

Isospin symmetry breaking and baryon-isospin correlations from Polyakov–Nambu–Jona-Lasinio model

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We present a study of the 1+1 flavor system of strongly interacting matter in terms of the Polyakov–Nambu–Jona-Lasinio model. We find that though the small isospin symmetry breaking brought in through unequal light quark masses is too small to affect the thermodynamics of the system in general, it may have significant effect in baryon-isospin correlations and have a measurable impact in heavy-ion collision experiments.

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I. INTRODUCTION

Signatures of phases of matter with deconfined color charges is under critical investigation for last few decades, both theoretically and experimentally. Quantum Chromodynamics (QCD) is the formulation for first principle studies of strongly interacting matter. Along with the local color symmetry, the quark sector has few global symmetries also. In the chiral limit for two light flavors u and d , we have global vector and axial vector symmetry $SU_V(2) \otimes SU_A(2)$. For non-zero quark masses, the axial symmetry $SU_A(2)$ is explicitly broken, while for non-zero quark mass difference vector (isospin) symmetry $SU_V(2)$ is explicitly broken. At low energies the isospin symmetry breaking (ISB) has relevance in many aspects of hadronic observables [1]. Apart from the quark mass difference, ISB effects may be brought in by electromagnetic contributions as well. Low energy $\pi - K$ scattering has been studied considering the inclusion of electromagnetic correction into the effective Lagrangian [2]. In the chiral quark model, ISB of valence and sea quark distributions in protons and neutrons has been studied in [3, 4] and thermodynamics has been discussed in [5]. ISB may also have significant effect in the context of existence of CP violating phase [6]. Some Lattice QCD investigation of the effect of unequal quark masses was done in Ref.[7] and recently the effect of ISB on different hadronic observables were studied in Ref.[8, 9]. Within the framework of chiral perturbation theory the isospin breaking effect in quark condensates has been studied considering $m_u \neq m_d$ and electromagnetic corrections as well, where the authors have given an analysis of scalar susceptibilities [10, 11]. Both of the above-mentioned effects have been incorporated also in Nambu–Jona-Lasinio (NJL) model [12] to study the influence of the isospin symmetry breaking on the orientation of chiral symmetry breaking.

In the context of high energy heavy ion collisions where strongly interacting matter is supposed to exist in a state of thermal and chemical equilibrium, the ISB effects have not been explored much. Fluctuations and correlations of conserved charges are important and sensitive probes for heavy ion physics. Most of the theoretical studies in this respect are in isospin symmetric limit (see *e.g.* [13–29]). Here we present our first case study of ISB effect on fluctuations and correlations of strongly interacting matter within the framework of the Polyakov loop enhanced Nambu–Jona-Lasinio (PNJL) model. We discuss the possible experimental manifestations of the ISB effects based on quite general considerations in the limit of small current quark masses.

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II. FORMALISM

In the last few years PNJL model has appeared in several forms and context to study the various aspects of phases of strongly interacting matter (see e.g. [20, 23, 30–37]). Here we use the form of the 2 flavor PNJL model with the Lagrangian as in Ref.[22, 23];

$$\begin{aligned} \mathcal{L}_{PNJL} = & - \mathcal{U}[\Phi[A], \bar{\Phi}[A], T] + \bar{\psi}(\not{D} - \hat{m})\psi \\ & + G_1[(\bar{\psi}\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \\ & + G_2[(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \end{aligned} \quad (1)$$

$\mathcal{U}[\Phi[A], \bar{\Phi}[A], T]$ is the effective potential expressed in terms of traced Polyakov loop and its charge conjugate:

$$\Phi = \frac{\text{Tr}L}{N_c} \quad \bar{\Phi} = \frac{\text{Tr}L^\dagger}{N_c}$$

Although Φ and $\bar{\Phi}$ are complex valued fields, in the mean field approximation their expectation values are supposed to be real [38]. In all the previous studies the u and d quarks were considered to be degenerate. Here we shall consider a mass matrix of the form:

$$\begin{aligned} \hat{m} &= m_1 \mathbb{1}_{2 \times 2} - m_2 \tau_3 \\ &= \begin{pmatrix} m_1 - m_2 & 0 \\ 0 & m_1 + m_2 \end{pmatrix} \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}. \end{aligned}$$

where, $\mathbb{1}_{2 \times 2}$ is the identity matrix in flavor space and τ_3 is the third Pauli matrix. Here m_u and m_d are the current masses of the u and d quarks respectively. While a non-zero m_1 breaks the chiral $SU_A(2)$ symmetry explicitly a non-zero m_2 does the same for the isospin $SU_V(2)$ symmetry. We shall restrict ourselves to $G_1 = G_2 = G$ which implies $m_2 = (M_d - M_u)/2$, where M_u and M_d are the constituent masses of the u and d quarks respectively.

