

Light neutrinos from massless texture and below TeV seesaw scale

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Abstract

We present general conditions on Dirac and Majorana mass terms under which a type-I seesaw mechanism can lead to three exactly massless neutrinos at the tree level. We depict several examples where the conditions are satisfied and relate some of them to an underlying $U(1)$ symmetry. We show that higher order corrections may generate the small observed masses and this may be achieved even when the heavy Majorana neutrinos are at the electroweak scale or a little higher.

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1 Introduction

There are several good indications of physics still to be unveiled associated with some new high scale above electroweak energies. In the seesaw mechanism this idea has been implemented for generating neutrino masses [1] which are expected to be much smaller than those for other elementary particles.

Here we shall consider one of the simplest and elegant versions of the type-I seesaw mechanism where the Standard Model (SM) is supplemented with three neutral fermion singlets (N_R). To avoid an unattractive suppression of Yukawa couplings much below unity, one requires the seesaw scale of the heavy Majorana mass of N_R to be of the order of $10^8 - 10^{16}$ GeV. Thus, a direct experimental test of this idea is not possible at present. However, there are some proposals for lowering this scale [2, 3, 4]. One such, in the context of the type-I seesaw mechanism, calls for a cancellation among different contributions to the light neutrino mass matrix.

In this work, we discuss in general terms the structure of the neutrino mass matrix for which three exactly massless neutrinos are possible at the tree level [5]. We show that some of these mass matrix structures can be traced to a $U(1)$ symmetry. We indicate how small masses can be generated for the light neutrinos, once this symmetry is softly broken, through higher order effects keeping right-handed neutrinos even at the electroweak scale.

Let us consider the model where alongside the three generations of leptons of the SM three right-handed (SM singlet) neutrinos, N_{Ri} ($i = 1, 2, 3$), have been added. The most general mass term for the neutrino fields is given by

$$\mathcal{L}_{mass} = \frac{1}{2} (\bar{\nu}_L, \bar{N}_R^c) M \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + h.c. \quad (1)$$

where M , the 6×6 neutrino mass matrix spanning $[\nu_e, \nu_\mu, \nu_\tau, N_{R1}, N_{R2}, N_{R3}]$, is

$$M = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}. \quad (2)$$

It is to be noted that, in general, M is not hermitian. If there is no Higgs triplet, as we choose, M_L is zero in the above and there is no type-II seesaw contribution to the neutrino mass. M_D is related to the Yukawa couplings to scalar doublets with vacuum expectation value. M_R is the Majorana mass term and is complex symmetric in general. The matrix elements in M_D are much smaller than the non-zero elements in M_R , the latter in general characterizing new high scale physics.

To obtain three massless neutrinos, M_D as well as M_R must be attributed with some special features. In the following section 2 we obtain the general conditions on M which would lead to one, two, or three exactly massless neutrinos. In section 3 we list the correlated structures of M_D and M_R which follow from the requirement of three massless neutrinos. A discussion of how a $U(1)$ symmetry would lead to the desired mass matrix textures is presented in section 4. In section 5 we then show how a small departure from the symmetry can lead to tiny non-zero neutrino masses at the two-loop level. The higher order nature of the correction allows the new symmetry breaking scale to be in a relatively low range. The testability of such a scenario is briefly outlined in section 6 before ending with our conclusions.

2 Conditions for massless neutrinos

The general structure of M_D and M_R can be written as:

$$M_D = \begin{pmatrix} x_1 & x_2 & x_3 \\ \alpha_1 x_1 & \alpha_2 x_2 & \alpha_3 x_3 \\ \beta_1 x_1 & \beta_2 x_2 & \beta_3 x_3 \end{pmatrix}, \quad (3)$$

and

$$M_R = \begin{pmatrix} M_1 & M_4 & M_5 \\ M_4 & M_2 & M_6 \\ M_5 & M_6 & M_3 \end{pmatrix}, \quad (4)$$

where the entries x_i, α_i, β_i , and M_i may be complex. The eigenvalues of M are obtained from the characteristic equation

$$Det [M^\dagger M - \lambda \text{diag}(1, 1, 1, 1, 1, 1)] = 0. \quad (5)$$

In this notation, we discuss next the conditions on M_D and M_R for one, two, or three massless neutrinos.

2.1 One massless neutrino

The existence of one zero eigenvalue requires $Det(M^\dagger M)$ to vanish. This implies $AA^* = 0$ where

$$A = Det[M_D] = \{\alpha_3(\beta_2 - \beta_1) + \alpha_2(\beta_1 - \beta_3) + \alpha_1(\beta_3 - \beta_2)\}^2 x_1^2 x_2^2 x_3^2. \quad (6)$$

This result does not depend at all on the structure of M_R . It follows that the necessary condition for at least one massless neutrino is any one of the following: (a) $\alpha_1 = \alpha_2 = \alpha_3$; (b) $\beta_1 = \beta_2 = \beta_3$; (c) $\alpha_j = \alpha_k$ and $\beta_j = \beta_k$, $j \neq k$; (d) at least one of the x_i 's is zero. (a) and (b) correspond to two rows of M_D being proportional, *i.e.*, by an appropriate redefinition of the doublet neutrino fields one of them can be entirely decoupled. For (c) the j, k columns are proportional while in (d) one column is vanishing³. (d) can be obtained from (c) by redefining the right-handed neutrino fields; in effect, one of the three right-handed neutrinos is decoupled for these alternatives.

