Investigating Perturbative Unitarity in Presence of Anomalous Couplings

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We perform a model independent analysis of the helicity amplitudes at high energy for all the $2 \to 2$ scattering processes involving gauge and Higgs bosons in the presence of anomalous WWV, $WWVV$, VVH, $VVHH$ ($V \equiv Z, \gamma$ and $W^{\pm}$), $HHHH$ and HHH interactions. We obtain the perturbative unitarity constraints on anomalous couplings by demanding the vanishing of terms proportional to s^2 and $s^{3/2}$ in the helicity amplitudes. Using these constraints, we also compute the upper bound on all the anomalous couplings from terms linear in s.

Further, assuming all anomalous couplings to have arisen only from dimension six operators, we show that the perturbative unitarity violation can be evaded up to ~ 9 TeV corresponding to the best fit values of f_{WW}/Λ^2 and f_{BB}/Λ^2 from the combined analysis of Tevatron and LHC data.

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I. INTRODUCTION

With the discovery of a new massive ($\sim$ 125 GeV) scalar particle by both ATLAS and CMS [1] and with most of the observations and consistency checks indicating that the new particle has a large overlap with the Higgs boson of the Standard Model (SM) [2], it is now possible to directly explore the fabric of the electroweak symmetry breaking mechanism. In order to find out how the $SU(2)_L \times U(1)_Y$ symmetry is broken in nature, one needs to measure precisely the strength of the self-interactions of the Higgs boson and its interactions with the gauge bosons as well as the fermions. Although there are enough reasons to expect the presence of new physics beyond SM, a good agreement of the SM predictions with the experiments so far imply that any new model must reduce to the SM at low energies. Thus even if the SM is only a low energy effective theory valid upto some energy scale Λ, the observation of any departure from the SM predicted values of the interaction strengths in the bosonic sector can give hints of new physics (NP). The specific form of the NP which will supersede the SM is not yet known. However, a model independent approach can be adopted either to observe the signatures of the NP spectrum, if any, though too heavy to be produced in the present colliders, or realize their effects in the precision measurements of the novel interactions among the known SM particles *via* loop corrections. There are two conceptual model independent prescriptions. One approach to an effective field theory is to extend the theory by adding higher dimensional operators constructed out of the SM fields. The other approach is by writing Lagrangian containing all possible Lorentz structures that can contribute to a given process but with fewest number of derivatives [3, 4].

With the present data, we can safely assume that the observed new scalar state belongs indeed to a light electroweak doublet scalar and that the $SU(2)_L \times U(1)_Y$ symmetry is linearly realized in the effective theory. The effect of any NP at energy below the cut-off scale can be parametrized as effective interactions in a theory whose particle content is the same as in the SM [3]. In the effective field theory (EFT) framework [4], operators constructed out of the SM fields and of dimension higher than four are added to the SM Lagrangian[1]. These higher dimensional operators are suppressed by appropriate powers of the cut-off scale Λ.

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{n>4} \sum_{i} \frac{f_i^{(n)}}{\Lambda^{(n-4)}} \mathcal{O}_i^{(n)}, \tag{1}$$

where $\mathcal{L}_{\rm SM}$ denotes the renormalizable SM Lagrangian and $\mathcal{O}_i^{(n)}$s are the gauge invariant operators of mass dimension n. The index i runs over all operators (consistent with the symmetries of the SM) of the given mass dimension, and the coefficients $f_i^{(n)}$ are dimensionless parameters, which are determined once the full theory is known. The scale Λ can be regarded as the scale of new physics and is large compared with the experimentally-accessible energies. Thus the dominant extended operators will be those of the lowest dimensionality and hence we will be concerned with the dimension six operators in this article. Recently attempt has been made to constrain the coefficients of dimension six operators with the Higgs boson data from LHC in references [5, 6].

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[1] In the SM Lagrangian, all operators are restricted to be of mass dimension four or less.

In the anomalous coupling approach, the most general effective Lagrangian is written assuming Lorentz invariance, Bose symmetry and $SU(3)_C$ and electromagnetic gauge invariance. This might contain additional Lorentz structures that are not present in the SM Lagrangian. The deviation from the SM value of the coefficient corresponding to a given Lorentz structure induces an anomalous coupling, which has no inherent scale dependence. Usually, an effective Lagrangian contains all possible Lorentz structures, each constructed with the fewest number of derivatives. One can construct an infinite number of additional terms by adding derivatives [7] but to be conservative, one retains the terms containing least number of derivatives. However, with constant anomalous couplings, unitarity is broken at some scale and to circumvent this problem sometimes arbitrary momentum dependent form factors are introduced in the vertex. This is the momentum space analogue of the infinite number of terms in the Lagrangian approach that can be constructed by including more derivatives.

Thus, the effective Lagrangian in both approaches contain infinite number of terms and if the entire series are considered, both approaches would be equivalent.

We attempt to address the problem of preserving perturbative unitarity of all $2 \to 2$ scattering processes in the gauge and the Higgs boson sector in the presence of anomalous couplings. We shall also relate the analysis with the dimension six operators.

With just the gauge sector Feynman diagrams, the VV scattering amplitudes within the SM grow with energy and eventually violate unitarity. If the symmetry breaking is due to a light Higgs boson, the Higgs Mechanism removes this famous bad high energy behaviour and restores unitarity. The nuances of non-Abelian gauge structure of SM ensures the cancellation of order s^2 terms ($\sqrt{s}$ being the centre of mass energy) among the gauge mediated diagrams while order s terms cancel among the gauge and Higgs boson mediated diagrams and hence the perturbative unitarity is preserved. Thus any appreciable deviation in the s dependence of the scattering amplitudes from that within the SM provides a rather sensitive test of the anomalous couplings in high energy VV, VH or HH scattering experiments.

Recently the authors of reference [8] performed a similar analysis as ourselves after the preliminary version of our article had appeared in the arXiv [9]. Taking a cue from the study of reference [10] they dropped the most dominant helicity amplitudes which are either proportional to s^2 or $s^{3/2}$ assuming that these terms will be automatically cancelled by demanding $SU(2)_L \times U(1)_Y$ gauge invariant sum rules and consequently arrived at the perturbative unitarity conditions with terms linearly proportional to s only. However, it is important to note that the sum rules derived in reference [10] for Higgsless models with infinite KK modes and also in recently reviewed article on the bulk Higgs models in reference [11] are neither valid for the SM nor for the SM with finite number of light Higgs models. As a consequence, we observe that, a priori, these sum rules do not hold true for dimension six operators involving light Higgs bosons irrespective of whether they are $SU(2)_L \times U(1)_Y$ gauge invariant or not. Thus, the helicity amplitudes that grow with the centre of mass energy as s^n ($n \geq 0$) do not get cancelled automatically.

In this article, we investigate the high energy behaviour of the scattering amplitudes for the following sixteen distinct scattering processes: $W^+W^- \to W^+W^-$, $W^+W^- \to Z(\gamma)Z(\gamma)$, $W^+W^- \to Z\gamma$, $ZZ \to Z(\gamma)Z(\gamma)$, $Z(\gamma)Z(\gamma) \to Z\gamma$, $\gamma\gamma \to \gamma\gamma$, $W^+W^- \to Z(\gamma)H$, $W^+W^- \to HH$, $Z(\gamma)Z(\gamma) \to HH$, $Z\gamma \to HH$, $HH \to HH$ in the presence of the anomalous trilinear gauge, quartic gauge, the Higgs–gauge boson and Higgs self couplings[2].

In particular, we ask if it is possible to preserve perturbative unitarity even in the presence of anomalous couplings in the gauge and Higgs sector and determine the values of these anomalous couplings allowed by unitarity constraints for all the scattering processes considered. Vector boson scattering has drawn a lot of attention earlier and many works exist in the literature that discuss the gauge boson scattering and unitarity problems [12]. The anomalous gauge and gauge-Higgs couplings and their limits in various models as well as in model-independent approach have also been studied in various papers [13–21].

The rest of the paper is organised as follows: We provide the framework of our calculations in Section II. In Section III we discuss the high energy behaviour of the partial wave amplitudes for various scattering processes mentioned above and obtain the unitarity constraints on the linear combination of anomalous couplings. In Section IV, we relate all the anomalous couplings to the coefficients of dimension six operators. A summary of our results is given in Section VI. The notation of our calculations are given in the Appendix A.