The thermodynamic potential in the mean field approximation is given by,

$$\begin{aligned} \Omega = & 2G_1(\sigma_u^2 + \sigma_d^2) + 4G_2\sigma_u\sigma_d + \mathcal{U}'[\Phi[A], \bar{\Phi}[A], T] - 6 \sum_{f=u,d} \int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_f \\ & - 2T \sum_{f=u,d} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln \left[1 + 3\Phi e^{-\frac{(E_f - \mu_f)}{T}} + 3\bar{\Phi} e^{-2\frac{(E_f - \mu_f)}{T}} + e^{-3\frac{(E_f - \mu_f)}{T}} \right] \\ & - 2T \sum_{f=u,d} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln \left[1 + 3\bar{\Phi} e^{-\frac{(E_f + \mu_f)}{T}} + 3\Phi e^{-2\frac{(E_f + \mu_f)}{T}} + e^{-3\frac{(E_f + \mu_f)}{T}} \right] \end{aligned} \quad (2)$$

where σ_u and σ_d are the condensates for u and d quarks respectively. Here $\mathcal{U}'[\Phi[A], \bar{\Phi}[A], T]$ is the modified Polyakov loop potential [23];

$$\frac{\mathcal{U}'}{T^4} = \frac{\mathcal{U}}{T^4} - \kappa \ln J[\Phi, \bar{\Phi}]$$

where $J[\Phi, \bar{\Phi}] = \frac{27}{24\pi^2} (1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2)$ is known as Vandermonde determinant and κ is a phenomenological parameter, taken to be 0.1 here.

The behavior of different charge susceptibilities can be studied from corresponding chemical potential derivatives of the pressure (P) obtained from the thermodynamic potential. In general the n^{th} order diagonal and off-diagonal susceptibilities are respectively given by, $\chi_n^X = \frac{\partial^n (P/T^4)}{\partial (\mu_X/T)^n}$ and $\chi_{ij}^{XY} = \frac{\partial^{i+j} (P/T^4)}{\partial (\mu_X/T)^i \partial (\mu_Y/T)^j}$ with $i + j = n$. These susceptibilities are related to the fluctuations and correlations of conserved charges X and Y with corresponding chemical potentials μ_X and μ_Y . Given a two flavor system the global charge conservation is expected for baryon number B , (third component of) isospin I_3 and electric charge Q . In the isospin symmetric limit one can easily check that the $B - I$ correlation vanishes exactly. For an explicit isospin symmetry breaking, this correlation may be non-zero. Therefore it is an interesting and important observable that we wish to study here.

III. RESULTS AND DISCUSSIONS

Here we consider the average quark mass $m_1 = (m_u + m_d)/2$ fixed at 0.0055 GeV and study the effect of ISB with three representative values of $m_2 = (m_d - m_u)/2$. The parameter set in the NJL sector has been determined separately for the different values of m_2 and the differences in the parameter values were found to be practically insignificant. The bulk thermodynamic properties of the system expressed through pressure, energy density, specific heat, speed of sound etc. did not show significant dependence on m_2 . Even the diagonal susceptibilities were almost identical to those at the isospin symmetric limit. However, interesting differences were observed for the off-diagonal susceptibilities in the $B - I$ sector. We first discuss the results for $\mu_B = 0$ and then move to finite μ_B .

A. Off-diagonal Susceptibilities for $\mu_B = 0$

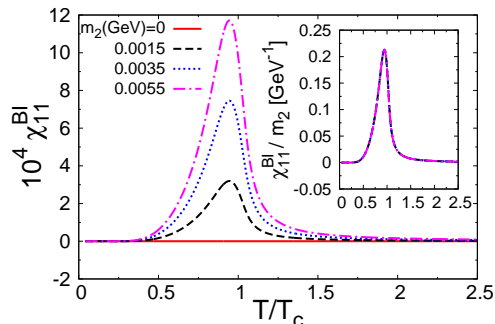


FIG. 1: (Color online) Second order off-diagonal susceptibility in $B - I$ sector at $\mu_B = 0$.

In Fig.1 the second order off-diagonal susceptibility χ_{11}^{BI} , is plotted against T/T_c for different values of m_2 . Here T_c is the crossover temperature obtained from the inflection point of the scalar order parameters - the mean values of chiral condensate and Polyakov Loop [21–23]. As expected we find $\chi_{11}^{BI} = 0$ for $m_2 = 0$. For non-zero m_2 we find χ_{11}^{BI} to have non-zero values that change non-monotonically with the increase in temperature. At low temperatures the excitations are suppressed due to large constituent masses as well as confining effects of the Polyakov loop. As the constituent masses and confining effects decrease with the increase in temperature, the correlations are enhanced. The peak value appears very close to T_c . Thereafter as the constituent masses become small with respect to the corresponding temperature, the correlation drops and approaches zero at very high temperatures.

The sensitivity of χ_{11}^{BI} on m_2 is clearly visible. An exciting feature observed here is that there is an almost linear scaling of χ_{11}^{BI} with m_2 . This is shown in the inset of Fig.1.

