin-condensates with strange and (ud) quarks.

$$f = \frac{< \bar{s}s >}{\bar{u}u}, \tag{5}$$

$$\phi = \frac{< \bar{s}\sigma_{\mu\nu}s >}{< \bar{u}\sigma_{\mu\nu}u >} \tag{6}$$

For the expression for the μ_{Ω^-} sumrule we refer the reader to the paper by Lee [3] which we reproduce here for the sake of completeness, in terms of the Borel parameter M and the intermediate state contribution A :

$$
\frac{9}{28}e_s L^{4/27} E_1 M^4 - \frac{15}{7}e_s f\phi m_s \chi a L^{-12/27} E_0 M^2 + \frac{3}{56}e_s b L^{4/27} - \frac{18}{7}e_s f m_s a L^{4/27}
$$
$$
-\frac{9}{28}e_s f\phi(2\kappa+\xi) m_s a L^{4/27} - \frac{6}{7}e_s f^2\phi\chi a^2 L^{12/27} - \frac{4}{7}e_s f^2 \kappa_v a^2 L^{28/27}\frac{1}{M^2}
$$
$$
-\frac{1}{14}e_s f^2\phi(4\kappa+\xi)a^2 L^{28/27}\frac{1}{M^2} + \frac{1}{4}e_s f^2\phi\chi m_0^2 a^2 L^{-2/27}\frac{1}{M^2}
$$
$$
-\frac{9}{28}e_s f m_s m_0^2 a L^{-10/27}\frac{1}{M^2} + \frac{1}{12}e_s f^2 m_0^2 a^2 L^{14/27}\frac{1}{M^4}
$$
$$
= \tilde{\lambda}_\Omega^2 \left(\frac{\mu_\Omega}{M^2} + A\right) e^{-M_\Omega^2/M^2}. \tag{7}
$$

Here
$$
E_n(x) = 1 - e^{-x}\sum_n \frac{x^n}{n!},\ x = w_B^2/M_B^2 \tag{8}
$$
where w_B is the continuum, and
$$
L = \frac{ln(M^2/\Lambda_{QCD}^2)}{ln(\mu^2/\Lambda_{QCD}^2)} \tag{9}
$$
For evaluating the magnetic moment we use the above equation and divide by the equation for the mass sumrule given earlier by Lee [10]. Thus we eliminate the parameter λ_Ω^- in the spirit of [1] and we get an excellent fit to the resulting numbers in the form $\mu_{\Omega^-} + A/M^2$. We find that the results are not very sensitive to κ_v, the so called factorization violation parameter, defined through
$$
< \bar{u}u\bar{u}u >= \kappa_v < \bar{u} >^2 . \tag{10}
$$
Neither are the results very sensitive to the parameters κ and ξ. We use the crucial parameters a and b from [1], since they must fit the octet baryon moment-differences $(\mu_p - \mu_n)$ and $(\mu_{\Sigma^+} - \mu_{\Sigma^-})$. It was shown in [1] that by using the empirical scaling of the $\tilde{\lambda}$ with the $(baryon\ mass)^3$ - these differences depend only of a and b, and one gets $a = 0.475\ GeV^3$ and $b = 1.695\ GeV^4$. Further, to fit the difference $(\mu_{\Xi^0} - \mu_{\Xi^-})$, m_s was set to be 170 MeV in [1] and we use this value.

Table 1 shows the dependence of the magnetic moments on the parameters. Obviously $\mu_{\Delta^{++}}$ does not depend on f and ϕ. It is clear that μ_{Ω^-} also does not depend so much on f but it is sensitive to both ϕ and χ, and it appears that $(\chi = 6.5, \phi = 0.6)$ and $(\chi = 5.5, \phi = 0.7)$ are preferred values, close to the experimental number $\mu_{\Omega^-} = 2.019 \pm 0.054\,\mu_N$ [7]. The $\mu_{\Delta^{++}}$ is known only approximately, $4.52 \pm 0.95\,\mu_N$ [11] and a better determination will enable us to pinpoint χ.

It is satisfactory to see that there is no conflict between experiment and QCDSR since sum rules are a 'first principle method', although it is based partly on phenomenology.

In summary we find that using the constrained values of the parameters a and b [1] one can get a good fit to the known decuplet magnetic moments. The moments do not depend sensitively on the factroization violation parameter but may be used to pinpoint the susceptibility χ and ϕ, the ratio of the spin condensate for strange and ud quarks.

Table 1. The values of the parameters and the corresponding magnetic moments.

κ	ξ	χ	κ_v	f	ϕ	μ_{Ω^-}	$\mu_{\Delta^{++}}$
0.70	-1.5	-6.5	1.0	0.83	0.6	-2.007	3.702
0.75	-1.5	-6.5	1.0	0.83	0.6	-2.005	3.697
0.80	-1.5	-6.5	1.0	0.83	0.6	-2.002	3.691
0.75	-1.4	-6.5	1.0	0.83	0.6	-1.983	3.670
0.75	-1.6	-6.5	1.0	0.83	0.6	-2.026	3.724
0.75	-1.5	-7.0	1.0	0.83	0.6	-2.146	3.964
0.75	-1.5	-6.0	1.0	0.83	0.6	-1.884	3.457
0.75	-1.5	-6.5	1.5	0.83	0.6	-1.928	3.588
0.75	-1.5	-6.5	1.0	0.83	0.7	-2.750	3.697
0.75	-1.5	-5.5	1.0	0.83	0.7	-2.011	3.217
0.75	-1.5	-6.5	1.0	0.88	0.6	-2.020	3.697

References

[1] J. Dey, M. Dey and M. Sinha Roy, Phys. Lett. B 443 (1998) 293.

[2] J. Dey, M. Dey and J. LeTourneux, Phys. Rev. D34 (1986) 2104.

[3] F. X. Lee, hep-ph/9707411.

[4] D. B. Leinweber, T. Draper and R. M. Woloshyn, Phy. Rev. D 46 (1992) 3067.

[5] F. Schlumpf, Phys. Rev. D 48 (1993) 4478.

[6] H. C. Kim, M. Praszalowicz and K. Goeke, hep-ph/9706531.

[7] N. B. Wallace et al., Phys. Rev. Lett. 74 (1995) 3732.

[8] M. A. Shifman, A. I Vainshtein and Z. I. Zakharov, Nucl. Phys. B 147 (1979) 385, 448.

[9] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232 (1984) 109; C. B. Chiu, S. J. Wilson, J. Pasupathy and J. P. Singh, Phys. Rev. D 36 (1987) 1553.

[10] F. X. Lee, Phys.Rev. D 57 (1997) 322.

[11] A. Bosshard et al., Phys. Rev. D 44 (1991) 1992.