II. FORMALISM–ANOMALOUS COUPLINGS

Within the SM, the interactions among the bosons of the electroweak theory are determined entirely by the gauge symmetry. Any deviations from the SM couplings are, therefore, evidences of new physics. As mentioned in previous Section, these deviations from the SM predictions may be parametrized in a model independent way in terms of effective Lagrangian. The terms of the effective Lagrangian relevant for the processes considered by

[2] Note that the helicity amplitudes of other scattering processes such as $W^\pm W^\pm \to W^\pm W^\pm$, $W^\pm Z(\gamma) \to W^\pm Z(\gamma)$ and $W^\pm Z(\gamma) \to W^\pm \gamma(Z)$ are related to the ones mentioned here by crossing symmetry. Thus the high energy behaviour of these follow the same suit as for the processes analysed in the article.

us may be written as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{WWV} + \mathcal{L}_{\rm eff}^{WWVV'} + \mathcal{L}_{\rm eff}^{V_1V_2H} + \mathcal{L}_{\rm eff}^{V_1V_2HH} + \mathcal{L}_{\rm eff}^{H^3} + \mathcal{L}_{\rm eff}^{H^4}, \quad (2)$$

where $\mathcal{L}_{\rm eff}^{WWV}$ and $\mathcal{L}_{\rm eff}^{WWVV'}$ gives rise to triple gauge couplings (TGC) and quartic gauge couplings (QGC) involving two W bosons respectively. $\mathcal{L}_{\rm eff}^{V_1V_2H}$ and $\mathcal{L}_{\rm eff}^{V_1V_2HH}$ lead to anomalous vertices involving the Higgs boson and electroweak gauge bosons while $\mathcal{L}_{\rm eff}^{H^3}$ and $\mathcal{L}_{\rm eff}^{H^4}$ respectively generate cubic and quartic Higgs self couplings. Note that we are not considering effective Lagrangian involving only (three or four) neutral gauge bosons as these are absent at tree level in SM and also (in anticipation) because these are not generated by dimension six operators as we shall see in Section IV.

Restricting our study to CP-even vertices only, the triple gauge vertices involving two W bosons can be parametrized as [7]

$$\mathcal{L}_{\rm eff}^{WWV} = {\rm i} g_{WWV}\Big[g_1^V\left(W^+_{\mu\nu}W^{-\mu}V^\nu - W^+_\mu V_\nu W^{-\mu\nu}\right) + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{m_W^2} W^-_{\mu\nu}W^{+\nu\rho}V^\mu_\rho\Big], \quad (3)$$

with $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. Also, $g_{WW\gamma} = -e = -g\sin\theta_W$ and $g_{WWZ} = -e\cot\theta_W = -g\cos\theta_W$, θ_W being the weak mixing angle. In the SM, at tree level, $g_1^V = \kappa_V = 1$ and $\lambda_V = 0$. Writing each TGC as sum of the SM part and the anomalous part, these vertices involve six C and P conserving anomalous couplings but demanding the electromagnetic gauge invariance requires that $g_1^\gamma = 1$ leaving five anomalous TGCs, namely $\Delta g_1^Z \equiv g_1^Z - g_{1,SM}^Z$, $\Delta\kappa_\gamma \equiv \kappa_\gamma - \kappa_{\gamma,SM}$, $\Delta\kappa_Z \equiv \kappa_Z - \kappa_{Z,SM}$, λ_γ and λ_Z . The constraints on the TGC from LEP [22] and LHC [23] are obtained by assuming the relations

$$\lambda_Z = \lambda_\gamma = \lambda, \quad (4)$$

$$\Delta\kappa_Z = \Delta g_1^Z - \Delta\kappa_\gamma \tan^2\theta_W, \quad (5)$$

We shall also assume these constraints to be valid in our analysis in next Section.

The effective interactions of four electroweak gauge bosons (QGC), may be parametrized in terms of two Lorentz invariant structures, given by the Lagrangian [24]

$$\mathcal{L}_{\rm eff}^{WWVV'} = c_0^{VV'}\mathcal{O}_0^{VV'} - c_1^{VV'}\mathcal{O}_1^{VV'}, \quad (6)$$

where

$$\begin{aligned}
\mathcal{O}_0^{WW} &= \frac{1}{2}g^{\alpha\gamma}g^{\beta\delta}[W^+_\alpha W^+_\beta W^-_\gamma W^-_\delta],\\
\mathcal{O}_1^{WW} &= \frac{1}{2}g^{\alpha\beta}g^{\gamma\delta}[W^+_\alpha W^+_\beta W^-_\gamma W^-_\delta],\\
\mathcal{O}_0^{VV} &= g^{\alpha\gamma}g^{\beta\delta}[W^+_\alpha W^-_\beta V_\gamma V_\delta];\ V = Z/\gamma,\\
\mathcal{O}_1^{VV} &= g^{\alpha\beta}g^{\gamma\delta}[W^+_\alpha W^-_\beta V_\gamma V_\delta];\ V = Z/\gamma,\\
\mathcal{O}_0^{Z\gamma} &= (g^{\alpha\gamma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\gamma})[W^+_\alpha W^-_\beta Z_\gamma A_\delta],\\
\mathcal{O}_1^{Z\gamma} &= 2g^{\alpha\beta}g^{\gamma\delta}[W^+_\alpha W^-_\beta Z_\gamma A_\delta].
\end{aligned} \quad (7)$$

The SM tree level quartic couplings associated with these Lorentz structures are given as

$$\begin{aligned}
-c_{0,SM}^{WW} &= -c_{1,SM}^{WW} = c_{0,SM}^{ZZ}/\cos^2\theta_W = c_{1,SM}^{ZZ}/\cos^2\theta_W\\
&= c_{0,SM}^{\gamma\gamma}/\sin^2\theta_W = c_{1,SM}^{\gamma\gamma}/\sin^2\theta_W\\
&= c_{0,SM}^{Z\gamma}/(\cos\theta_W\sin\theta_W) = c_{1,SM}^{Z\gamma}/(\cos\theta_W\sin\theta_W)\\
&= g^2.
\end{aligned} \quad (8)$$

Accordingly we can define eight anomalous quartic gauge couplings (AQGC) as $\Delta c_i^{VV'} \equiv c_i^{VV'} - c_{i,SM}^{VV'}$ with $i = 0, 1$, corresponding to all Lorentz structures listed in (7). It is to be noted that, to restrict our parameter space, we are not considering the Lorentz structures involving derivatives of the gauge fields. Further, we shall not consider AQGC $\Delta c_0^{\gamma\gamma}$ and $\Delta c_1^{\gamma\gamma}$ which means that we assume the couplings of two photon fields with two the W fields to be same as in the SM. The reason will be clear in Section IV when we see that these are not generated by the dimension six operators considered by us.

Similarly, demanding only Lorentz invariance, the most general form of CP-even coupling between a pair of gauge bosons (V_1 and V_2) and the Higgs boson is given by [25]

$$\left(\Gamma_{\rm eff}^{V_1V_2H}\right)_{\mu\nu} = g m_W\left[a_1^{V_1V_2H}g_{\mu\nu} + \frac{a_2^{V_1V_2H}}{m_Z^2}\left(p_{2\mu}p_{1\nu} - g_{\mu\nu}\,p_1\cdot p_2\right)\right]. \quad (9)$$

where V_1V_2H corresponds to $\gamma\gamma H$, $Z\gamma H$, ZZH and W^+W^-H vertices.

Identically the most general Lorentz invariant CP-even couplings between a pair of gauge bosons (V_1 and V_2) and a pair of Higgs bosons may be parametrised as

$$\left(\Gamma_{\rm eff}^{V_1V_2HH}\right)_{\mu\nu} = \frac{g^2}{2}\left[a_1^{V_1V_2HH}g_{\mu\nu} + \frac{a_2^{V_1V_2HH}}{m_Z^2}\left(p_{2\mu}p_{1\nu} - g_{\mu\nu}\,p_1\cdot p_2\right)\right] \quad (10)$$

In (9) and (10), p_1, p_2 are the incoming momenta of the two gauge bosons. At tree level in SM, we have

$$\begin{aligned}
a_{1,\,\rm SM}^{ZZH} &= a_{1,\,\rm SM}^{ZZHH} = \sec^2\theta_W/2,\\
a_{1,\,\rm SM}^{WWH} &= a_{1,\,\rm SM}^{WWHH} = 1,\\
a_{1,\,\rm SM}^{Z\gamma H} &= a_{1,\,\rm SM}^{\gamma\gamma H} = a_{1,\,\rm SM}^{Z\gamma HH} = a_{1,\,\rm SM}^{\gamma\gamma HH} = 0\,,\\
\text{and}\quad a_{2,\,\rm SM}^{V_1V_2H} &= a_{2,\,\rm SM}^{V_1V_2HH} = 0\,.
\end{aligned} \quad (11)$$

Thus, writing $a_1^{V_1V_2H(H)} = a_{1,\,\rm SM}^{V_1V_2H(H)}(1 + \Delta a_1^{V_1V_2H(H)})$, we have eight anomalous VVH (four $\Delta a_1^{V_1V_2H}$ and four $a_2^{V_1V_2H}$) couplings and similarly eight V_1V_2HH couplings. However, the present and low energy data indicates that the effective Lagrangian should better preserve the SM gauge symmetries, which then requires that the tree level SM prediction for VVH and $VVHH$ may not be modified. Hence we take

$$\Delta a_1^{ZZH} = \Delta a_1^{WWH} = \Delta a_1^{\gamma\gamma H} = \Delta a_1^{Z\gamma\ H} = 0, \quad (12)$$

$$\Delta a_1^{ZZHH} = \Delta a_1^{WWHH} = \Delta a_1^{\gamma\gamma HH} = \Delta a_1^{Z\gamma HH} = 0. \quad (13)$$

reducing the total number of anomalous VVH and $VVHH$ couplings to eight.