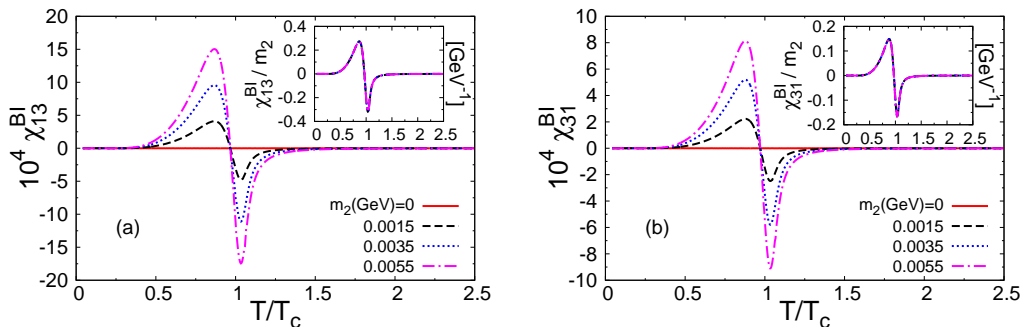


FIG. 2: (Color online) Behavior of 4th order off diagonal susceptibility for different m_2 .

The fourth order off-diagonal susceptibilities in the $B - I$ sector are χ_{13}^{BI} , χ_{31}^{BI} and χ_{22}^{BI} . The m_2 dependence of χ_{22}^{BI} was found to be insignificant. For $\mu_B = 0$, the T dependence for the other two susceptibilities along with their m_2 scaling is shown in Fig.2. The qualitative features of the variation of χ_{13}^{BI} and χ_{31}^{BI} with temperature may be understood by noting that these quantities are correlators between χ_{11}^{BI} with those of the isospin fluctuation χ_2^I and the baryon fluctuation χ_2^B respectively. In our earlier studies [21–23], we found that the both χ_2^I and χ_2^B increase

monotonically with increasing temperature. On the other hand χ_{11}^{BI} first increases up to $T \sim T_c$ and then decreases with increase in T , as shown in Fig.1. Therefore one expects that χ_{11}^{BI} has a positive correlation with χ_2^I and χ_2^B below T_c and is anti-correlated above T_c .

To understand the presence of m_2 scaling for some correlators and absence in others we first note that the different $B - I$ correlators may be expressed in terms of those in the flavor space. The corresponding relation between the chemical potentials are $\mu_u = \frac{1}{3}\mu_B + \frac{1}{2}\mu_I$ and $\mu_d = \frac{1}{3}\mu_B - \frac{1}{2}\mu_I$. This implies,

$$\chi_{11}^{BI} = \frac{1}{6}(\chi_2^u - \chi_2^d). \quad (3)$$

The flavor diagonal susceptibilities can be expanded in a Taylor series of the quark masses around $m_u = m_d = 0$.

$$\chi_2^f(m_u, m_d) = \sum_{n=0}^{\infty} \sum_{i=0}^n a_{i,j}^f m_u^i m_d^j \quad (4)$$

where, $a_{i,j}^f = \frac{1}{i!j!} \left[\frac{\partial^n \chi_2^f}{\partial m_u^i \partial m_d^j} \right]_{m_u=m_d=0}$ are the Taylor coefficients, with $i + j = n$ and $f \in u, d$. Here $a_{0,0}^u$ and $a_{0,0}^d$ are respectively u and d flavor susceptibilities in the chiral limit; hence they are equal. Moreover, response of χ_2^u to a change in m_u (m_d) and that of χ_2^d to a change in m_d (m_u) are identical in the chiral limit. Thus we have $a_{i,j}^u = a_{j,i}^d$, $\forall i, j$. Therefore we get,

$$\begin{aligned} & \chi_2^u(n^{th} \text{ order}) - \chi_2^d(n^{th} \text{ order}) \\ &= \sum_{i=0}^n \alpha_i m_u^i m_d^i (m_d^{n-2i} - m_u^{n-2i}). \end{aligned} \quad (5)$$

where $\alpha_i = a_{i,n-i}^u = a_{n-i,i}^d$. It is clear that for any given n and i , the R.H.S. contains a factor $(m_d - m_u)$. Therefore χ_{11}^{BI} (Eq.3) is proportional to m_2 if the terms for $n \geq 3$ are sub-dominant in Eq.4. This is what we observed for the range of m_2 considered here.

For the higher order correlators one can similarly write,

$$\chi_{13}^{BI} = \frac{1}{24}(\chi_4^u - \chi_4^d + 2\chi_{13}^{ud} - 2\chi_{31}^{ud}) \quad (6)$$

$$\chi_{31}^{BI} = \frac{1}{54}(\chi_4^u - \chi_4^d - 2\chi_{13}^{ud} + 2\chi_{31}^{ud}) \quad (7)$$

$$\chi_{22}^{BI} = \frac{1}{36}(\chi_4^u + \chi_4^d - 2\chi_{22}^{ud}) \quad (8)$$

For all these quantities the first two terms on R.H.S. were found to be dominant. Considering again the Taylor expansion of χ_4^f (but obviously with Taylor coefficients different from that of χ_2^f) in quark masses, χ_{13}^{BI} and χ_{31}^{BI} were found to be proportional to m_2 . Since χ_{22}^{BI} contains sum of fourth order flavor fluctuations instead of their difference, no m_2 scaling appeared in this case.