2.2 Two massless neutrinos

Demanding the coefficient of λ in eq. (5) to be zero along with (6) it is possible to get two massless neutrinos. For brevity we do not present the expression for this coefficient here. Instead, we list various possible solutions for two massless neutrinos.

Solution 2.1:

$$\alpha_1 = \alpha_2 = \alpha_3; \quad \beta_1 = \beta_2 = \beta_3. \quad (7)$$

Suitably redefining the left-handed neutrinos this results in two of them being decoupled in the mass matrix and hence massless.

³All $\alpha_i = 0$ or all $\beta_i = 0$ ($i = 1, 2, 3$) corresponds to the vanishing of a row of M_D .

Solution 2.2:

$x_i = 0$; for some i and one of the following conditions:

$$\begin{aligned}
(i) \quad & M_i = 0; \\
(ii) \quad & x_j = 0; \quad j \neq i, \\
(iii) \quad & \alpha_j = \alpha_k; \quad \beta_j = \beta_k; \quad i \neq j \neq k \neq i; \quad i, j, k = 1, 2, 3.
\end{aligned} \tag{8}$$

(ii) corresponds to effectively only one coupled right-handed neutrino and therefore only one massive light neutrino. (iii) is also the same upto a redefinition of the fields.

Solution 2.3:

$$\alpha_1 = \alpha_2 = \alpha_3; \quad B = 0, \tag{9}$$

where

$$\begin{aligned}
B = & (\beta_1 - \beta_2)^2 M_3 x_1^2 x_2^2 - 2(\beta_1 - \beta_2) x_1 \{(\beta_1 - \beta_3) M_6 x_1 + (\beta_3 - \beta_2) M_5 x_2\} x_3 x_2 + \\
& \{(\beta_1 - \beta_3)^2 M_2 x_1^2 + 2(\beta_1 - \beta_3)(\beta_3 - \beta_2) M_4 x_2 x_1 + (\beta_2 - \beta_3)^2 M_1 x_2^2\} x_3^2.
\end{aligned} \tag{10}$$

Solution 2.1 is but a special case of this one; it is listed separately for its later relevance. In general, any M_i in (10) can be constrained using eq. (9).

Solution 2.4:

This is obtained from Solution 2.3 after replacing all α_i by β_i and vice versa. (11)

Solution 2.5:

$$\begin{aligned}
\alpha_i = \alpha_j; \quad \beta_i = \beta_j; \quad i \neq j; \quad i, j = 1, 2, 3, \quad \text{and} \\
(M_R)_{ii} = (M_R)_{jj} = (M_R)_{ij} = (M_R)_{ji} = 0.
\end{aligned} \tag{12}$$

Thus the right-handed neutrinos of the i, j -type are coupled only to the k -th right-handed neutrino while the coupling strengths of the left-handed i, j -type neutrinos to the right-handed neutrinos bear the constant ratio $x_i : x_j$.

The solutions 2.1-2.4 for two massless neutrinos presented above are valid for general M_R and as such hold for both singular or non-singular M_R . The condition in 2.5 requires $Det[M_R] = 0$.

2.3 Three massless neutrinos

To obtain three massless neutrinos the coefficient of λ^2 in eq. (5) is also to be zero apart from setting λ -independent and λ^1 terms to zero. For every alternative for two massless neutrinos, *i.e.*, eqs. (8) - (12), we examine what additional requirement will yield a third massless neutrino. Interestingly, it turns out that, excepting for the 2.2(ii) alternative on which we comment below, in all other options solution 2.1 - eq. (7) - must be required supplemented with the following relation:

$$\begin{aligned}
(M_2 M_3 - M_6^2) x_1^2 + (M_1 M_3 - M_5^2) x_2^2 + (M_1 M_2 - M_4^2) x_3^2 + 2(M_6 M_5 - M_3 M_4) x_1 x_2 \\
+ 2(M_4 M_5 - M_1 M_6) x_2 x_3 + 2(M_4 M_6 - M_2 M_5) x_1 x_3 = 0.
\end{aligned} \tag{13}$$

Eq. (13) is the master constraint condition on different elements of the matrix M_R . For some specific cases it restricts the different x_i appearing in M_D ; in some alternatives one or more of the x_i may even be zero.

In case 2.2(ii), where $x_i = 0$ and $x_j = 0$, one does *not* require eq. (7); indeed $\alpha_{i,j}$ and $\beta_{i,j}$ cannot even be defined. Simply satisfying the master constraint eq. (13) is sufficient. Note that if eq. (7) is valid then by a suitable redefinition of the ν_i fields $x_i = 0$ and $x_j = 0$ can be achieved. However, the converse is not necessary.

One may look at the condition in (13) in the following way. If eq. (7) (or $x_i = 0$ and $x_j = 0$) is valid the matrix M_D is of rank 1. In that case, with suitable unitary transformation on M_D one may reduce M in (2) to a 4×4 non-zero block

$$M' = \begin{pmatrix} 0 & x_1 & x_2 & x_3 \\ x_1 & M_1 & M_4 & M_5 \\ x_2 & M_4 & M_2 & M_6 \\ x_3 & M_5 & M_6 & M_3 \end{pmatrix}, \quad (14)$$

and now the massless condition of the third neutrino requires the vanishing of $Det[M'^\dagger M']$, *i.e.*, simply

$$Det[M'] = 0. \quad (15)$$

This is just eq. (13). If M_R is non-singular then another way of looking at eq. (13) is to consider the seesaw formula for the 3×3 light neutrino mass matrix as the power series expansion:

$$m_\nu = -M_D M_R^{-1} M_D^T + \frac{1}{2} M_D M_R^{-1} (M_D^T M_D^* M_R^{-1*} + M_R^{-1*} M_D^\dagger M_D) M_R^{-1} M_D^T + \dots \quad (16)$$

In that case, from (16) one can see the massless condition for all three neutrinos to all orders will correspond to the vanishing of the leading order term [6], *i.e.*,

$$M_D M_R^{-1} M_D^T = 0. \quad (17)$$

If we use eq. (7) on the left-hand side of this equation then we get a 3×3 matrix each element of which has a common factor. This common factor is nothing but the left-hand-side of eq. (13). Thus, (17) together with (7) implies eq. (13).