The Higgs boson self coupling measurements and their deviations from the SM expectations provide the hint of alternative scenarios of symmetry breaking with light Higgs boson. These interactions can be parametrised as follows [26]

$$\mathcal{L}_{\rm eff}^{H^3} = -\frac{m_H^2}{2v}\left[\left(1+\Delta b_1^{H^3}\right)H^3 - 3\frac{b_2^{H^3}}{m_H^2}H(\partial_\mu H)(\partial^\mu H)\right], \quad (14)$$

$$\mathcal{L}_{\rm eff}^{H^4} = -\frac{m_H^2}{8v^2}\left[\left(1+\Delta b_1^{H^4}\right)H^4 - 6\frac{b_2^{H^4}}{m_H^2}H^2(\partial_\mu H)(\partial^\mu H)\right]. \quad (15)$$

In this parametrisation, there are four anomalous Higgs boson self couplings, namely, $\Delta b_1^{H^3}$, $\Delta b_1^{H^4}$, $b_2^{H^3}$and $b_2^{H^4}$all of which are zero in the SM at the tree level.

Considering all anomalous couplings to be constant and perturbative unitarity be preserved upto a given energy scale, we attempt to compute the upper bound on all the anomalous couplings discussed in this Section. Since we assume the Higgs boson to be SM-like, all couplings at tree level are assumed to be close to their SM values.

III. PARTIAL WAVE ANALYSIS

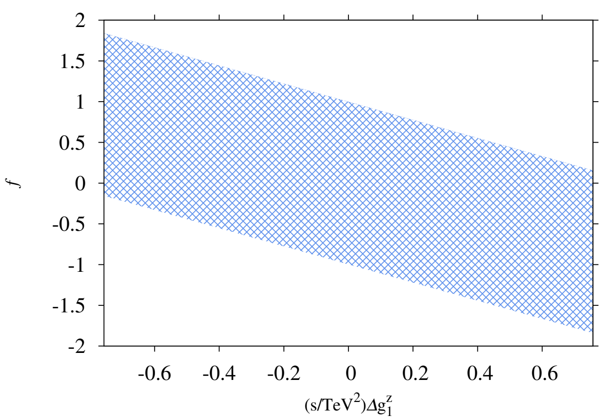

FIG. 1: *The shaded enclosed region on the plane of f and Δg_1^Z corresponds to the constraints from $\mathcal{A}^0_{0,0,0,0}(WW \to ZZ)$, $\mathcal{A}^0_{0,0,\pm,\pm}(ZZ \to Z\gamma)$ and $\mathcal{A}^0_{0,0,\pm,\pm}(WW \to Z\gamma)$.*

Partial wave analysis of scattering processes is one of the often used methods to constrain unknown parameters in a theory [12]. For a given $2 \to 2$ scattering process $a(p_a,\,\lambda_a)+b(p_b,\,\lambda_b) \to c(p_c,\,\lambda_c)+d(p_d,\,\lambda_d)$, the invariant transition amplitude $\mathcal{M}_{fi}$ can be decomposed in terms of partial wave amplitudes $\mathcal{A}^J_{\lambda_a\,\lambda_b\,\lambda_c\,\lambda_d}(s)$ as [27]

$$\mathcal{M}_{fi}(s,\,\Omega) = 16\pi\sum_J (2J+1)\mathcal{A}^J_{\lambda_a\,\lambda_b\,\lambda_c\,\lambda_d}(s)D^{J*}_{\lambda\lambda'}(\phi,\theta,0) \quad (16)$$

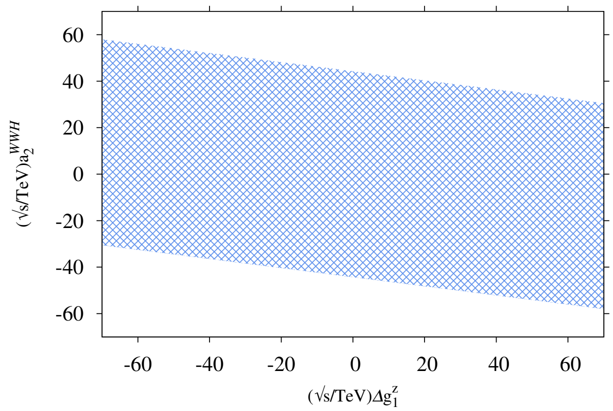

FIG. 2: *The shaded region is an infinite band on the $(\sqrt{s}a_2^{\rm WWH} - \sqrt{s}\Delta g_1^{\rm Z})-$ plane constrained by inequation given in (27).*

where $\lambda = \lambda_a - \lambda_b$, $\lambda' = \lambda_c - \lambda_d$, and $\Omega \equiv (\theta,\phi)$ is the solid angle. $D^J_{\lambda\lambda'}$ is the standard rotation matrix and we have chosen (in the c.m. frame), $\vec{p}_a = -\vec{p}_b = \vec{p}_i = |\vec{p}|\hat{z}$ while $\vec{p}_c = -\vec{p}_d = \vec{p}_f$ to be along direction (θ,ϕ). The partial wave amplitudes may be obtained from transition amplitude $\mathcal{M}_{fi}$ by inverting equation (16) and using orthogonality relation of rotation matrices as

$$\mathcal{A}^J_{\lambda_a\,\lambda_b\,\lambda_c\,\lambda_d}(s) = \frac{1}{64\pi^2}\int d\Omega\; D^J_{\lambda\lambda'}(\phi,\theta,0)\mathcal{M}_{fi}(s,\,\Omega). \quad (17)$$

The unitarity of S-matrix which is equivalent to $T^\dagger - T = iT^\dagger T$, with $S = I + iT$, requires that even the most dominant partial amplitude $\mathcal{A}^J$ should satisfy

$$|\mathcal{R}e\,(\mathcal{A}^J(s))| \le 1/2. \quad (18)$$

For a given J, the high energy behaviour (*i.e.* behaviour at energies much higher than m, the mass of the heaviest particle involved in the scattering process) of the amplitude $\mathcal{A}^J(s)$ may be studied by expanding the amplitudes in powers of s/m^2 as

$$\mathcal{A}^J(s) = \sum_{n=-\infty}^{\infty} c_n^J\;\left[\frac{s}{m^2}\right]^{n/2}. \quad (19)$$

Terms generated for $n < 0$ approach zero at high energies (i.e. for $\sqrt{s} \gg m$). Terms for $n > 0$ grow with energy and hence the unitarity condition will require either the coefficient c_n^J to vanish or satisfy

$$\sum_{n>0} c_n^J\;[s/m^2]^{n/2} \le \frac{1}{2} \quad \text{for} \quad \sqrt{s} \gg m. \quad (20)$$