B. Off diagonal Susceptibilities for $\mu_B \neq 0$

In Fig.3, the variation of χ_{11}^{BI} with μ_B is shown for four different temperatures. The features vary widely over the different ranges of temperature and chemical potential. At $T \sim 2T_c$, χ_{11}^{BI} is positive, and slowly decreases with increasing μ_B . Close to T_c , χ_{11}^{BI} drops sharply to zero, becomes negative and then again slowly approaches zero. Going down somewhat below T_c there is an initial increase in χ_{11}^{BI} for some range of μ_B , and thereafter it follows the behavior at T_c . Finally at very low temperatures the change in sign of χ_{11}^{BI} is marked by a discontinuity, arising due to a first order phase boundary which exists in this range of T and μ_B .

These various features can be understood by expressing $\chi_{11}^{BI} = \frac{\partial}{\partial \mu_B} \left(\frac{\partial P}{\partial \mu_I} \right) = \left(\frac{\partial n_I}{\partial \mu_B} \right)$, where n_I is the isospin number density. It is worth noting that although we have considered $\mu_I = 0$ throughout the present study, a non-zero isospin number is generated due to non-vanishing m_2 . So let us study the behavior of isospin number density with changing baryon chemical potential. Now $n_I = (n_u - n_d)/2$, where n_u and n_d are respectively the u and d quark densities.

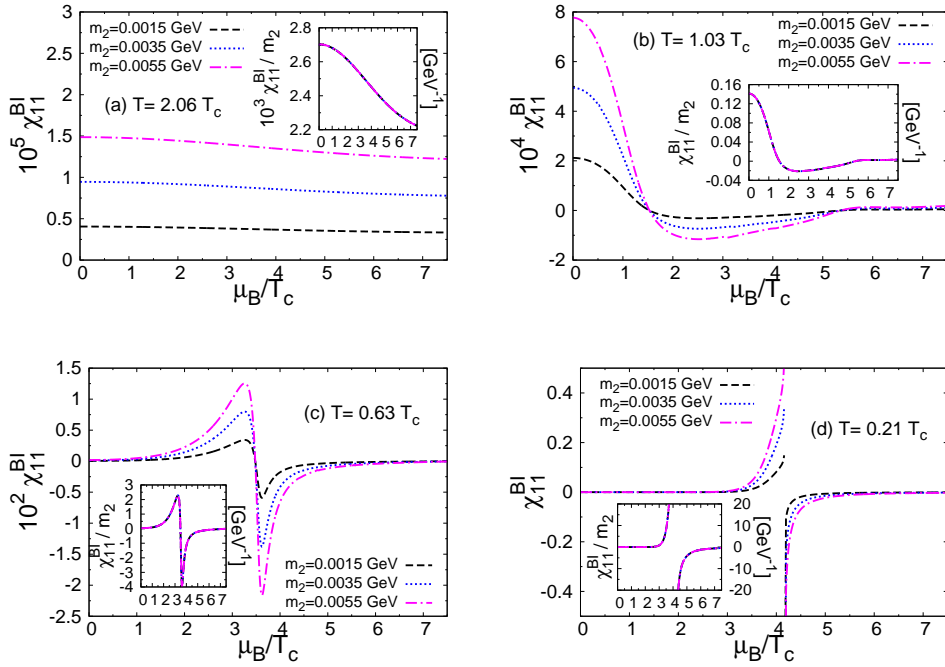


FIG. 3: (Color online) χ_{11}^{BI} along baryon chemical potential at different temperatures.

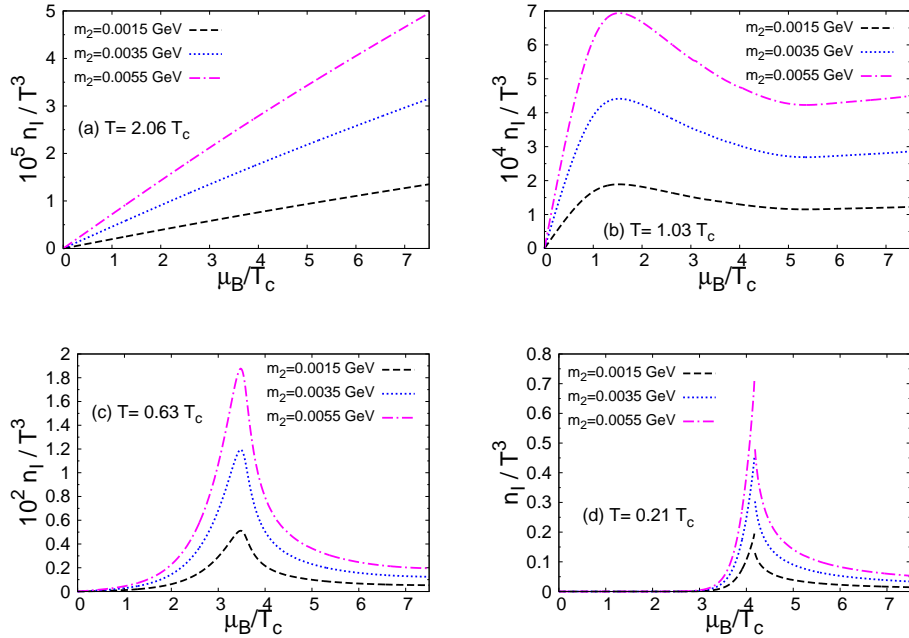


FIG. 4: (Color online) Isospin number density along μ_B at the same temperatures as in Fig.3.