3 Correlated Dirac and Majorana sectors

Next, based on eqs. (7) and (13) we discuss various possible structures of M_R and M_D that together result in three massless neutrinos. We consider cases where one or more of the x_i are non-vanishing. We ask the question what can one say about M_R for three massless neutrinos for a chosen form of M_D *irrespective of the specific non-zero values of x_i* . Determinant of M_R is permitted to be zero or non-zero. We discuss them separately.

It is noteworthy that if any two of the x_i are nonzero (and certainly if all three are non-zero) then for arbitrary values of these x_i there is no solution satisfying eq. (13) unless $Det[M_R] = 0$. Such examples are taken up after considering the only option when $Det[M_R]$ can be non-vanishing.

3.1 The $\text{Det}[M_R] \neq 0$ case

There is only one class of possibilities in this category.

(i) Only one x_i nonzero: Considering $x_1 = x_2 = 0$ in M_D and x_3 is arbitrary, (13) implies the following forms for M_D and M_R :

$$M_D = \begin{pmatrix} 0 & 0 & x_3 \\ 0 & 0 & \alpha x_3 \\ 0 & 0 & \beta x_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & \pm\sqrt{M_1 M_2} & M_5 \\ \pm\sqrt{M_1 M_2} & M_2 & M_6 \\ M_5 & M_6 & M_3 \end{pmatrix}, \quad (18)$$

provided that

$$M_6\sqrt{M_1} \neq \pm M_5\sqrt{M_2}. \quad (19)$$

Above, the \pm sign is chosen matching the \pm sign in (18). This inequality condition is required only to satisfy $\text{Det}[M_R] \neq 0$. One may choose any of M_1, M_2, M_5, M_6 equal to zero keeping in mind eq. (19). There is no condition on M_3 and it may take zero or non-zero values. The two different signs in the off-diagonal elements signify the possibility of different CP phases of heavy Majorana neutrinos satisfying the three massless neutrino condition.

One may consider $x_1 = x_3 = 0$ or $x_2 = x_3 = 0$ in (13) to find other possible forms of M_R . They follow from the previous example through suitable permutations. For $x_2 = x_3 = 0$ in M_D and x_1 arbitrary the possible form of M_R is obtained by replacing M_6 by $\sqrt{M_2 M_3}$ and satisfying the inequality $M_4\sqrt{M_3} \neq \pm M_5\sqrt{M_2}$; one may consider some of M_2, M_3, M_4, M_5 equal to zero. There is no condition on M_1 . One may note that the cancellation structure of M_R given in eq. (26) in ref. [5] can be obtained in this case if one sets $M_2 = M_5 = 0$. Similar solutions exist for $x_1 = x_3 = 0$ in M_D .

It is important to bear in mind that in all the above cases eq. (7) cannot be imposed.

3.2 The $\text{Det}[M_R] = 0$ case

The conventional see-saw formula breaks down when $\text{Det}[M_R] = 0$. It needs to be stressed, however, that the formula is but an approximation. Here we are using the diagonalisation of the full (6×6) neutrino mass matrix, without taking recourse to the see-saw formula, to arrive at the mass eigenvalues and therefore $\text{Det}[M_R] = 0$ causes no difficulty.

(i) Only one x_i nonzero: In the cases where two of the x_i are zero (discussed above) if we replace the inequalities, *e.g.*, in (19), by equality sign then they correspond to $\text{Det}[M_R] = 0$. This would imply correlations within the elements of M_R beyond what is necessary and sufficient for three massless neutrinos.

(ii) Two x_i nonzero: Next, let us consider cases when only one of the x_i is zero. As for example, choosing $x_1 \neq 0, x_2 \neq 0, x_3 = 0$ in (13), three massless neutrinos for arbitrary choices of $x_{1,2}$ requires that only the (12) block of M_R be non-zero:

$$M_D = \begin{pmatrix} x_1 & x_2 & 0 \\ \alpha x_1 & \alpha x_2 & 0 \\ \beta x_1 & \beta x_2 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & M_4 & 0 \\ M_4 & M_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (20)$$

Similarly, for only $x_2 = 0$ the (13) block and for only $x_1 = 0$ the (23) block of M_R is non-zero.

(iii) All x_i nonzero: Finally, when in addition to the requirement of eq. (7) all x_i in M_D are non-zero then the solution for M_R is given by:

$$M_D = \begin{pmatrix} x_1 & x_2 & x_3 \\ \alpha x_1 & \alpha x_2 & \alpha x_3 \\ \beta x_1 & \beta x_2 & \beta x_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & \pm\sqrt{M_1 M_2} & \pm\sqrt{M_1 M_3} \\ \pm\sqrt{M_1 M_2} & M_2 & \pm\sqrt{M_2 M_3} \\ \pm\sqrt{M_1 M_3} & \pm\sqrt{M_2 M_3} & M_3 \end{pmatrix}. \quad (21)$$

Above, alternate sign choices reflect the possibility of different CP phases of heavy Majorana neutrinos. The democratic form of M_R emerges as a special case when $M_1 = M_2 = M_3$ with positive sign in the off-diagonal elements in (21).