Anomalous couplings	Unitarity Bound	in unit of 1 TeV$^2/s$	Associated Partial Amplitude(s)		Anomalous Couplings related to Δg_1^Z	Unitarity Bound in unit of 1 TeV$^2/s$
$\lvert\Delta g_1^Z\rvert$	$\leq \frac{4\sin^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 0.756$	$\mathcal{A}^0_{0000}(WW \to ZZ)$	(B8)		
					$\lvert\Delta\kappa_Z\rvert$	≤ 0.756
$\lvert\Delta c_1^{Z\gamma}\rvert$	$\leq \frac{2\tan\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 0.898$	$\mathcal{A}^0_{00\pm\pm}(WW \to Z\gamma)$	(B5)	$\lvert\Delta c_0^{ZZ}\rvert$	≤ 1.51
			$\mathcal{A}^0_{\pm\pm00}(ZZ \to ZZ)$	(B1)		
$\lvert a_2^{ZZH}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{00\pm\pm}(ZZ \to ZZ)$	(B1)	$\lvert\Delta c_1^{ZZ}\rvert$	≤ 1.51
			$\mathcal{A}^0_{00\pm\pm}(WW \to ZZ)$	(B7)		
$\lvert a_2^{\gamma\gamma H}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{00\pm\pm}(ZZ \to \gamma\gamma)$	(B2)	$\lvert\Delta c_0^{Z\gamma}\rvert$	≤ 0.756
$\lvert a_2^{Z\gamma H}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{00\pm\pm}(ZZ \to Z\gamma)$	(B3)		
			$\mathcal{A}^0_{\pm\pm00}(WW \to WW)$	(B9)	$\lvert\Delta c_0^{WW}\rvert$	≤ 1.16
$\lvert a_2^{WWH}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{00\pm\pm}(WW \to WW)$	(B9)		
			$\mathcal{A}^0_{\pm\pm00}(WW \to ZZ)$	(B6)	$\lvert\Delta c_1^{WW}\rvert$	≤ 1.51
$\lvert a_2^{WWHH}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{\pm,\pm}(WW \to HH)$	(B12)		
$\lvert a_2^{ZZHH}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{\pm,\pm}(ZZ \to HH)$	(B14)		
$\lvert a_2^{Z\gamma HH}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{\pm,\pm}(Z\gamma \to HH)$	(B16)		
$\lvert a_2^{\gamma\gamma HH}\rvert$	$\leq \frac{8\tan^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.97$	$\mathcal{A}^0_{\pm,\pm}(\gamma\gamma \to HH)$	(B17)		
$\lvert b_2^{H^3}\rvert$	$\leq \frac{16\sin^2\theta_W}{3\,\alpha}\left(\frac{m_W^2}{s}\right)$	$\simeq 1.01$	$\mathcal{A}^0_{00}(WW \to HH)$	(B13)		
			$\mathcal{A}^0_{00}(ZZ \to HH)$	(B15)		

TABLE I: *Left panel of the table exhibit the unitarity constraints on the 11 linearly independent anomalous couplings and their corresponding partial wave amplitudes as given in appendix B. These bounds are obtained by retaining only one non-zero anomalous coupling at a time for all the processes analysed in this article. Right panel of the table show the stringent upper bound of anomalous couplings which are linearly dependent on* Δg_1^Z *as given in equation* (23).

In this article, we compute the partial wave helicity amplitudes for all vector boson scattering processes $V_1 V_2 \to V_3 V_4$ and also for all processes where one or more vector bosons V_i are replaced by the scalar Higgs and their corresponding helicity λ_i by zero as mentioned in Section I (with the choice of momenta and polarisations listed in Appendix A). We further investigate the high energy behaviour of these amplitudes as a function of the 23 anomalous couplings (five TGC, six QGC, four VVH, four $VVHH$, two H^3 and two H^4) by expanding them in powers of energy *viz.* s/m^2.

The partial wave amplitudes for the processes considered by us grow with energy ($\sqrt{s}$) as either $\mathcal{O}(s^2/m^4)$ or $\mathcal{O}(s^{3/2}/m^3)$. The unitarity condition given in equation (20) would then impose

$$c_1\left(\frac{s}{m^2}\right)^{1/2} + c_2\left(\frac{s}{m^2}\right) + c_3\left(\frac{s}{m^2}\right)^{3/2} + c_4\left(\frac{s}{m^2}\right)^{2} \leq \frac{1}{2}\,, \tag{21}$$

where each of c_i is a linear combination of the anomalous couplings. Working along the perturbative unitarization of gauge dynamics in the SM (as mentioned in Section I) and beyond the SM scenarios (for example in reference [10]), we expect that the cancellation of s^2 and $s^{3/2}$ terms in all the $2 \to 2$ scattering amplitudes can be realized among the guage mediated diagrams even in the presence of anomalous couplings involving the Higgs boson and gauge bosons. This provides us a clue as well as a conservative choice for satisfying equation (21). Hence, we enforce this choice by demanding the linear combinations c_3 and c_4 to be zero. The resulting relations among the anomalous couplings are then exploited along with the equation (5) to reduce number of independent parameters and obtain

$$\lambda_Z = \lambda_\gamma = \Delta\kappa_\gamma = 0 \tag{22}$$

$$2\Delta g_1^Z = \Delta c_0^{ZZ} = \Delta c_1^{ZZ} = 2\Delta c_0^{Z\gamma} = 2\Delta\kappa_Z = \frac{\Delta c_0^{WW}}{\cos^2\theta_W} = \frac{\Delta c_1^{WW}}{\cos^2\theta_W} \tag{23}$$

Thus, of the above ten anomalous couplings, three vanish and rest seven are related among themselves leaving us with only one independent coupling which we take to be Δg_1^Z . We are now left with a set of following fourteen linearly independent anomalous couplings:

$$\Delta g_1^Z,\ \Delta c_1^{Z\gamma},\ a_2^{\gamma\gamma H},\ a_2^{Z\gamma H},\ a_2^{ZZH},\ a_2^{WWH},\ a_2^{\gamma\gamma HH},\ a_2^{Z\gamma HH},\ a_2^{ZZHH},\ a_2^{WWHH},\ \Delta b_1^{H^3},\ \Delta b_1^{H^4},\ b_2^{H^3} \text{ and } b_2^{H^4}. \tag{24}$$

A. Unitarity Bound

After using the relations given by (22)-(23), the most divergent partial wave helicity amplitudes at high energies of all the scattering processes considered by us are at most either of $\mathcal{O}(s/m^2)$ or $\mathcal{O}(\sqrt{s}/m)$ and are listed in equations (B1)-(B17) of Appendix B. Note that the higher partial wave amplitudes $\mathcal{A}^J$ with $J > 0$ grow with energy slowly compared to $\mathcal{A}^0$ and thus give less stringent bounds on the couplings. Hence only the lowest partial wave amplitudes $\mathcal{A}^0$ are listed in the appendix. Higher partial scattering amplitudes ($J > 0$) are listed only for the cases where they provide independent bounds on the anomalous couplings.

However, the quartic Higgs boson self couplings $b_2^{H^4}$ which appears only in $HH \to HH$ scattering process do not show any bad high energy behaviour and hence it cannot be constrained from the energy dependent unitarity argument given in (20). On the same note anomalous triple $\Delta b_1^{H^3}$ and quartic $\Delta b_1^{H^4}$ Higgs couplings which do not contribute to any amplitude that grows with energy, cannot be constrained from perturbative unitarity.

With the help of relations (20), (22) and (23) we are now equipped to extract the unitarity constraint $|\mathcal{R}e(\mathcal{A}^J(s))| \leq 1/2$ either on the individual anomalous couplings or on the linear combination of anomalous couplings from the remaining all non-zero partial wave amplitudes which are of the $\mathcal{O}(s/m^2)$ or $\mathcal{O}(\sqrt{s}/m)$. We calculate the absolute upper bound of the anomalous couplings, by considering the effect on the high energy behaviour of the partial wave amplitudes for all the processes simultaneously keeping one anomalous coupling at a time and report the most stringent ones for the independent couplings given in equation (24) in the left panel of Table I. The bounds on other six dependent anomalous couplings related to the Δg_1^Z via (23) may be derived from the obtained constraints and are given in the right panel of the same Table. While computing the upper bound on anomalous couplings we have used [28]

$$\alpha^{-1}(m_Z) = 127.916\,,\quad \sin^2\theta_W(m_Z) = 0.23116\,,\quad m_Z = 91.1879 \text{ GeV}. \tag{25}$$

Extending our analysis, we allow simultaneous variation of two or more non-zero anomalous couplings and search for a constrained region in the parameter space. Thus, we consider all such partial wave amplitudes that depend upon more than one anomalous coupling.

We observe that the anomalous couplings Δg_1^Z , $\Delta c_1^{Z\gamma}$ and $a_2^{Z\gamma H}$ affect the partial amplitudes given by equation (B5): $\mathcal{A}^0_{00\pm\pm}(WW \to Z\gamma)$. The constrained parameter region obtained from the perturbative unitarization of these partial wave amplitudes along with the constraints from $\mathcal{A}^0_{0000}(WW \to ZZ)$ and $\mathcal{A}^0_{00\pm\pm}(ZZ \to Z\gamma)$ (as listed in Table I) is given by

$$-1 - \frac{\alpha s}{2\tan\theta_W m_W^2}\Delta g_1^Z \leq f \leq 1 - \frac{\alpha s}{2\tan\theta_W m_W^2}\Delta g_1^Z;$$
$$\text{with}\quad f = \frac{\alpha}{2\tan\theta_W}\frac{s}{m_W^2}\left[\frac{1}{4\tan\theta_W}a_2^{Z\gamma H} - \Delta c_1^{Z\gamma}\right]. \tag{26}$$

This region is displayed in Figure 1. Similarly, the bounds on Δg_1^Z and a_2^{WWH} are not independent as they are also related by various amplitudes for the process $WW \to WW$ given in equation (B11). The region on the ($a_2^{WWH} - \Delta g_1^Z$) plane simultaneously allowed by the non-violation of unitarity of these amplitudes and the constraints from amplitudes $\mathcal{A}^0_{0000}(WW \to ZZ)$, $\mathcal{A}^0_{\pm\pm00}(WW \to WW)$, and $\mathcal{A}^0_{\pm\pm00}(WW \to ZZ)$ is given by

$$-C_1 + 2(4 - 3\sec^2\theta_W)\Big(\frac{\sqrt{s}}{m_W}\Delta g_1^Z\Big) \leq \Big(\frac{\sqrt{s}}{m_W}a_2^{WWH}\Big) \leq C_1 + 2(4 - 3\sec^2\theta_W)\Big(\frac{\sqrt{s}}{m_W}\Delta g_1^Z\Big)$$
$$\text{with}\quad C_1 = \frac{32\sqrt{2}\tan^2\theta_W}{\pi\alpha}. \tag{27}$$

This constrained parameter space is plotted in Figure 2.

