

# On local indistinguishability of orthogonal pure states by using a bound on distillable entanglement

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## Abstract

We show that the four states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$ ,  $c|01\rangle + d|10\rangle$  and  $\bar{d}|01\rangle - \bar{c}|10\rangle$  cannot be discriminated with certainty if only local operations and classical communication (LOCC) are allowed and if only a single copy is provided, except in the case when they are simply  $|00\rangle$ ,  $|11\rangle$ ,  $|01\rangle$  and  $|10\rangle$  (in which case they are trivially distinguishable with LOCC). We go on to show that there exists a continuous range of values of  $a$ ,  $b$ ,  $c$  and  $d$  such that even three states among the above four are not locally distinguishable, if only a single copy is provided. The proof follows from the fact that logarithmic negativity is an upper bound of distillable entanglement.

Entanglement [1] has always been a storehouse of surprises. It has been the vehicle in demonstration of several paradoxes [2, 3, 4, 5]. However in the past few years, entanglement has been found to be useful in information processing and communication between possibly distant parties which *a priori* share an entangled state [6]. Examples include quantum cryptography [7], dense coding [8], quantum teleportation [9], enhanced communication [10]. As the focus is on what can and cannot be implemented between separated parties when they *a priori* share some entangled state, greater attention is being given on what can and cannot be done with entangled states, when we act locally on them. For example, it has recently been shown that given a single copy from a set of any *two* multipartite orthogonal pure states, it is always possible to distinguish between them even if one is acting only locally [11]. It is therefore natural to probe the question of local distinguishability of a set containing more than two orthogonal (in general, entangled) states. Our attempt in this paper is to investigate the question of local indistinguishability of bipartite ( $2 \otimes 2$ ) states which is a recent interest in understanding entanglement [11, 12, 13, 14].

In general, more than two orthogonal states cannot be discriminated. For example, any three of the four Bell states

$$\begin{aligned} |B_1\rangle &= \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle), \\ |B_2\rangle &= \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle), \\ |B_3\rangle &= \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle), \\ |B_4\rangle &= \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle), \end{aligned} \tag{1}$$

cannot be deterministically discriminated by using local operations and classical communication (LOCC) when only a single copy is provided [14]. However the problem of discrimination of an arbitrary set of three or four orthogonal (in general, entangled) states in  $2 \otimes 2$  seems to be quite formidable.

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In this paper we probe the question of local distinguishability of the following set of four (orthogonal) states:

$$\begin{aligned}
|A_1\rangle &= a|00\rangle + b|11\rangle, \\
|A_2\rangle &= \bar{b}|00\rangle - \bar{a}|11\rangle, \\
|A_3\rangle &= c|01\rangle + d|10\rangle, \\
|A_4\rangle &= \bar{d}|01\rangle - \bar{c}|10\rangle
\end{aligned} \tag{2}$$

We show that these four states cannot be discriminated deterministically by using LOCC when only a single copy is provided, except when the  $|A_i\rangle$ 's are just  $|00\rangle$ ,  $|11\rangle$ ,  $|01\rangle$  and  $|10\rangle$  (in which case they are trivially distinguishable with LOCC) [15]. We are therefore faced with the question as to whether any *three* among the above four states in (2) are locally distinguishable. We show that for a certain continuous range of values of  $a$ ,  $b$ ,  $c$  and  $d$ , even *three* of the above four states in (2) cannot be deterministically discriminated if only LOCC are allowed and if only a single copy is provided. However this continuous range of values of  $a$ ,  $b$ ,  $c$  and  $d$  do *not* include values to reproduce the set of four Bell states from the set of  $|A_i\rangle$ 's and hence our results do not reproduce the result obtained in [14]. Without loosing any generality, we assume here that  $|a| \geq |b|$  and  $|c| \geq |d|$ .