In summary, for the most general possible form of M_R with $Det[M_R] \neq 0$ the type of solution is given by (18) where M_D is highly constrained with two x_i being zero. On the other hand, if we opt for the most general form of M_D consistent with solution 2.1 then the form for M_R is given by (21) in which all off-diagonal elements are fixed in terms of the diagonal entries and $Det[M_R] = 0$. (20) is an intermediate situation between these extremes.

4 A $U(1)$ symmetry

So far we have outlined the manner in which M_D of eq. (3), already restricted through eq. (7), and M_R of (4) are further constrained through eq. (13) in order to obtain three massless neutrinos. The natural next question is whether there is any symmetry at the root of the cancellation of different contributions to the light neutrino mass matrix, thus rendering three neutrinos massless. Below we exhibit a $U(1)$ symmetry which could imply the textures of M_D and M_R in eqs. (18), (20) and (21). In the next section, we further show that in this $U(1)$ model when the symmetry is softly broken, the loop level corrections generate small neutrino masses.

We consider a $U(1)$ symmetry of the basic Lagrangian [7]. Following ref. [2] a new scalar doublet $\chi = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix}$ is introduced with $U(1)$ quantum number +1. The Standard Model (SM) doublet Higgs, ϕ , has $U(1)$ charge 0. All SM quarks and leptons are assigned the same $U(1)$ charge (=1) so that they receive their masses through the Yukawa coupling to the ϕ , while a coupling to χ is forbidden for them. Assigning zero $U(1)$ charge to the three neutral fermion singlets N_{iR} , such a symmetry has been used in [2] to forbid the neutrino Dirac masses from arising from ϕ and only χ can contribute here. Arranging $\langle \chi \rangle \ll \langle \phi \rangle$ the smallness of the neutrino Dirac mass is ensured. As we discuss in the following, in this work too we use the scalar χ to generate small neutrino Dirac masses. Further, we choose the $U(1)$ charges of the N_{iR} appropriately to reproduce the desired textures of M_D and M_R . Thus the $U(1)$ symmetry serves a dual role.

We consider the following $U(1)$ transformations:

$$L \rightarrow e^{i\gamma n_L} L; \quad l_R \rightarrow e^{i\gamma n_R} l_R; \quad N_R \rightarrow e^{i\gamma n_\nu} N_R, \quad (22)$$

where γ is real and n_L , n_R and n_ν are hermitian matrices acting on flavor space. L_j are the left-handed lepton doublets, l_{Rj} the right-handed charged lepton singlets, and N_{Rj} the right-handed neutrino fields. Except these and the scalars (ϕ , χ) no other fields transform under this $U(1)$. In the basis where the charged lepton mass matrix is diagonal one can take $n_L = n_R = \text{diag}(n_1, n_2, n_3)$ where $n_{1,2,3}$ are $U(1)$ charges of e , μ , and τ respectively.

The lepton sector masses arise from the Lagrangian:

$$\mathcal{L}_Y = -Y_{ij}\bar{L}_i\phi l_{Rj} - Y_{ij}^\nu\bar{L}_i\tilde{\chi}N_{Rj} - \frac{1}{2}\bar{N}_{Ri}^c M_{Rij}N_{Rj}, + h.c. \quad (23)$$

where $\tilde{\chi} = i\sigma_2\chi^*$. The assignments of n_L , n_R and n_ν determine the non-vanishing elements of Y_{ij} , Y_{ij}^ν and M_{Rij} . With only $U(1)$ symmetry one cannot derive relationships among the non-zero Yukawa couplings Y_{ij}^ν . We now take up the different cases in turn.

(i) Only one x_i nonzero: Here the mass matrices, M_D and M_R will be as in (18) when $x_1 = x_2 = 0$. Choosing, for example,

$$(i) n_\nu = \text{diag}(-2, 0, 2); \quad (ii) n_\nu = \text{diag}(0, -2, 2); \quad (iii) n_\nu = \text{diag}(-2, -2, 2), \quad (24)$$

one can respectively obtain the mass matrices

$$(i) M_R = \begin{pmatrix} 0 & 0 & M_5 \\ 0 & M_2 & 0 \\ M_5 & 0 & 0 \end{pmatrix}; \quad (ii) M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & 0 & M_6 \\ 0 & M_6 & 0 \end{pmatrix}; \quad (iii) M_R = \begin{pmatrix} 0 & 0 & M_5 \\ 0 & 0 & M_6 \\ M_5 & M_6 & 0 \end{pmatrix}, \quad (25)$$

all of which are in the class of (18). Of these, $\text{Det}[M_R]$ is non-vanishing for the first two while it is zero for the third. (24) along with the choice:

$$n_L = n_R = \text{diag}(1, 1, 1) \quad (26)$$

ensures that $\langle\chi\rangle$ reproduces the M_D in (18) and that $\langle\phi\rangle$ does not contribute. These three basic textures are the most general possibilities apart from trivial permutation of various Majorana neutrino fields.