The number density of a given flavor at a constant temperature is governed by the corresponding mass as well as the chemical potential. The isospin number should be positive as the u quark mass is smaller than that of d quark. With increase in baryon chemical potential this difference is expected to increase giving an increasing n_I . This expected feature is found to hold in the low temperatures for a range of μ_B as can be seen from Fig.4. However there is a subsequent drop in isospin number as the constituent quark masses suddenly start to fall beyond a critical μ_B , gradually becoming insignificant as the constituent masses reduce to the current mass values. The rise and fall of n_I explains the complete behavior of χ_{11}^{BI} for $T < T_c$. In fact the same explanation applies for the other two temperatures in the following way. Close to T_c the constituent masses of the quarks are again approaching the current mass values.

n_I is increasing with μ_B , but too slowly and therefore χ_{11}^{BI} , given by the slope, starts dropping. The latter part still follows the behavior of $T < T_c$. By $2T_c$ the current mass is almost achieved and n_I increases almost linearly with a very small slope with respect to μ_B . The corresponding χ_{11}^{BI} is positive and decreasing very slowly.

An amazing fact remains that the scaling of the correlators with m_2 survives for all conditions of T and μ_B . This is shown in the insets of Fig.3. A major implication is that all higher order derivatives of n_I with respect to μ_B would also show similar scaling behavior. This can be seen by expanding χ_{11}^{BI} in a Taylor series in μ_B about $\mu_B = 0$ as,

$$\chi_{11}^{BI}(\mu_B) = \chi_{11}^{BI}(0) + \frac{\mu_B^2}{2!} \chi_{31}^{BI}(0) + \frac{\mu_B^4}{4!} \chi_{51}^{BI}(0) + \dots \quad (9)$$

In the above series odd order terms vanish due to CP symmetry. Since $\chi_{11}^{BI}(\mu_B)$ on the L.H.S. scales with m_2 , the same can be expected to hold true individually for all the coefficients on the R.H.S. up to any arbitrary order. The first two Taylor coefficients have already been shown to follow the scaling relation in Fig.1 and Fig.2(right panel) respectively.

C. Further implications of ISB in Heavy Ion Collisions

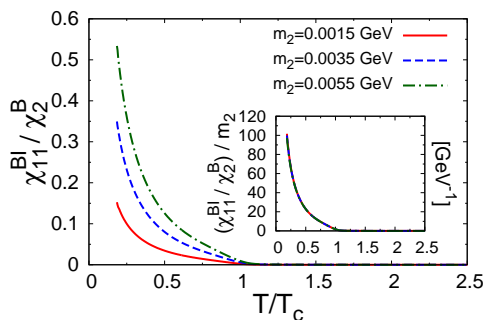


FIG. 5: (Color online) Ratio of B–I correlation to baryon number fluctuation at $\mu_B = 0$.

Correlation between conserved charges, is an experimentally measurable quantity obtained from event-by-event analysis in heavy-ion collisions [39]. To compare with experiments it is often useful to consider ratios such as $R_2 = \chi_{11}^{BI}/\chi_2^B = C_{BI}/C_{BB}$ [39, 40]. Here $C_{XY} = \frac{1}{N_E} \sum_{i=1}^{N_E} X_i Y_i - (\frac{1}{N_E} \sum_{i=1}^{N_E} X_i) \cdot (\frac{1}{N_E} \sum_{i=1}^{N_E} Y_i)$, where N_E is the total number of events considered and X_i and Y_i are the event variables corresponding to the conserved charges in a given event i . Ratios of this kind are practically useful in eliminating uncertainties in the estimates of the measured volume of the fireball. Relevance of similar ratios of fluctuations have also been discussed in the Lattice QCD framework [41, 42].

The temperature variation of R_2 obtained here is shown in Fig.5. It decreases monotonically and approaches zero above T_c . This is expected as the baryon number fluctuation increases much more rapidly than the $B - I$ correlation below T_c , and thereafter χ_{11}^{BI} goes to zero while χ_2^B attains a non-zero value.

Though not completely monotonic, R_2 goes down to extremely small values close to the phase/crossover boundary for $\mu_B \neq 0$. This is shown in Fig. 6. If freeze-out of the particles produced in heavy-ion collisions occurs very close to phase/crossover boundary after the system has passed through the partonic phase then R_2 will have very small values. A systematic study of this ratio can thus indicate how close one could approach the phase boundary in heavy-ion collisions. In fact a small negative value of R_2 for intermediate energy experiments where the temperature is supposed to be quite low would be an exciting indicator of a phase transition.