IV. DIMENSION SIX OPERATORS

As discussed in Section I, model independent NP effects can also be investigated by adding gauge invariant higher dimensional operators to the SM Lagrangian. The present precision of the data allows us to parametrize the deviations of the SM couplings in terms of coefficients of these higher dimension operators. We consider operators upto dimension six for our analysis *i.e.* upto the order of $1/\Lambda^2$ in the expansion given in equation (1). A complete list of such operators is listed in the classic paper of reference [29] and are classified again in reference [30].

Restricting ourselves to the CP-even dimension six operators which are relevant for the scattering processes considered in this article, *i.e* the ones that modify the Higgs and electroweak gauge bosons couplings, we get the following ten operators:

$$\begin{aligned}
\mathcal{O}_{WWW} &= \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}^{\mu}_{\rho}], \\
\mathcal{O}_{W} &= (D_\mu\Phi)^\dagger\hat{W}^{\mu\nu}(D_\nu\Phi), \\
\mathcal{O}_{B} &= (D_\mu\Phi)^\dagger\hat{B}^{\mu\nu}(D_\nu\Phi), \\
\mathcal{O}_{BB} &= \Phi^\dagger\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi, \\
\mathcal{O}_{WW} &= \Phi^\dagger\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi, \\
\mathcal{O}_{BW} &= \Phi^\dagger\hat{W}^{\mu\nu}\Phi\hat{B}_{\mu\nu}, \\
\mathcal{O}_{\Phi,1} &= ((D_\mu\Phi^\dagger)\Phi)\left(\Phi^\dagger D^\mu\Phi\right), \\
\mathcal{O}_{\Phi,2} &= \frac{1}{2}\partial_\mu\left(\Phi^\dagger\Phi\right)\partial^\mu\left(\Phi^\dagger\Phi\right), \\
\mathcal{O}_{\Phi,3} &= -\frac{1}{3}\left(\Phi^\dagger\Phi\right)^3, \\
\mathcal{O}_{\Phi,4} &= (D_\mu\Phi)^\dagger(D^\mu\Phi)\Phi^\dagger\Phi. \qquad (28)
\end{aligned}$$

Here Φ is the Higgs doublet represented in the unitary gauge as

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \qquad (29)$$

The covariant derivative along with field strength tensors $\hat{W}^{\mu\nu}$ and $\hat{B}^{\mu\nu}$ are defined as

$$\begin{aligned}
D_\mu &= \partial_\mu + \frac{i}{2}g\tau^I W^I_\mu + \frac{i}{2}g' B_\mu, \\
\hat{B}_{\mu\nu} &= \frac{i}{2}g'(\partial_\mu B_\nu - \partial_\nu B_\mu), \qquad (30) \\
\text{and } \hat{W}_{\mu\nu} &= \frac{i}{2}g\tau^I(\partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\epsilon_{IJK}W^J_\mu W^K_\nu).
\end{aligned}$$

It may be noted that, of the ten operators listed in equation (28), only one operator, namely, $\mathcal{O}_{\Phi,3}$ gives an additional contribution to the scalar Higgs boson potential and hence modifies the minima of the SM potential. This, in turn, modifies the SM vacuum expectation value $v^2_{\rm SM} = -\mu^2/\lambda$ to

$$\frac{v^2}{2} \simeq \left(\frac{v^2_{\rm SM}}{2}\right)\left[1 - \left(\frac{f_{\phi 3}}{4\Lambda^2}\right)v^2_{\rm SM}\right]. \qquad (31)$$

Further, inclusion of the operators $\mathcal{O}_{\Phi,1}$, $\mathcal{O}_{\Phi,2}$, and $\mathcal{O}_{\Phi,4}$ modifies the kinetic term of the Higgs field, leading to a redefinition of the Higgs boson field and the Higgs boson mass m_H [8] as

$$H \simeq \left[1 + \frac{v^2}{4\Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2} + f_{\Phi,4})\right] h; \qquad (32)$$

$$m^2_H \simeq 2\lambda v^2\left[1 - \frac{v^2}{2\Lambda^2}\left(f_{\Phi,1} + 2f_{\Phi,2} + f_{\Phi,4} + \frac{f_{\Phi,3}}{\lambda}\right)\right]. \qquad (33)$$

The operators $\mathcal{O}_{BW}$ and $\mathcal{O}_{\Phi,1}$ have a tree level effect on precision electroweak observables and therefore are subject to very strict constraints [14]. Hence we do not constrain these operators in our analysis.

A. Relation to Anomalous Couplings

A given dimension six operator can be expanded in terms of a set of independent Lorentz structures appearing with the same coefficient f_i. On comparing with the effective Lagrangian given in equation (2), we can express the anomalous couplings as a linear combination of the coefficients f_i (see reference [13] and [14]). Below we give relations of TGC and QGC with the coefficients of the above mentioned dimension six operators:

$$\begin{aligned}
\lambda_\gamma &= \lambda_Z = f_{WWW}\frac{3g^2m^2_W}{2\Lambda^2}, \qquad (34) \\
\Delta\kappa_Z &= (f_W u - f_B\tan^2\theta_W)\frac{m^2_W}{2\Lambda^2}, \qquad (35) \\
\Delta\kappa_\gamma &= (f_W + f_B)\frac{m^2_W}{2\Lambda^2}, \quad \text{and} \qquad (36) \\
\Delta c^{ZZ}_0 &= \Delta c^{ZZ}_1 = 2\Delta c^{Z\gamma}_0 = 2\Delta c^{Z\gamma}_1 = \frac{\Delta c^{WW}_0}{\cos^2\theta_W} = \frac{\Delta c^{WW}_1}{\cos^2\theta_W} \\
&= -2\Delta g^Z_1 = -f_W\frac{m^2_Z}{\Lambda^2}. \qquad (37)
\end{aligned}$$

Based on the analysis performed in previous section we attempt to constrain the coefficients of dimension six operators. Combining the relations (37) with the unitarity constraints from (23) we get

$$\Delta g^Z_1 = \Delta c^{VV'}_i = 0. \qquad (38)$$

Further using (22), (34), (35) and (36), we get $f_{WWW} = f_W = f_B = 0$.

We now parameterize the coefficients of remaining five operators in terms of five dimensionless parameters defined as

$$\begin{aligned}
&d_2 = \frac{m^2_W}{\Lambda^2}f_{\Phi,2}; \quad d_3 = \frac{m^2_W}{\Lambda^2}f_{\Phi,3};, \quad d_4 = \frac{m^2_W}{\Lambda^2}f_{\Phi,4}; \\
&d = -\frac{m^2_W}{\Lambda^2}f_{WW} \quad \text{and} \quad d_B = -\frac{m^2_W}{\Lambda^2}\tan^2\theta_W f_{BB}. \qquad (39)
\end{aligned}$$

We re-write all the anomalous VVH and $VVHH$ Higgs-gauge bosons couplings and Higgs boson self interactions in terms of these dimensionless parameters d_i's and TGC (see refs. [14, 16, 26, 31]) [3].

$$\Delta a_1^{WWH} = \Delta a_1^{ZZH} = \frac{\sin^2\theta_W}{4\pi\alpha}\left(3d_4 - 2d_2\right), \quad (40)$$

$$\Delta a_1^{WWHH} = \Delta a_1^{ZZHH} = \frac{\sin^2\theta_W}{4\pi\alpha}\left(5d_4 - 2d_2\right), \quad (41)$$

$$a_2^{WWH} = a_2^{WWHH} = 2\sec^2\theta_W\Big[d + \cos^2\theta_W\,\Delta g_1^Z\Big], \quad (42)$$

$$a_2^{ZZH} = a_2^{ZZHH} = 2\sec^2\theta_W\Big[d\cos^2\theta_W + d_B\sin^2\theta_W + \Delta g_1^Z\cos 2\theta_W + \Delta\kappa_\gamma\tan^2\theta_W\Big], \quad (43)$$

$$a_2^{Z\gamma H} = a_2^{Z\gamma HH} = 2\tan\theta_W\Big[d - d_B + \Delta g_1^Z - \frac{\Delta\kappa_\gamma}{2\cos^2\theta_W}\Big], \quad (44)$$

$$a_2^{\gamma\gamma H} = a_2^{\gamma\gamma HH} = 2\sec^2\theta_W\Big[d\sin^2\theta_W + d_B\cos^2\theta_W\Big]. \quad (45)$$