As noted after eq. (18), there are solutions where rather than (x_1, x_2) other pairs vanish. It is simple to change the $U(1)$ quantum number assignments in (24) to achieve these forms of M_D and M_R . One set, $x_2 \neq 0$, gives a previously not discussed structure for three massless neutrinos while another, $x_1 \neq 0$, reproduces the model obtained earlier in ref. [5].

(ii) Two x_i nonzero: Let us now turn to the M_D and M_R in (20). Here, two x_i in M_D are non-zero. One gets these M_R and M_D with, for example, the assignments $n_\nu = \text{diag}(0, 0, 2)$ and $n_L = n_R = \text{diag}(-1, -1, -1)$ to get $x_3 = 0$.

(iii) All x_i nonzero: The form of M_R in (21) requires the choice $n_\nu = \text{diag}(0, 0, 0)$. This along with $n_L = n_R = \text{diag}(-1, -1, -1)$ reproduces the desired texture of M_D . Of course, the relationships between the matrix elements cannot be obtained through the $U(1)$ symmetry.

At this stage it is worth noting that the forms of M_D and M_R in eq. (21) lead to three massless neutrinos for *arbitrary* values of $x_{1,2,3}$ and $M_{1,2,3}$. A particular choice, $x_3 = 0$ and $M_1 = M_2 = 0$, i.e.,

$$M_D = \begin{pmatrix} x_1 & x_2 & 0 \\ \alpha x_1 & \alpha x_2 & 0 \\ \beta x_1 & \beta x_2 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \quad (27)$$

can be accomplished by the $U(1)$ symmetry by choosing $n_L = n_R = \text{diag}(1, 1, 1)$ and $n_\nu = \text{diag}(2, 2, 0)$. Needless to say, other similar special cases of eq. (21) with $x_2 = 0$, $M_3 = M_1 = 0$ and $x_1 = 0$, $M_2 = M_3 = 0$ can also be derived from the $U(1)$ symmetry.

5 Non-zero neutrino masses

Present experimental data, *e.g.*, direct mass measurements, neutrino oscillations, etc., indicate that light neutrinos have a mass below about 0.1 eV. To generate such tiny masses it is required to break the above noted $U(1)$ symmetry of \mathcal{L}_Y – eq. (23). Particularly, one may include a soft symmetry breaking term [2]:

$$\mu_1^2 (\phi^\dagger \chi + \chi^\dagger \phi). \quad (28)$$

This would result in charged physical Higgs bosons given by

$$h^\pm = \frac{v\chi^\pm - u\phi^\pm}{\sqrt{v^2 + u^2}}, \quad (29)$$

where $v = \langle \phi^0 \rangle$ and $u = \langle \chi^0 \rangle$ and the $U(1)$ conserving terms in the scalar potential $V(\phi, \chi)$ can be chosen⁴ so as to ensure $u \ll v$. The $U(1)$ charge assignments require the quark and charged lepton masses to arise through v while the smaller neutrino masses originate from u . The $U(1)$ violating soft interaction (28) will induce contributions to M_D and M_R from higher order corrections which may result in deviations from the textures responsible for three massless neutrinos and give rise to small neutrino masses. Here, we pick two specific textures of M_D and M_R given in eqs. (21) and (27) which result in three massless neutrinos. We consider one- and two-loop corrections to these textures and show how light neutrino masses and splittings are generated. The loop corrections are calculated in the *weak* basis, i.e., to M_D , M_L , and M_R , and the consequences that follow are explored.

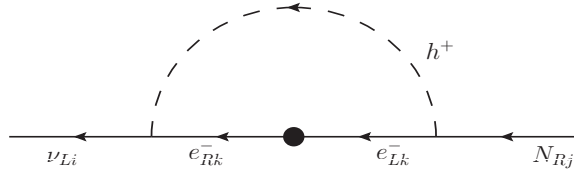


Figure 1: One-loop Feynman diagram contributing to M_D .

5.1 One-loop effects

At one-loop level a typical contribution is the diagram in Fig. 1 which contributes to M_D . This is given by:

$$\Delta M_{Dij}^{(1)} \approx \sum_k \frac{Y_{ik} Y_{kj}^\nu m_{e_k}}{32 \pi^2 m_{h^+}^2} \sin 2\kappa [m_{e_k}^2 \overline{\ln}(m_{e_k}^2) - m_{h^+}^2 \overline{\ln}(m_{h^+}^2)] , \quad (30)$$

⁴See, for example, eq. (11) in ref. [2].

where $\overline{\ln}(x) = \ln(x/Q^2)$, Q being the renormalization scale, and

$$\sin \kappa = u/\sqrt{v^2 + u^2}. \quad (31)$$

In spite of breaking the $U(1)$ symmetry softly, however, this one-loop effect does not change the texture of the matrix M_D as we note below.

Recall from eq. (23) that at the tree level, $M_{Dij} = Y_{ij}^\nu u$. To satisfy eq. (7), which is a key requirement for massless neutrinos, one must have $M_{D2j}/M_{D1j} = Y_{2j}^\nu/Y_{1j}^\nu$ and $M_{D3j}/M_{D1j} = Y_{3j}^\nu/Y_{1j}^\nu$ to be j independent⁵. This relationship is preserved after inclusion of the one-loop corrections.

To see this, note that the contribution from eq. (30) is $\Delta M_{Dij}^{(1)} = \Sigma_k X_{ik} f_k Y_{1j}^\nu$, where X_{ik} encapsulates the entire factor multiplying Y_{kj}^ν . Thus, $(M_{Dij} + \Delta M_{Dij}^{(1)}) / (M_{Dkj} + \Delta M_{Dkj}^{(1)})$ remains independent of j .