We first prove that the four states  $\{|A_i\rangle\}$  in (2) cannot be discriminated with certainty if only LOCC are allowed and if only a single copy is provided, except when the  $|A_i\rangle$ 's are  $|00\rangle$ ,  $|11\rangle$ ,  $|01\rangle$  and  $|10\rangle$ . To prove it, we exploit a property of a function called logarithmic negativity ( $E_N(\rho)$ ) [16] of the state parameters of a bipartite state  $\rho$ . It is defined as  $E_N(\rho) \equiv \log_2 \|\rho^{TA}\|_1$  for a state  $\rho_{AB}$  of two parties A and B. The trace norm of a square matrix  $\sigma$  is denoted by  $\|\sigma\|_1$ , and defined as  $\|\sigma\|_1 \equiv \text{Tr}[(\sigma^\dagger \sigma)^{1/2}]$ . And here  $\rho^{TA}$  is the partial transpose [17] of  $\rho_{AB}$  with respect to the part A of the bipartite state  $\rho_{AB}$ . It turns out that we can express  $E_N(\rho)$  as  $E_N(\rho) = \log_2(1 + 2N(\rho))$ , where  $N(\rho)$  is the absolute value of the sum of the negative eigenvalues of  $\rho^{TA}$ . The property of logarithmic negativity that we use here is that it is an upper bound of distillable entanglement [16]. This property has recently been used in demonstration of irreversibility in asymptotic manipulations of entanglement [18].

Consider the following state shared between Alice (A), Bob (B), Charu (C) and Debu (D), with all four at distant locations:

$$\rho = \frac{1}{4} \sum_{i=1}^4 P[|A_i\rangle_{AB} |B_i\rangle_{CD}]$$

Here the  $|A_i\rangle$ 's are given by equation (2) and  $|B_i\rangle$ 's are given by equation (1) [19]. Suppose that it is possible to distinguish the four states  $\{|A_i\rangle\}$  with certainty even if only LOCC are allowed and only a single copy is provided. Then it immediately follows from the structure of the shared state  $\rho$  that Alice and Bob (without meeting) would be able to help Charu and Debu to share a Bell state with certainty.

This means that the distillable entanglement of  $\rho$ , in the AC:BD cut, is at least 1 ebit.

Now the logarithmic negativity  $E_N(\rho) \equiv \log_2 \|\rho^{TAC}\|_1$  of the state  $\rho$ , in the AC:BD cut, is

$$\log_2 (|a|^2 + |c|^2),$$

which is strictly less than unity, except when the  $|A_i\rangle$ 's are  $|00\rangle$ ,  $|11\rangle$ ,  $|01\rangle$  and  $|10\rangle$  [20].

However,  $E_N(\rho)$  is an upper bound of distillable entanglement [16]. This implies that the distillable entanglement of the state  $\rho$  in the AC:BD cut must be strictly less than 1 ebit. But as we have already stated, the assumption of local distinguishability of the  $|A_i\rangle$ 's forces the distillable entanglement of  $\rho$  in the AC:BD cut to be at least 1 ebit. This is a contradiction. Thus our assumption on the local distinguishability of the  $|A_i\rangle$ 's is proved to be wrong. In other words, we have proved that the four (orthogonal) states  $\{|A_i\rangle\}$  cannot be distinguished locally, with certainty, if only a single copy is provided (except in the trivial case when the states are  $|00\rangle$ ,  $|11\rangle$ ,  $|01\rangle$  and  $|10\rangle$ ).

We now go on to prove that there exists a certain continuous range of values of  $a$ ,  $b$ ,  $c$  and  $d$ , for which three of the four states from the set  $\{|A_i\rangle\}$  in equation (2) cannot be deterministically discriminated if only LOCC are allowed and if only a single copy is provided.

Consider the following state shared between Alice, Bob, Charu and Debu, with all four at distant locations:

$$\eta = \frac{1}{3} \sum_{i=1}^3 P[|A_i\rangle_{AB} |B_i\rangle_{CD}]$$

$|A_i\rangle$ 's are given by equation (2) and  $|B_i\rangle$ 's are given by equation (1) [21]. Again we suppose that it is possible to locally distinguish with certainty, the three states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$  and  $c|01\rangle + d|10\rangle$ , even if only a single copy is provided. And again, as earlier, it implies that the distillable entanglement of the state  $\eta$  (in the AC:BD cut) is more than or equal to 1 ebit.

The logarithmic negativity [16] of  $\eta$ , in the AC:BD cut is

$$\log_2 \left\{ \frac{1}{3} \left( \sqrt{1 + 16|ab|^2 - 4|cd|^2} + 2\sqrt{1 - 4|ab|^2 + |cd|^2} \right) + 1 \right\}.$$

There would again arise a contradiction, if this expression is strictly less than unity.