The m_2 scaling that we observed for χ_{11}^{BI} or R_2 is most likely model independent as it is expected on very general grounds for small current quark masses as discussed above. Therefore, at any temperature and chemical potential, one can use the m_2 scaling to estimate the mass asymmetry of constituent fermions in a physical system as,

$$m_2^{\text{expt}} = \frac{R_2^{\text{expt}}(T, \mu_B)}{R_2^{\text{th}}(T, \mu_B)} \times m_2^{\text{th}} \quad (10)$$

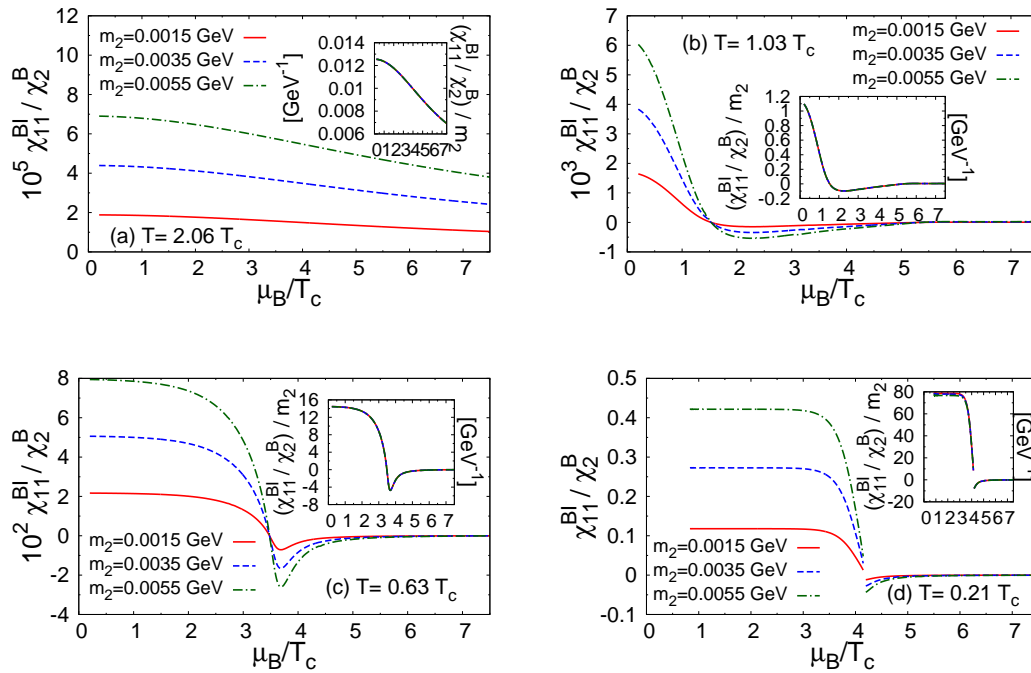


FIG. 6: (Color online) Ratio of B–I correlation to baryon number fluctuation along μ_B at the same temperatures as in Fig.3.

where, ‘expt’ and ‘th’ denotes the experimentally measured and theoretically calculated values of the corresponding quantities respectively. To the best of our knowledge this is the first theoretical attempt which indicates that quark mass asymmetry in thermodynamic equilibrium can be directly measured from heavy-ion collision experiments.

For the fourth order correlators, an important point to note is that for fractional baryon number of the constituents, $|\chi_{13}^{BI}|/|\chi_{31}^{BI}| > 1$. It is easy to check that the inequality is reversed for integral baryon number *i.e.* for protons and neutrons. However from Fig.2 we see that the former inequality persists well below T_c . This may well be an artifact of the PNJL model. Therefore it would be in principle interesting to crosscheck the corresponding results from Lattice QCD. Enhanced statistics of present and future experiments may make it possible to measure this extremely sensitive probe. The direction of the above inequality would be important in deciding if partonic matter may have been produced in the medium.

We expect that the measurement of these correlations in experiments pose a big challenge. Firstly, at low temperatures where R_2 is large, both the numerator and denominator are quite small, making the measurement difficult. Experimentally these fluctuations are measured from the cumulants of the multiplicity distribution at chemical freeze-out [43]. For example, around highest RHIC energy, particle ratios are expected to be frozen at $T \sim 0.170$ GeV and $\mu_B \sim 0.020$ GeV and for those values of temperature and baryon chemical potential χ_2^B has been measured very accurately. To measure χ_{11}^{BI} at same level of accuracy, assuming normally distributed population, naively the statistics needs to be increased by a factor of 10^6 w.r.t. the same for existing calculation in case of χ_2^B which seems to be difficult at the present stage. In view of this the situation is somewhat better at say $\sqrt{s_{NN}} \sim 8$ GeV, where the freezeout is expected for $T \sim 0.140$ GeV and $\mu_B \sim 0.420$ GeV. In this case absolute values of both the numerator and denominator of R_2 are well within the measurable regime for future experimental facilities like BES-II in RHIC and CBM in FAIR which will have fairly high statistics for low energy runs and possibly can overcome this problem.