There is a similar correction to M_D from h^0 exchange. In this, or a similar one due to W^\pm exchange, there is an overall factor of M_D . Thus this additional piece will also leave the texture of M_D unchanged and is not pursued any further.

Likewise, there will be one-loop contributions to M_L through h^0 or W^\pm exchange. These are proportional to M_L . Since we have chosen $M_L = 0$ at the tree level the loop corrections also vanish.

There are diagrams similar to Fig. 1 involving W^\pm exchange contributing to M_R [8]. These corrections involve the couplings between ν_L and N_R arising from the second term of \mathcal{L}_Y in (23) and are proportional to $\langle \chi \rangle$. In general, they are expected to be very small. A larger contribution may arise from the one loop diagram similar to Fig. 1 with h^+ exchange with mass insertion on the external leg. The one-loop contribution to M_R due to this is given by (assuming $m_{h^+}^2 > M_{Rij}^2$)

$$\Delta M_{Rij}^{(1)} \approx \sum_{k,m} C_1 \frac{Y_{kj}^\nu Y_{mk}^{\nu\dagger} \cos^2 \kappa}{16 \pi^2 m_{h^+}^2} \left[m_{h^+}^2 + M_{Rij}^2 + m_{e_k}^2 \frac{m_{h^+}^2}{M_{Rij}^2} \overline{\ln}(m_{e_k}^2) - m_{h^+}^2 \overline{\ln}(m_{h^+}^2) \right] M_{Rim} \quad (32)$$

where

$$\begin{aligned} C_1 &\approx 1 \text{ for } M_{Rn} \ll M_{Rij} \\ &\approx \frac{M_{Rij}}{M_{Rn}} \text{ otherwise} \end{aligned}$$

and M_{Rn} is a typical eigenvalue of M_R .

Let us now turn to the effect of these corrections on the light neutrino masses. Consider first the M_D and M_R given in eq. (27). Incorporating the one-loop corrections these become

$$M_D = \begin{pmatrix} x'_1 & x'_2 & 0 \\ \alpha' x'_1 & \alpha' x'_2 & 0 \\ \beta' x'_1 & \beta' x'_2 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} M'_1 & M'_4 & 0 \\ M'_4 & M'_2 & 0 \\ 0 & 0 & M'_3 \end{pmatrix}, \quad (33)$$

where the primes indicate the one-loop corrected values. Notice that the form of (33) satisfies the conditions in 2.2(iii) and thus will lead to *two* massless neutrinos. Therefore, one must check whether two-loop effects can remove the remaining degeneracy.

⁵This can be represented as $Y_{ij}^\nu = f_i Y_{1j}^\nu$ where $f_{(1,2,3)} \equiv (1, \alpha, \beta)$.

Similarly, for the M_D and M_R given in eq. (21) inclusion of the one-loop contributions results in

$$M_D = \begin{pmatrix} x'_1 & x'_2 & x'_3 \\ \alpha'x'_1 & \alpha'x'_2 & \alpha'x'_3 \\ \beta'x'_1 & \beta'x'_2 & \beta'x'_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} M'_1 & M'_4 & M'_5 \\ M'_4 & M'_2 & M'_6 \\ M'_5 & M'_6 & M'_3 \end{pmatrix}. \quad (34)$$

As in the previous example, the effect of these corrections is to make one of the neutrinos massive. M_D in (34) satisfies the condition 2.1 for *two* massless neutrinos.

The result that *one* of the neutrinos acquires mass through the one-loop corrections is similar to that of [4] where a model with only SM interactions and right-handed singlet neutrinos is examined. In order to remove the remaining neutrino mass degeneracy, as required by the atmospheric and solar observations, we turn to two-loop effects now. In both examples above, the degeneracy is due to the textural property of M_D . So, it is enough to focus on the two-loop effects on this matrix.

5.2 Two-loop effects

We find that at the two-loop level there are diagrams which yield contributions which deviate from eq. (7). Consider, for example, the two-loop Feynman diagram in Fig. 2 (in which N_k and N_l are the k -th and l -th flavour eigenstates). The contribution to M_D from such

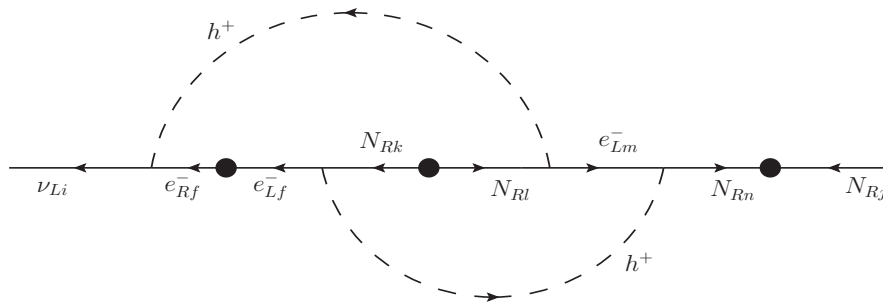


Figure 2: Two-loop Feynman diagram contributing to M_D .

