

Ground states and excited states of hypernuclei in Relativistic Mean Field approach

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Abstract

Hypernuclei have been studied within the framework of Relativistic Mean Field theory. The force FSU Gold has been extended to include hyperons. The effective hyperon-nucleon and nucleon-nucleon interactions have been obtained by fitting experimental energies in a number of hypernuclei over a wide range of mass. Calculations successfully describe various features including hyperon separation energy and single particle spectra of single- Λ hypernuclei throughout the periodic table. We also extend this formalism to double- Λ hypernuclei.

1 Introduction

Hypernuclei are the first kind of flavoured nuclei in the direction of other exotic systems. One of the main reasons of interest in hypernuclear physics lies in the characteristics of the hyperon-nucleon(YN) and hyperon-hyperon(YY) interactions, crucial inputs to describe the structure of these strange nuclei. Obviously, measurements of YN and YY cross sections would give direct information on the interactions. However, such experiments are very difficult due to the short lifetime of the hyperons; till date no scattering data are available on the YY interaction while very limited data are available for the Ξ N interactions.

As the Λ is the lightest among the hyperons, Λ hypernuclei have been investigated more thoroughly than similar systems. Extensive experimental studies involving (π^+, K^+) , $(e, e' K^+)$, (K^-, π^-) or (γ, K^+) reactions have measured the binding energy, shell structure and other properties of single- Λ hypernuclei over a wide range of the periodic table, (see, e.g. Hashimoto *et al.*[1] for a recent review of the experimental scenario). Considerable amount of details about the Λ N interaction have already been extracted. For example, It has been established[2, 3] that the spin-orbit part of this force is weaker than that of the NN system.

On the other hand, existing experimental information about Ξ or double- Λ hypernuclear systems are extremely insufficient to draw any strong conclusion about Ξ N or the $\Lambda\Lambda$ interactions, respectively. Though a large amount of theoretical studies have been performed already to describe these systems (see, for example, some Relativistic Mean Field (RMF) works[4, 5, 6]) the situation is not clearly understood, especially due to the lack of experimental data. In this regard the importance of these systems is increasing even more now-a-days with the program for search of H-dibaryon at J-PARC and also the SKS and the

proposed S-2S experimental facilities at KEK/J-PARC concentrating especially on Ξ hypernuclei.[7]

It is, therefore, important to develop reliable theoretical tools to investigate the structure of these systems. Both relativistic and non-relativistic descriptions were used for the purpose. SU(3) symmetric field theories[8, 9] and the quark-meson coupling model[10, 11, 12] were developed to investigate hypernuclei. A density dependent relativistic hadron (DDRH) field theory was used by Keil *et al.*[13] to describe Λ hypernuclei. Among the non-relativistic approaches, one can name the shell model calculation,[14] the semi-empirical mass formulas,[15, 16, 17] the phenomenological single-particle fields,[18, 19] and the Skyrme-Hartree-Fock model (SHF),[20, 21] etc.

The RMF approach, which is very useful in describing the properties of normal nuclei, was used to study hypernuclear systems in various works.[4, 5, 6, 22, 23, 24] The ΛN interaction can either be extracted from microscopic methods like G-matrix calculations, or adjusted by fitting the experimental data. In this work, we perform a study of Λ hypernuclei within the framework of the RMF theory using the FSU Gold Lagrangian density[25] where the parameters of the hyperon interaction are determined by fitting the experimental separation energies of several hypernuclei in the mass region 16 to 208. Ideally, one should have looked to minimize the χ^2 value. However, for a number of nuclei, the theoretical binding energy values from mean field calculations are not expected to be sufficiently accurate to approach the experimental values within the experimental error. Thus, a χ^2 minimization will lead to over-dependence on only a few experimental values, particularly in the lighter hypernuclei. Hence, the parameter set that produces the minimum root mean square (rms) deviation for the experimental separation energies are adopted to calculate various other properties of hypernuclei throughout the periodic table, and compared with experimental data, whenever possible. Unless otherwise mentioned, all our results are obtained from this parameter set.