The other experimental challenge is the detection and measurement of neutrons which along with the protons are supposed to be the highest contributor to the baryon-isospin correlations. Incidentally the Large Area Neutron Detector facility has already been developed at GSI, Darmstadt, Germany, where one can measure neutron properties in heavy-ion collision experiments up to an incident energy of 1 GeV per nucleon [44–46]. Hopefully with further developments in detection technology, relevant data for neutrons may be available for higher collision energies in near future.

At this point it seems relevant to mention that the issue of neutron detection has arisen earlier even for the measurement of baryon fluctuation itself. In case of baryon number cumulants, methods are given [47–50] to reconstruct and estimate the effect of unobserved neutrons as well as other effects like finite acceptance and global conservation of baryon number. In Ref.[47, 48] the key ingredient is the observation that due to some late stage processes the isospins of different nucleon species are almost uncorrelated which makes it possible to write the actual baryon number cumu-

lants in terms of the observed proton number cumulants. It is argued that for low values of $\sqrt{s_{NN}}$ this randomization of isospin is not favored and neutron and proton number distribution will not be factorized in the final state due to the existence of primordial isospin correlation. This is precisely what we observe in our framework. From Fig.1 and Fig.3 it can be easily seen that as we go down by $\sqrt{s_{NN}}$ (*i.e.* decreasing T and increasing μ_B), correlation through isospin between different baryon species, *i.e.* χ_{11}^{BI} will increase. Therefore a direct measurement of neutrons is desirable even for measuring baryon number fluctuations at low energies apart from the baryon-isospin correlations.

Another question that still remains is whether the isospin asymmetry brought in through nonzero electric charge may disturb the scaling and inclusion of this QED effect will be another complete study in itself and will be reported later.

IV. CONCLUSION

In the present paper we have investigated the effect of isospin symmetry breaking through the unequal masses of u and d flavor. The work is done within the framework of 1+1 flavor PNJL model. The main result found is the observation that the off-diagonal susceptibilities in the $B-I$ sector depend on a small current quark-mass difference, whereas the bulk thermodynamic properties of the system (pressure, energy density, specific heat, speed of sound) do not show such dependence. The relevance of conserved charge fluctuations to the study of the transition region of strongly interacting matter is unquestionable. We showed that the $B-I$ correlations may give important information of the state of matter created in the heavy-ion collision experiments. Whereas the correlation remains positive for small μ_B , it may become negative in the high μ_B partonic phase. The change of sign of the correlation seems to be completely model independent.

Also the typical scaling behavior of these correlations with the quark mass difference m_2 has been argued to be model independent as long as the current masses are small. This scaling may enable one to estimate the quark mass asymmetry in heavy-ion experiments.

Another model independent observable that we discussed is for the fourth order correlators. Depending on whether the ratio $|\chi_{13}^{BI}|/|\chi_{31}^{BI}|$ is greater than 1 or not, one may infer if a partonic phase has been created and survived till freeze-out in the heavy-ion experiments. In our study the ratio was always found to be greater than 1, which may be model artifact.

A physically more realistic scenario of course requires the incorporation of strange quarks and/or QED effects which will be reported elsewhere.

The experimental observation of baryon-isospin correlation is a challenging job. Hopefully the future relativistic heavy-ion experiments with appropriate neutron detectors and high statistics data would be able to address this important issue.

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- [1] A. Rusetsky, PoS CD **09**, 071 (2009) [arXiv:0910.5151 [hep-ph]].
 - [2] A. Nehme, P. Talavera, Phys. Rev. D**65**, 054023 (2002).
 - [3] B.-Q. Ma, Phys. Lett. B**274**, 111 (1992).
 - [4] H. Song, X. Zhang, B.-Q. Ma, Phys. Rev. D**82**, 113011 (2010).
 - [5] E. S. Fraga, L. F. Palhares and C. Villavicencio, Phys. Rev. D**79**, 014021 (2009); L. F. Palhares, E. S. Fraga and C. Villavicencio, Nucl. Phys. A**820**, 287c (2009).
 - [6] M. Creutz, PoS LAT **2005**, 119 (2006) [hep-lat/0508012].
 - [7] R. V. Gavai and S. Gupta, Phys. Rev. D**66**, 094510 (2002).
 - [8] G.M. de Divitiis *et al* [RM123 collaboration], JHEP **1204**, 124 (2012).
 - [9] R. Horsley, J. Najjar, Y. Nakamura, D. Pleiter, P.E.L. Rakow, G. Schierholz and J.M. Zanotti [QCDSF-UKQCD Collaboration], Phys. Rev. D**86**, 114511 (2012).
 - [10] A. G. Nicola, R. T. Andrés, Phys. Rev. D**83**, 076005 (2011).
 - [11] A. G. Nicola, R. T. Andrés, J. Phys. G:Nucl. Part. Phys.**39** 015004 (2012).
 - [12] T. Fujihara, T. Inagaki and D. Kimura, Prog. Theor. Phys. **117** 139 (2007).