diagrams is estimated to be:

$$\Delta M_{Dij}^{(2)} \approx \frac{C_2}{(4\pi)^4} \sum_{f,k,l,m,n} Y_{if} Y_{fk}^\nu Y_{ml}^\nu Y_{mn}^{\nu*} \sin \kappa \cos^3 \kappa m_{e_f} M_{Rkl}^* M_{Rnj} \operatorname{Re} \left[I_1(m_{h^+}^2, m_{e_m}^2, m_{e_f}^2, m_{h^+}^2, M_{Rn}^2) \right], \quad (35)$$

where

$$\begin{aligned} C_2 &\approx 1 \text{ for } M_{Rn} \ll M_{Dij} \\ &\approx \frac{M_{Dij}}{M_{Rn}} \text{ otherwise} \end{aligned}$$

and M_{Rn} is a typical eigenvalue of M_R . In the above expression,

$$I_1(m_{h^+}^2, m_{e_m}^2, m_{e_f}^2, m_{h^+}^2, M_{Rn}^2) = A^2 \int d^d k \int d^d q \frac{1}{(k^2 + m_{h^+}^2)(q^2 + m_{e_m}^2)((k-p)^2 + m_{e_f}^2)((q-p)^2 + m_{h^+}^2)((k-q)^2 + M_{Rn}^2)}, \quad (36)$$

and $A = (2\pi\mu)^{2\epsilon}/\pi^2$ is the loop factor and integrals are regularized by dimensional reduction to $d = 4 - 2\epsilon$ dimensions.

To estimate [9] the integral I_1 we shall ignore the external invariant momentum $-p^2$ (as M_{Dij} is small) and charged lepton masses with respect to the scalar, h^\pm , and right-handed neutrino masses. Then the integral I_1 in (34) can be approximated (in \overline{MS} scheme) for $m_{h^+}^2 > M_{Rn}^2$ as

$$Re[I_1] \approx \frac{1}{x^2} \left[2(x-y) \{ Li_2(y/x) - \overline{\ln}(x-y) \ln(x/y) \} + (x-3y/4)(\overline{\ln}(x))^2 + (2x+y)\overline{\ln}(x) \overline{\ln}(y) + y \zeta(2) + (y/2) (\overline{\ln}(y))^2 \right], \quad (37)$$

where $x = m_{h^+}^2$, $y = M_{Rn}^2$. For $m_{h^+}^2 < M_{Rn}^2$ it can be approximated as

$$Re[I_1] \approx \frac{1}{x^2} \left[2(y-x) \{ Li_2(x/y) - \overline{\ln}(y-x) \ln(y/x) \} + (3y/2-x) \times \left\{ (\overline{\ln}(y))^2 + 2\zeta(2) \right\} - y\overline{\ln}(x) \overline{\ln}(y) - (y/2-x) (\overline{\ln}(x))^2 \right]. \quad (38)$$

In the asymptotic limit [10]

$$I_1 \approx \frac{\pi^2}{3x} \quad \text{for } m_{h^+}^2 \gg M_{Rn}^2, \\ \approx \frac{(\ln^2(y/x) + \pi^2/3 - 1)}{y} \quad \text{for } m_{h^+}^2 \ll M_{Rn}^2. \quad (39)$$

One can see that unlike the one-loop result, the two-loop contribution can change the texture of M_D as the j -dependence emerges from M_{Rnj} as well as from Y_{nj}^ν . So the relationships $\alpha_1 = \alpha_2 = \alpha_3$ and $\beta_1 = \beta_2 = \beta_3$ will cease to apply. The novelty of neutrino mass generated in this manner is that there is a seesaw mechanism operative and then there is suppression⁶ due to $\sin \kappa$ which is approximately proportional to $\frac{\langle \chi^0 \rangle}{\langle \phi^0 \rangle}$.

We now proceed to examine the effect of these contributions on neutrino masses. Consider first the M_D given in eq. (27). With two-loop corrections it is

$$M_D = \begin{pmatrix} x_1'' & x_2'' & 0 \\ \alpha_1 x_1'' & \alpha_2 x_2'' & 0 \\ \beta_1 x_1'' & \beta_2 x_2'' & 0 \end{pmatrix}, \quad (40)$$

where now $\alpha_1 \neq \alpha_2$, $\beta_1 \neq \beta_2$, and the double primes indicate two-loop corrected values. This form of M_D results in *one* massless neutrino ($x_3 = 0$). We have used the M_D of eq.

⁶Note that such a suppression occurs at the tree level in ref [2] but there is further reduction here as the contribution leading to non-zero light neutrino masses arises not from tree level Yukawa couplings nor from one-loop but from two-loop diagrams.

Parameter	i		
	1	2	3
x_i (in eV)	158	564	640
α_i	10^1	$10^1 + 5.12 \times 10^{-4}$	$10^1 + 7.68 \times 10^{-4}$
β_i	1.2×10^1	$1.2 \times 10^1 + 3.84 \times 10^{-4}$	$1.2 \times 10^1 + 2.56 \times 10^{-4}$

Table 1: The parameter choices for M_D . The tree level term and the higher order corrections are indicated separately.

(40) and M_R of eq. (33) to obtain the light neutrino masses. Though two light neutrinos are indeed massive, we find that it is not possible to reproduce the observed mass splittings in this case.

On the other hand, the second example we have been considering, namely M_D and M_R in (21), can produce a mass spectrum in line with observations. Including one- and two-loop corrections, one has

$$M_D = \begin{pmatrix} x''_1 & x''_2 & x''_3 \\ \alpha_1 x''_1 & \alpha_2 x''_2 & \alpha_3 x''_3 \\ \beta_1 x''_1 & \beta_2 x''_2 & \beta_3 x''_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} M''_1 & M''_4 & M''_5 \\ M''_4 & M''_2 & M''_6 \\ M''_5 & M''_6 & M''_3 \end{pmatrix}. \quad (41)$$

with $\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_1$, $\beta_1 \neq \beta_2 \neq \beta_3 \neq \beta_1$.