Using (12) and (13), along with (40), (41) we get

$$d_2 = d_4 = 0. \quad (46)$$

Anomalous couplings inducing Higgs boson self interactions H^3 and H^4 are related to the parameters d_i's as

$$\Delta b_1^{H^3} = \frac{\sin^2\theta_W}{12\pi\alpha}\left[-6d_2 - 3d_4 + \frac{8v^2}{m_H^2}d_3\right], \quad (47)$$

$$\Delta b_1^{H^4} = \frac{\sin^2\theta_W}{2\pi\alpha}\left[-2d_2 - d_4 + \frac{8v^2}{m_H^2}d_3\right], \quad (48)$$

$$b_2^{H^3} = b_2^{H^4} = \frac{\sin^2\theta_W}{3\pi\alpha}\left[2d_2 + d_4\right]. \quad (49)$$

However, the constraint given in equation (46) guarantees the vanishing of the anomalous couplings $b_2^{H^3}$ and $b_2^{H^4}$. As a consequence $\Delta b_1^{H^3}$ and $\Delta b_1^{H^4}$ depend only on the dimensionless parameter d_3.

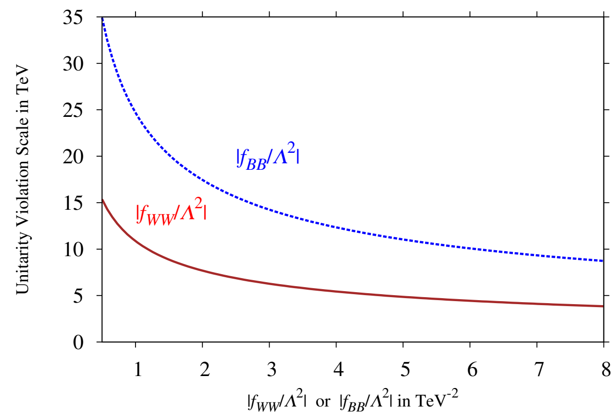

FIG. 4: *Variation of unitarity violation energy with coefficients f_{WW}/Λ^2 and f_{BB}/Λ^2. These coefficients are varied within limits derived from combined analysis of LHC and Tevatron data at 90 % CL.*

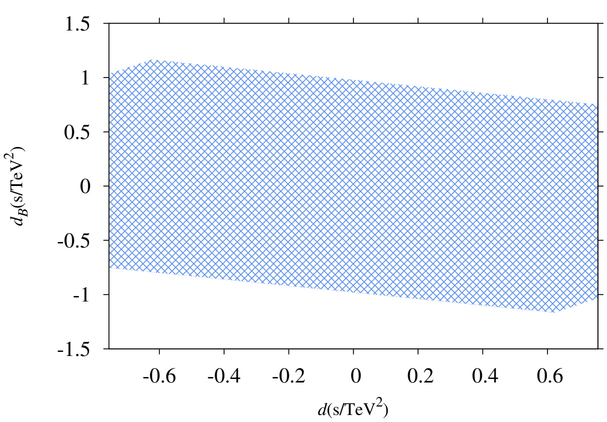

FIG. 3: *The shaded enclosed region on the* $(d - d_B)-$ *plane corresponds to unitarity constraints from partial wave amplitudes as given in equations* (50)-(53)

Thus operator analysis has reduced the number of linearly independent parameters to three, namely d_3, d and d_B, which are essentially the dimensionless coefficients of operators $\mathcal{O}_{\Phi,3}$, $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$ respectively. This is in contrast to fourteen linearly independent anomalous couplings based on the partial wave analysis of the Lorentz structures given in equation (24) of previous Section. Out of these three, non-violation of perturbative unitarity constraints only d and d_B while d_3 remains unconstrained as it does not appear as a coefficient of $\mathcal{O}(s/m^2)$ or $\mathcal{O}(\sqrt{s}/m)$ terms in any of the partial wave amplitudes.

With the vanishing of anomalous TGC and QGC (38), the dimensionless coefficients d and d_B are related to four VVH (or $VVHH$) couplings, taking any two at a time. We depict the allowed region constrained by unitarity of all partial wave amplitudes on $(d,\, d_B)-$plane in Figure 3. The enclosed region is constrained by following four inequalities arising from (B6), (B9) & (B12), (B5) & (B16), (B4) & (B17) and (B7) & (B14) respectively

$$|d| \le \frac{4\sin^2\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right), \quad (50)$$

$$|d_B - d| \le \frac{4\tan\theta_W}{\alpha}\left(\frac{m_W^2}{s}\right), \quad (51)$$

$$|d_B\cot^2\theta_W + d| \le \frac{4}{\alpha}\left(\frac{m_W^2}{s}\right), \quad (52)$$

$$|d_B + d\cot^2\theta_W| \le \frac{4m_W^2}{\alpha s}. \quad (53)$$

Unitarity bounds on anomalous VVH and $VVHH$ couplings can thus be translated to d and d_B from the boundary of the enclosed shaded region of Figure 3. Translating in terms of the coefficients of the dimension six operators,

[3] Note that we do not take into account the operators $\mathcal{O}_{\Phi,1}$ and $\mathcal{O}_{BW}$ as mentioned earlier

we get the most stringent bounds from (51) and (52) to be

$$\left|\frac{f_{WW}}{\Lambda^2}\right| \leq \frac{4\sin^2\theta_W}{\alpha}\left(\frac{1}{s}\right) = \left(\frac{118}{s}\right) \quad \text{and} \tag{54}$$

$$\left|\frac{f_{BB}}{\Lambda^2}\right| \leq \frac{4(1+\tan\theta_W)}{\sec^2\theta_W\,\alpha\, s} = \left(\frac{609}{s}\right). \tag{55}$$

However, keeping only one coupling at a time, the most stringent unitarity bound on f_{WW}/Λ^2 remains same as given in equation (54) while upper bound on f_{BB}/Λ^2 is further lowered and is given as

$$\left|\frac{f_{BB}}{\Lambda^2}\right| \leq \frac{4m_W^2}{\alpha s} = \left(\frac{512}{s}\right). \tag{56}$$

V. CONSTRAINTS FROM EXPERIMENTS

In this section we discuss the experimental constraints on anomalous couplings. Adhering to the conditions given in equations (4) and (5), the existing LEP limit on TGC [22] along with recent data from LHC [23] are summarized in the Table II. However, one can obtain

Anomalous TGC	LEP	LHC
$\lvert\Delta g_1^Z\rvert$	0.020	0.095
$\lvert\lambda_Z\rvert$	0.022	0.048
$\lvert\Delta\kappa_\gamma\rvert$	.042	0.22

TABLE II: *Experimental Limits on anomalous Triple gauge boson couplings assuming the custodial $SU(2)$ symmetry.*

much less stringent bound on these couplings by relaxing the custodial and gauge symmetry and a similar analysis have been performed with LHC data [32] to give

$$\begin{aligned}
-0.135 \leq &\Delta\kappa_\gamma \leq 0.190,\\
-0.373 \leq &\Delta g_1^Z \leq 0.562,\\
-0.078 \leq &\Delta\kappa_Z \leq 0.092,\\
-0.152 \leq &\lambda_\gamma \leq 0.146,\\
-0.074 \leq &\lambda_Z \leq 0.073.
\end{aligned} \tag{57}$$

LEP bounds on the coefficient of operators $\mathcal{O}_{BB}$ and $\mathcal{O}_{WW}$ involving Higgs gauge boson coupling can be read out from Figure 6 of reference [31] (for $m_H = 125$ GeV) and expressed in terms of the upper limit on the magnitude of $|d_B| \lesssim 0.05$ and $|d| \lesssim 0.2$. Using these upper limits on d_B and d we find that unitarity is not violated upto 4.8 and 2 TeV respectively.