This implies that whenever we have

$$4|ab|^2 - |cd|^2 > 3/4, \quad (3)$$

the states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$  and  $c|01\rangle + d|10\rangle$  would be locally indistinguishable with certainty, if only a single copy is provided. It is obvious from the expression on the left hand side of (3), that the condition would not change if  $c|01\rangle + d|10\rangle$  is replaced by  $\bar{d}|01\rangle - \bar{c}|10\rangle$ . And the condition would change to

$$4|cd|^2 - |ab|^2 > 3/4, \quad (4)$$

if we investigate the local indistinguishability of  $c|01\rangle + d|10\rangle$ ,  $\bar{d}|01\rangle - \bar{c}|10\rangle$  and any one of  $a|00\rangle + b|11\rangle$  and  $\bar{b}|00\rangle - \bar{a}|11\rangle$ .

It is interesting to note that none of these inequalities satisfy the values of  $a$ ,  $b$ ,  $c$ ,  $d$  such that the Bell states can be obtained from the  $|A_i\rangle$ 's. Therefore the result that any three Bell states cannot be discriminated with certainty if only LOCC are allowed and if only a single copy is provided [14], is not reproduced by the results of this paper. This fact is quite plausible, because we have considered logarithmic negativity [16] as an upper bound of distillable entanglement, while in ref. [14], relative entropy of entanglement [22] was taken as an upper bound of distillable entanglement [23].

It is interesting to consider the following cases:

**Case (1.1.a)** From (3), it follows that the three states  $(1/\sqrt{2})(|00\rangle + |11\rangle)$ ,  $(1/\sqrt{2})(|00\rangle - |11\rangle)$  and  $c|01\rangle + d|10\rangle$  (or  $\bar{d}|01\rangle - \bar{c}|10\rangle$ ) are locally indistinguishable with certainty if only a single copy is provided, for *all* values of  $c$  and  $d$  except when  $|cd| = 1/2$ .

In particular, the states  $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ ,  $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$  and  $|01\rangle$  (or  $|10\rangle$ ) are locally indistinguishable with certainty if only a single copy is provided.

**Case (1.1.b)** It was shown in [14] that the three states  $(1/\sqrt{2})(|00\rangle + |11\rangle)$ ,  $(1/\sqrt{2})(|00\rangle - |11\rangle)$  and  $c|01\rangle + d|10\rangle$  (or  $\bar{d}|01\rangle - \bar{c}|10\rangle$ ) are locally indistinguishable with certainty if only a single copy is provided, for *all* values of  $c$  and  $d$  when  $|cd| = 1/2$  [24].

Combining the cases (1.1.a) and (1.1.b), we have the following result:

**Case (1.1)** The three states  $(1/\sqrt{2})(|00\rangle + |11\rangle)$ ,  $(1/\sqrt{2})(|00\rangle - |11\rangle)$  and  $c|01\rangle + d|10\rangle$  (or  $\bar{d}|01\rangle - \bar{c}|10\rangle$ ) are locally indistinguishable with certainty if only a single copy is provided, for *all* values of  $c$  and  $d$ .

Similarly we have the following case, which follows from equation (4) and ref. [14]:

**Case (1.2)** The three states  $(1/\sqrt{2})(|01\rangle + |10\rangle)$ ,  $(1/\sqrt{2})(|01\rangle - |10\rangle)$  and  $a|00\rangle + b|11\rangle$  (or  $\bar{b}|00\rangle - \bar{a}|11\rangle$ ) are locally indistinguishable with certainty if only a single copy is provided, for *all* values of  $a$  and  $b$  [24].

On the other hand, one can see that

**Case (2.1.a)** the three states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$  and  $(1/\sqrt{2})(|01\rangle + |10\rangle)$  (or  $(1/\sqrt{2})(|01\rangle - |10\rangle)$ ) are (trivially) distinguishable with certainty by LOCC if  $ab = 0$  even in the single copy case.

And (as in ref. [14]) one can show that

**Case (2.1.b)** the three states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$  and  $(1/\sqrt{2})(|01\rangle + |10\rangle)$  (or  $(1/\sqrt{2})(|01\rangle - |10\rangle)$ ) are indistinguishable with certainty by LOCC if  $|ab| = 1/2$ , in the single copy case.

Similarly we have the following cases:

**Case(2.2.a)** The three states  $c|01\rangle + d|10\rangle$ ,  $\bar{d}|01\rangle - \bar{c}|10\rangle$  and  $(1/\sqrt{2})(|00\rangle + |11\rangle)$  (or  $(1/\sqrt{2})(|00\rangle - |11\rangle)$ ) are (trivially) distinguishable with certainty by LOCC if  $cd = 0$  even in the single copy case.