We find that the M_D and M_R in eq. (41) do yield the correct neutrino mass spectrum when the various parameters are assigned appropriate values. As a typical example, the parameters may be chosen as given in the following.

The $U(1)$ symmetry is broken in the scalar potential. As discussed in [2], keeping the strength of the soft breaking term in eq. (28) $|\mu_1^2| \approx 10 \text{ GeV}^2$, the remaining terms in the scalar potential can be chosen such that $u = \langle \chi^0 \rangle \sim 1 \text{ MeV}$. Recalling that $v = \langle \phi^0 \rangle \sim 174 \text{ GeV}$, this implies $\sin \kappa \sim 5.7 \times 10^{-6}$. We consider $m_h^+ \sim 250 \text{ GeV}$ and $m_\chi = 200 \text{ GeV}$. We keep the Yukawa couplings Y_{ij}^ν to be about $\mathcal{O}(10^{-3})$ and Y_{ij} is lesser than about 0.1. The product of these couplings with u sets the scale for the tree-level entries of M_D .

Including the higher order contributions the parameters defining M_D are shown in Table 1. The matrix M_R , including loop corrections, is given in eq. (42).

$$M_R = \begin{pmatrix} 9 \times 10^{10} + 1700 & 9.48683 \times 10^{10} + 1000 & 1.08167 \times 10^{10} + 51200 \\ 9.48683 \times 10^{10} + 1000 & 1 \times 10^{11} + 2500 & 1.14018 \times 10^{11} + 1500 \\ 1.08167 \times 10^{11} + 51200 & 1.14018 \times 10^{11} + 1500 & 1.3 \times 10^{11} + 1000 \end{pmatrix} \text{ eV}. \quad (42)$$

M_R in (42) corresponds to M_R in (21) with $M_1 = 90 \text{ GeV}$, $M_2 = 100 \text{ GeV}$ and $M_3 = 130 \text{ GeV}$. One may note here that in our model due to the loop suppression as well as the $\sin \kappa$ factor in the light neutrino mass, the mass scale for heavy right handed neutrinos - the seesaw scale - could be as low as the standard electroweak scale. This is the main focus of our work.

Here the one loop corrections to M_R make $\text{Det}[M_R]$ non-zero and large and as such one may use type-I seesaw formula to obtain the light neutrino masses. With these choices the light neutrino masses are about 0.045 eV, 0.0092 eV and 0.0 eV reproducing the two mass squared differences of about $8 \times 10^{-5} \text{ eV}^2$ and $2 \times 10^{-3} \text{ eV}^2$ respectively in the normal hierarchical neutrino mass pattern. The one loop correction to massless texture of M_R and two

loop corrections to massless texture of M_D essentially set the two different scales of mass squared differences of light neutrinos. However, these specific choices of parameters do not reproduce appropriate mixing. We have not made an exhaustive survey and expect that other parameter choices may lead to even more acceptable solutions.

6 Testability

The above estimations indicate that in these models the charged higgs, h^\pm , and the right-handed neutrinos, N_{Rj} , could well be within the range of the LHC. Depending on the ordering of m_{h^\pm} and M_{Rn} the signals would be different. If $m_{h^\pm} > M_{Rn}$ then on Drell-Yan pair production of h^\pm one may expect the observable decay chain $h^\pm \rightarrow l_i^\pm N_{Rj}$ followed by $N_{Rj} \rightarrow l_k^\pm W^\mp$. The decay of h^\pm may not be much suppressed. If the N_{Rj} is long-lived due to small mixing the right-handed neutrinos may well decay outside the detector; so just a pair of oppositely charged leptons with missing energy will be observed. Otherwise, the Majorana nature of N_{Rj} can lead, in addition to a W^\pm pair, to four leptons of which three may be of same sign. From such signatures at the collider the strength of the coupling Y_{ij}^ν , which plays significant role in determining the neutrino mass, and also Y_{ij} may be estimated. On the other hand, if $m_{h^\pm} < M_{Rn}$ then one must consider $N_{Rj} \rightarrow h^\pm l_i^\mp$, if right-handed neutrinos are produced *via* their small mixing with ν_L [11]. The more important signal in this case will be through Drell-Yan pair production of h^\pm which will lead to two charged tracks with matching p_T since the decay $h^\pm \rightarrow l_i^\pm \nu_j$ is suppressed by $\sin^2 \kappa$. The above indicates that there is a possibility to cross-check the neutrino mass parameters from neutrino oscillation experiments and from collider signatures.

7 Summary

The main emphasis of our paper is twofold: (a) We identify the Dirac and right-handed Majorana neutrino mass textures which lead to one, two, or three massless neutrinos. (b) We demonstrate that one may not need a new high seesaw scale for small neutrino mass. Right-handed neutrino fields even at the electroweak scale and a scalar doublet with a $U(1)$ symmetry could accomplish this naturally. When the $U(1)$ symmetry is broken, one of the massless neutrinos acquires a mass at the one-loop level while the remaining two become massive when two-loop contributions are included. Since the two small mass splittings arise at different orders in perturbation theory their relative sizes can be reproduced. These ideas admit exploration at the LHC.

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