Further, adding LHC data provides stringent limits, particularly when the Higgs to two photon decay signal strength is taken into account [33]. In this reference, the authors study the constraints on the dimension six operators by analysing LHC data from Higgs decays $H \to \gamma\gamma$, $H \to WW$ and $H \to ZZ$ channels. The one parameter bounds on $\epsilon_{WW} \equiv v^2 f_{WW}/\Lambda^2$ and $\epsilon_{BB} \equiv v^2 f_{BB}/\Lambda^2$ from ATLAS and CMS data obtained by them for diphoton channel at 95 % CL translate into

$$\begin{aligned}
&\frac{f_{WW}}{\Lambda^2}, \frac{f_{BB}}{\Lambda^2} \in [-3.47, 0.496]\,\text{TeV}^{-2},\\
&\qquad \text{Diphoton Channel : ATLAS}\\
&\frac{f_{WW}}{\Lambda^2}, \frac{f_{BB}}{\Lambda^2} \in [-3.80, 0.826]\,\text{TeV}^{-2}.\\
&\qquad \text{Diphoton Channel : CMS}
\end{aligned} \tag{58}$$

Combining the analysis from ATLAS and CMS, we compute the lowest energy scale where unitarity would be violated in the presence of these dimension six operators. Taking one operator at a time, the unitarity violation scale becomes ~ 6 TeV and 13 TeV respectively for f_{WW}/Λ^2 and f_{BB}/Λ^2. We depict the variation of the unitarity violating scale with $|f_{WW}/\Lambda^2|$ and $|f_{BB}/\Lambda^2|$ in Figure 4. In this figure, we have considered both these coefficients to vary within the allowed range given by the combined analysis.

A global fit to the existing LHC and Tevatron data has been performed in reference [34] allowing simultaneous determination of the parameters quantifying the Higgs boson couplings to the electroweak gauge bosons and the other SM particles. Using their best fit value 1.5 (-1.6) TeV^{-2} for f_{WW}/Λ^2 (f_{BB}/Λ^2), we observe that unitarity is not violated upto energies $\sim 9\,(19)$ TeV. Further if the operator f_{WW}/Λ^2 (f_{BB}/Λ^2) is allowed to be as large as the largest value of the 90% CL regions which is 8.2 (7.5) TeV^{-2} , the unitarity is preserved until $\sim 4\,(9)$ TeV.

VI. CONCLUSIONS

In this article, we have attempted to address perturbative unitarity of the vector boson scattering processes in the presence of anomalous couplings associated with the pure gauge sector (TGC and QGC), the Higgs boson - gauge boson sector (V_1V_2H, V_1V_2HH) and Higgs boson self interactions. We start with twenty three anomalous couplings involved in VV and/or HH scattering processes, taking all of them to be independent. Our observations are summarised below:

(a) We adopt the correct procedure for perturbative unitarization by analysing all dominant terms in the helicity amplitudes unlike reference [8]. At high energies, the helicity amplitudes corresponding to the gauge boson scattering in processes grow as $\mathcal{O}(s^2/m^4)$ and/or $\mathcal{O}(s^{3/2}/m^3)$. On demanding these divergent terms in the amplitudes to vanish identically, we are left with fourteen independent anomalous couplings given in equation (24).

However, three anomalous couplings which are related to Higgs boson self interactions, do not generate such terms in helicity amplitudes that grow with energy and hence they could not be constrained from the perturbative unitarity arguments.

On unitarizing all non-zero helicity amplitudes which grow as either $\mathcal{O}(s/m^2)$ or $\mathcal{O}(\sqrt{s}/m)$, we can successfully constrain remaining eleven anomalous couplings. In Figures 1, and 2 we plot the constrained regions of the linear combination of two anomalous couplings.

Upper limits on each of these couplings are computed taking one coupling to be operative at a time. The upper bounds of the independent and dependent anomalous couplings are presented in the Table I in units of (1 TeV^2/s). Inverting the argument, the Table can also be used to read off the energy scale upto which unitarity is not violated for a given value of coupling. Using the current LEP bound on Δg_1^z [22], $\left|\Delta g_1^z\right| \leq 0.016$ and reading out the constraint from the right column of the Table, we find that perturbative unitarity is not violated upto $\sqrt{s} \sim 7$ TeV.

(b) Assuming that the contribution to the anomalous couplings are restricted to have arisen from five CP-even dimension six operators, we find that the perturbative unitarity requires vanishing of all anomalous TGCs and QGCs. Study of the $VV-$ scattering processes shows that the SM along with anomalous couplings in the Higgs - gauge boson sector can preserve unitarity at least upto $\simeq 4$ TeV with 90 % CL. This result is also in agreement with the recent work in reference [35], where the authors have considered the anomalous Higgs coupling with the top quark and using the best fits of preliminary LHC Higgs data they show that unitarity can be preserved upto 4 TeV unless NP takes over.

Using the best fit values of the combined analysis with Tevatron and LHC data [34], we observe that the unitarity validation scale can be raised upto 9 TeV.

(c) Comparing our results with that of reference [8], we observe that, unlike theirs, we have only two linearly independent dimension six operators which fix the perturbative unitarity violation scale. In addition we provide the limits on all anomalous TGC, QGC, VVH, $VVHH$ and Higgs boson self couplings.

With more data from CMS and ATLAS at LHC, we expect to improve the unitarity bound on the anomalous couplings. Accordingly, the unitarity violation scale can be raised with the shrinking of the allowed region in anomalous couplings. On the contrary, if we find these anomalous couplings to be rather large than one needs to invoke a careful study of divergence cancellations with the inclusion of new physics spectrum, to respect unitarity.

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Appendix A: Notations and Conventions of Momenta and Polarizations

In our present article we study all $2 \to 2$ gauge boson scattering processes. Here we define the choice of momenta and polarizations vectors.

For all our scattering process the masses of initial particles are identical. Therefore, in CM reference frame, the momenta and polarization of initial particles are given as

$$\begin{aligned} k_1 \equiv& \frac{\sqrt{s}}{2}\left(1,0,0,\beta_V\right); \\ k_2 \equiv& \frac{\sqrt{s}}{2}\left(1,0,0,-\beta_V\right), \end{aligned} \tag{A1}$$

$$\begin{aligned} \epsilon^\pm(k_1) \equiv& \frac{1}{\sqrt{2}}\left(0,\pm 1,-i,0\right); \\ \epsilon^\pm(k_2) \equiv& \frac{1}{\sqrt{2}}\left(0,\mp 1,-i,0\right); \\ \epsilon^0(k_1) \equiv& \frac{\sqrt{s}}{2m_V}\left(\beta_V,0,0,1\right) \\ \epsilon^0(k_2) \equiv& \frac{\sqrt{s}}{2m_V}\left(\beta_V,0,0,-1\right). \end{aligned} \tag{A2}$$

where $\beta_V = \sqrt{1-4m_V^2/s}$, $\sqrt{s}$ being the CM energy and m_V the mass of the corresponding gauge boson (here W or Z).

Similarly, for the processes $WW \to \gamma\gamma$, $WW \to WW$, $WW \to ZZ$, $ZZ \to ZZ$ the momenta and transverse polarization of final particles are defined as

$$\begin{aligned} k_3 \equiv& \frac{\sqrt{s}}{2}\left(1,\beta_{V'}\mathrm{s}_\theta,0,\beta_{V'}\mathrm{c}_\theta\right); \\ k_4 \equiv& \frac{\sqrt{s}}{2}\left(1,-\beta_{V'}\mathrm{s}_\theta,0,-\beta_{V'}\mathrm{c}_\theta\right) \end{aligned} \tag{A3}$$

$$\begin{aligned} \epsilon^\pm(k_3) \equiv& \frac{1}{\sqrt{2}}\left(0,\pm\mathrm{c}_\theta,-i,\mp\mathrm{s}_\theta\right); \\ \epsilon^\pm(k_4) \equiv& \frac{1}{\sqrt{2}}\left(0,\mp\mathrm{c}_\theta,-i,\pm\mathrm{s}_\theta\right). \end{aligned} \tag{A4}$$

where V' implies W, Z or γ, $\mathrm{c}_\theta = \cos\theta$ and $\mathrm{s}_\theta = \sin\theta$, θ being scattering angle which the angle between $\mathbf{k}_1$ and $\mathbf{k}_3$.