- [13] S. Gottlieb, W. Liu, D. Toussaint, R. L. Renken and R. L. Sugar, Phys. Rev. Lett. **59**, 2247 (1987).
- [14] R.V. Gavai and S. Gupta, Phys. Rev. D**73**, 014004 (2006).
- [15] C. R. Alton *et. al.*, Phys. Rev. D**71**, 054508 (2007).
- [16] J. -P. Blaizot, E. Iancu, A. Rebhan, Phys. Lett. B**523** 143 (2001).
- [17] P. Chakraborty, M.G. Mustafa, M.H. Thoma, Eur. Phys. J. C**23**, 591 (2002).
- [18] P. Chakraborty, M.G. Mustafa, M.H. Thoma, Phys. Rev. D**68**, 085012 (2003).
- [19] C. Sasaki, B. Friman, and K. Redlich, Phys. Rev. D**75**, 054026 (2007).
- [20] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D**73**, 014019 (2006).
- [21] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa and R. Ray, Phys. Rev. D**73**, 114007 (2006).
- [22] S. Mukherjee, M. G. Mustafa and R. Ray; Phys. Rev. D**75**, 094015, (2007).
- [23] S.K. Ghosh, T.K. Mukherjee, M.G. Mustafa and R. Ray; Phys. Rev. D**77**, 094024, (2008).
- [24] K. Fukushima, Phys. Rev. D**77**, 114028, (2008).
- [25] B.-J. Schaefer, J. M. Pawłowski and J. Wambach, Phys. Rev. D **76** 074023 (2007); B.-J. Schaefer, M. Wagner and J. Wambach, Phys. Rev. D **81**, 074013 (2010); B.-J. Schaefer and M. Wagner, Phys.Rev. D**85** (2012) 034027.
- [26] A. Bhattacharyya, P. Deb, A. Lahiri, R. Ray, Phys. Rev. D**82**, 114028 (2010).
- [27] A. Bhattacharyya, P. Deb, A. Lahiri, R. Ray, Phys. Rev. D**83**, 014011 (2011).
- [28] N. Haque, M. G. Mustafa, M. H. Thoma, Phys. Rev. D**84**, 054009 (2010).
- [29] S. Chatterjee and K. A. Mohan, Phys. Rev. D**86**, 114021 (2012).
- [30] P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B **379** 163 (1996); Nucl. Phys. B (Proc. Suppl.) 47 519 (1996).
- [31] K. Fukushima, Phys. Lett. B **591** 277 (2004).
- [32] E. Megias, E. R. Arriola, and L. L. Salcedo, Phys. Rev. D **74**, 065005 (2006); **74**, 114014 (2006); J. High Energy Phys. 01 (2006) 073.
- [33] H. Ueda, T. Z. Nakano, A. Ohnishi, M. Ruggieri and K. Sumiyoshi, Phys. Rev. D**88**, 074006 (2013)
- [34] A. Bhattacharyya, S. K. Ghosh, S. Majumder and R. Ray, Phys. Rev. D**86**, 096006 (2012).
- [35] H. Abuki, M. Ciminale, R. Gatto, N. D. Ippolito, G. Nardulli and M. Ruggieri, Phys. Rev. D **78**, 014002 (2008).
- [36] H. Abuki, R. Anglani, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Rev. D**78**, 034034 (2008).
- [37] A. Ohnishi, H. Ueda, T. Z. Nakano, M. Ruggieri and K. Sumiyoshi, Phys. Lett. B **704**, 284 (2011).
- [38] A. Dumitru, R. D. Pisarski and D. Zschesche, Phys. Rev. D**72**, 065008 (2005).
- [39] S. Jeon and V. Koch, in Quark Gluon Plasma 3, Ed. R.C. Hwa and X. N. Wang, pg. 430 (World Scientific, Singapore, 2004), (arXiv:hep-ph/0304012).
- [40] M. M. Aggarwal *et. al.*, Phys. Rev. Lett. **105**, 022302 (2010).
- [41] S. Gupta, X. Luo, B. Mohanty, H. G. Ritter and N. Xu, Science **332**, 1525 (2011).
- [42] A. Bazavov *et. al.*, Phys. Rev. Lett. **109**, 192302 (2012).
- [43] L. Adamczyk *et al.* [STAR Collaboration], Phys. Rev. Lett. **112**, 032302 (2014).
- [44] Th. Blaich *et. al.* [LAND Collaboration], Nucl. Instrum. Meth. A**314**, 136 (1992).
- [45] P. Pawłowski *et. al.* [LAND Collaboration], Nucl. Instrum. Meth. A**694**, 47 (2012).
- [46] P. Russotto *et. al.*, Eur. Phys. J. A**50**, 38 (2014).
- [47] M. Kitazawa and M. Asakawa, Phys. Rev. C**85**, 021901(R) (2012).
- [48] M. Kitazawa and M. Asakawa, Phys. Rev. C**86**, 024904 (2012); Phys. Rev. C**86**, 069902(E) (2012).
- [49] A. Bzdak and V. Koch, Phys. Rev. C**86**, 044904 (2012).
- [50] A. Bzdak, V. Koch and V. Skokov, Phys. Rev. C**87**, 014901 (2013).