**Case(2.2.b)** The three states  $c|01\rangle + d|10\rangle$ ,  $\bar{d}|01\rangle - \bar{c}|10\rangle$  and  $(1/\sqrt{2})(|00\rangle + |11\rangle)$  (or  $(1/\sqrt{2})(|00\rangle - |11\rangle)$ ) are indistinguishable with certainty by LOCC if  $|cd| = 1/2$ , in the single copy case.

However the local distinguishability (with certainty and in the single copy case) of the states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$  and  $(1/\sqrt{2})(|01\rangle + |10\rangle)$  (or  $(1/\sqrt{2})(|01\rangle - |10\rangle)$ ) is still inconclusive for all values of  $a$  and  $b$  except in the cases when  $ab = 0$  or  $|ab| = 1/2$ . And similar is the situation for  $c|01\rangle + d|10\rangle$ ,  $\bar{d}|01\rangle - \bar{c}|10\rangle$  and  $(1/\sqrt{2})(|00\rangle + |11\rangle)$  (or  $(1/\sqrt{2})(|00\rangle - |11\rangle)$ ) for all values of  $c$  and  $d$  except when  $cd = 0$  or  $|cd| = 1/2$ .

In conclusion, we have shown that the four states  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$ ,  $c|01\rangle + d|10\rangle$  and  $\bar{d}|01\rangle - \bar{c}|10\rangle$  cannot be distinguished with certainty if only local operations and classical communication are allowed and if only a single copy is provided, except when they are simply  $|00\rangle$ ,  $|11\rangle$ ,  $|01\rangle$  and  $|10\rangle$  (in which case they are trivially distinguishable with LOCC). We went on to show that there exists a continuous range of values of  $a, b, c, d$  for which even *three* of the above four states are not locally distinguishable with certainty if only a single copy is provided. Precisely,  $a|00\rangle + b|11\rangle$ ,  $\bar{b}|00\rangle - \bar{a}|11\rangle$  and  $c|01\rangle + d|10\rangle$  (or  $\bar{d}|01\rangle - \bar{c}|10\rangle$ ) are not locally distinguishable with certainty, if only a single copy is provided, when  $4|ab|^2 - |cd|^2 > 3/4$ .

Let us add here that the relative entropy of entanglement [22]  $E_R(\eta)$ , of  $\eta = \frac{1}{3} \sum_{i=1}^3 P[|A_i\rangle_{AB} |B_i\rangle_{CD}]$  in the AC:BD cut, is  $2 - \log_2 3 < 0.42$  [25], for  $|A_i\rangle = |B_i\rangle$ , with  $|B_i\rangle$ 's being any three Bell states. But relative entropy of entanglement satisfies a strong continuity condition [26]. And if  $|A_1\rangle = a|00\rangle + b|11\rangle$ ,  $|A_2\rangle = \bar{b}|00\rangle - \bar{a}|11\rangle$  and  $|A_3\rangle = c|01\rangle + d|10\rangle$ , then the  $|A_i\rangle$ 's would be the Bell states for  $(a, b, c, d) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ , which is a boundary point of the set of points represented by  $4|ab|^2 - |cd|^2 \leq 3/4$ . This implies, via the continuity of  $E_R$ , that  $E_R(\eta) < 1$  would hold at least for some continuous subset of the region  $4|ab|^2 - |cd|^2 \leq 3/4$ . And the fact that relative entropy of entanglement is an upper bound of distillable entanglement [23] would result in the corresponding  $|A_i\rangle$ 's being locally indistinguishable (for a single copy) by the methodology used in this paper. However, the value of  $E_R(\eta)$  is not computable at present for values of  $(a, b, c, d)$  lying the relevant range [25]. It is probably true that local indistinguishability with certainty holds even when  $4|ab|^2 - |cd|^2 \leq 3/4$  (leaving out the trivial case of  $|00\rangle$ ,  $|11\rangle$  and  $|01\rangle$  (or  $|10\rangle$ )). But that would necessitate the consideration of a different upper bound of distillable entanglement, or a different method than is followed here.

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- [20] It is to be noted here that the quantity  $\log_2(|a|^2 + |c|^2)$  is non-negative, due to our assumptions that  $|a| \geq |b|$  and  $|c| \geq |d|$ .
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