Except for the photons all other final state massive gauge bosons have longitudinal polarisation which is defined as

$$\begin{aligned}\epsilon^0(k_3) &\equiv \frac{\sqrt{s}}{2m_{_{V'}}}\left(\beta_{_{V'}}, \mathrm{s}_{_\theta}, 0, \mathrm{c}_{_\theta}\right); \\ \epsilon^0(k_4) &\equiv \frac{\sqrt{s}}{2m_{_{V'}}}\left(\beta_{_{V'}}, -\mathrm{s}_{_\theta}, 0, -\mathrm{c}_{_\theta}\right).\end{aligned} \tag{A5}$$

The process $WW \to Z\gamma$ is unique as here the final state consist of particles with unequal masses. We define the momenta of the final state particles for $WW \to Z\gamma$ as

$$\begin{aligned}k_3 &\equiv \frac{1}{2\sqrt{s}}\Big((s+m_{_Z}^2), \\ &\quad (s-m_{_Z}^2)\mathrm{s}_{_\theta}, 0, (s-m_{_Z}^2)\mathrm{c}_{_\theta}\Big); \\ k_4 &\equiv \frac{1}{2\sqrt{s}}\Big((s-m_{_Z}^2), \\ &\quad -(s-m_{_Z}^2)\mathrm{s}_{_\theta}, 0, -(s-m_{_Z}^2)\mathrm{c}_{_\theta}\Big).\end{aligned} \tag{A6}$$

The transverse polarization of the final state particles $WW \to Z\gamma$ are same as those given in equation (A4), while the longitudinal polarization of the $Z(k_3)$ boson is given as

$$\begin{aligned}\epsilon^0(k_3) &\equiv \frac{1}{2\sqrt{s}m_{_Z}}\Big((s-m_{_Z}^2), \\ &\quad (s+m_{_Z}^2)\mathrm{s}_{_\theta}, 0, (s+m_{_Z}^2)\mathrm{c}_{_\theta}\Big).\end{aligned} \tag{A7}$$

Appendix B: Partial Wave Amplitudes

After using all conditions and constraints discussed in Sections II (equations (12)–(13)) and III (equations (22)–(23)), we are left with the following non-vanishing leading partial wave amplitudes of the processes we have considered in the high energy limit. Keeping only terms linear in anomalous couplings, we list below the terms of $\mathcal{O}(s/m^2)$ and $\mathcal{O}(\sqrt{s}/m)$ of these partial wave amplitudes $\mathcal{A}^J_{\lambda_a\lambda_b\lambda_c\lambda_d}$. Note that the $J=0$ partial wave amplitudes provide the most stringent unitarity bounds. Hence we provide only the amplitudes we have used to calculate the most conservative unitarity bounds on anomalous couplings.[4]

1. <u>$ZZ \to ZZ$</u>

$$\begin{aligned}\mathcal{A}^0_{\pm\pm00}(ZZ\to ZZ) &= \mathcal{A}^0_{00\pm\pm}(ZZ\to ZZ)) = \\ &= \frac{\alpha\,(s/m_{_W}^2)}{16\tan^2\theta_W}\,a_{_2}^{ZZH}\end{aligned} \tag{B1}$$

2. <u>$ZZ \to \gamma\gamma$</u>

$$\mathcal{A}^0_{00\pm\pm}(ZZ\to\gamma\gamma) = = \frac{\alpha\,(s/m_{_W}^2)}{16\tan^2\theta_W}\,a_{_2}^{\gamma\gamma H} \tag{B2}$$

3. <u>$ZZ \to Z\gamma$</u>

$$\mathcal{A}^0_{00\pm\pm}(ZZ\to Z\gamma) = \frac{\alpha\,(s/m_{_W}^2)}{16\tan^2\theta_W}\,a_{_2}^{Z\gamma H} \tag{B3}$$

4. <u>$\gamma\gamma \to \gamma\gamma$</u>

   This process takes place with Higgs exchange and since the $\gamma\gamma H$ coupling does not exist at tree level in SM, the helicity amplitudes will be all proportional to square of anomalous coupling $a_{_2}^{\gamma\gamma H}$. Thus there is no term that is linear in anomalous coupling and this process does not give any constraints at leading order.

5. <u>$\gamma\gamma \to Z\gamma$</u>

   Similar to $\gamma\gamma\to\gamma\gamma$, the helicity amplitudes of this process also depend upon the anomalous coupling $a_{_2}^{Z\gamma H}$ but all amplitudes are zero if only terms linear in coupling are retained.

6. <u>$W^+W^- \to \gamma\gamma$</u>

$$\mathcal{A}^0_{00\pm\pm}(WW\to\gamma\gamma) = \frac{-\alpha\,(s/m_{_W}^2)}{16\tan^2\theta_W}\,a_{_2}^{\gamma\gamma H} \tag{B4}$$

7. <u>$W^+W^- \to Z\gamma$</u>

$$\begin{aligned}\mathcal{A}^0_{00\pm\pm}(WW\to Z\gamma) &= \frac{\alpha\,(s/m_{_W}^2)}{16\tan^2\theta_W}\times \\ &\Big[-a_{_2}^{Z\gamma H} + 4\tan\theta_W(\Delta c_{_1}^{Z\gamma} - \Delta g_{_1}^{Z})\Big]\end{aligned} \tag{B5}$$

8. <u>$W^+W^- \to ZZ$</u>

$$\mathcal{A}^0_{\pm\pm00}(WW\to ZZ) = \frac{-\alpha\,(s/m_{_W}^2)}{16\,\tan^2\theta_W}\,a_{_2}^{WWH} \tag{B6}$$

$$\mathcal{A}^0_{00\pm\pm}(WW\to ZZ) = \frac{-\alpha\,(s/m_{_W}^2)}{16\tan^2\theta_W}\,a_{_2}^{ZZH} \tag{B7}$$

$$\mathcal{A}^0_{0000}(WW\to ZZ) = \frac{-\alpha\,(s/m_{_W}^2)}{8\,\sin^2\theta_W}\,\Delta g_{_1}^{Z} \tag{B8}$$

9. <u>$W^+W^- \to W^+W^-$</u>

$$\begin{aligned}\mathcal{A}^0_{00\pm\pm}(WW\to WW) &= \mathcal{A}^0_{\pm\pm00}(WW\to WW) \\ &= \frac{-\alpha\,(s/m_{_W}^2)}{16\,\tan^2\theta_W}\,a_{_2}^{WWH}\end{aligned} \tag{B9}$$

$$\mathcal{A}^0_{0000}(WW\to WW) = \frac{\alpha\,(s/m_{_W}^2)}{16\,\sin^2\theta_W}\,(4\sin^2\theta_W - 1)\,\Delta g_{_1}^{Z} \tag{B10}$$

[4] $\mathcal{A}^0_{\pm\pm00}$ means $\mathcal{A}^0_{++00} = \mathcal{A}^0_{--00}$

$$\mathcal{A}^1_{\mp000}(WW \to WW) = -\mathcal{A}^1_{0\mp00}(WW \to WW) = \mathcal{A}^1_{00\mp0}(WW \to WW) = -\mathcal{A}^1_{000\mp}(WW \to WW) = \frac{\alpha\,(\sqrt{s}/m_{_W})}{48\,\sin^2\theta_W}\left[(3-4\cos^2\theta_W)2\Delta g_{_1}^{^Z} + \cos^2\theta_W\, a_{_2}^{^{WWH}}\right] \tag{B11}$$

10. $\underline{W^+W^- \to HH}$

$$\mathcal{A}^0_{\pm,\pm}(WW \to HH) = \frac{-\alpha\,(s/m_{_W}^2)}{16\,\tan^2\theta_W}\, a_{_2}^{^{WWHH}} \tag{B12}$$

$$\mathcal{A}^0_{00}(WW \to HH) = \frac{-3\alpha\left(s/(m_{_W}^2\right)}{32\sin^2\theta_W}\, b_2^{^{H^3}} \tag{B13}$$

11. $\underline{ZZ \to HH}$

$$\mathcal{A}^0_{\pm,\pm}(ZZ \to HH) = \frac{-\alpha\,(s/m_{_W}^2)}{16\,\tan^2\theta_W}\, a_{_2}^{^{ZZHH}} \tag{B14}$$

$$\mathcal{A}^0_{00}(ZZ \to HH) = \frac{-3\alpha\left(s/(m_{_W}^2\right)}{32\sin^2\theta_W}\, b_2^{^{H^3}} \tag{B15}$$

12. $\underline{Z\gamma \to HH}$

$$\mathcal{A}^0_{\pm\pm}(Z\gamma \to HH) = \frac{-\alpha\,(s/m_{_W}^2)}{16\,\tan^2\theta_W}\, a_{_2}^{^{Z\gamma HH}} \tag{B16}$$

13. $\underline{\gamma\gamma \to HH}$

$$\mathcal{A}^0_{\pm\pm}(\gamma\gamma \to HH) = \frac{-\alpha\,(s/m_{_W}^2)}{16\,\tan^2\theta_W}\, a_{_2}^{^{\gamma\gamma HH}} \tag{B17}$$

14. $\underline{HH \to HH}$

The amplitudes for this process do not grow with energy and hence it is not used to put any unitarity constraints.

We have listed above the minimal set of partial wave amplitudes for a given process. partial wave amplitudes $\mathcal{A}^J$ for $J > 0$ are listed only when corresponding partial wave amplitudes $\mathcal{A}^0$ are zero and where $J > 0$ amplitudes give independent bound on certain couplings while $J = 0$ amplitudes fail to do so. Other partial wave amplitudes that are not listed above either contain terms lower than $\mathcal{O}(\sqrt{s}/m)$ or they provide less stringent unitarity conditions involving same combination of anomalous couplings.

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