We extend this formalism to the study of the $\Lambda\Lambda$ and Ξ , i.e. $S = -2$ systems. In these nuclei, experimental information is extremely scarce and we have to often rely on the naive quark model to extract the coupling constants.

The paper is organized as follows. In section 2, we present a brief discussion of the FSU Gold Lagrangian density and the method followed for the description of the hypernucleus. We also discuss the parameters involved in the effective YN and YY forces, and the procedure applied to determine them. Section 3 is dedicated to our results. Finally, in section 4, we summarize our conclusions.

2 Formalism

RMF calculations have been able to explain different features of stable and exotic nuclei like ground state binding energy, deformation, radius, excited states, spin-orbit splitting, neutron halo etc.[26] There are a number of different Lagrangian densities as well as a number of different parametrization. In the present work the FSU Gold Lagrangian density has been employed.[25] While similar in spirit

to most other forces, it contains two additional non-linear meson-meson interaction terms in the Lagrangian density, whose main virtue is a softening of both the EOS of symmetric matter and the symmetry energy. As a result, the new parametrization becomes more effective in reproducing a few nuclear collective modes,[25] namely the breathing mode in ^{99}Zr and ^{208}Pb , and the isovector giant dipole resonance in ^{208}Pb . However, to the best of our knowledge, the effectiveness of this force in the hypernucleonic sector has not yet been established. Motivated by this fact we study single and double Λ hypernuclei, as well as Ξ hypernuclei using FSU Gold force. We compare some of our results with those obtained with the NLSH force.[27]

2.1 Model

The standard FSU Gold Lagrangian density is given by the following form:

$$\begin{aligned} \mathcal{L}_{total} = & \bar{\psi}(i\gamma_\mu\partial^\mu - M_n)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{4}\vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} \\ & + g_{\sigma n}\bar{\psi}\psi\sigma - \bar{\psi}\gamma_\mu(g_{\omega n}\omega^\mu + \frac{g_{\rho n}}{2}\vec{\tau} \cdot \vec{\rho}^\mu + \frac{e}{2}A_\mu(1 + \tau_3))\psi \\ & - \frac{k}{3!}(g_{\sigma n}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma n}\sigma)^4 + \frac{\zeta}{4!}(g_{\omega n}^2\omega_\mu\omega^\mu)^2 + \Lambda_v(g_{\rho n}^2\vec{\rho}_\mu \cdot \vec{\rho}^\mu)(g_{\omega n}^2\omega_\mu\omega^\mu) \end{aligned} \quad (1)$$

where

$$\begin{aligned} A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ \Omega_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \\ \vec{\rho}_{\mu\nu} &= \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g_\rho(\vec{\rho}_\mu X \vec{\rho}_\nu) \end{aligned} \quad (2)$$

The different terms are the standard ones used by Todd-Rutel *et al.*[25]. The suffix ‘ n ’ in the coupling constants refers to nucleons.

The hyperon couple to the non-strange mesons as well as the strange mesons σ^* and ϕ . Hence, the hyperon is introduced into the system through the hypernuclear sector of the Lagrangian density by adding the following term to the Lagrangian density in (1).

$$\begin{aligned} \mathcal{L}_h = & \bar{\psi}_h[i\gamma_\mu\partial^\mu - M_h + g_{\sigma h}\sigma + g_{\sigma^* h}\sigma^* \\ & - g_{\omega h}\gamma^\mu\omega_\mu - g_{\rho h}\gamma^\mu\vec{\tau} \cdot \vec{\rho}^\mu - g_{\phi h}\gamma^\mu\phi_\mu + \frac{e}{2}\gamma^\mu A_\mu(1 + \tau_3)]\psi_h \end{aligned} \quad (3)$$

Here ψ_h represents a hyperon field and M_h is the mass of the hyperon. The coupling to the mesons are given by the coupling constants with a suffix ‘ h ’. The Λ hyperon being charge-neutral and isoscalar does not couple to the photon or the ρ meson. While solving the coupled equations, the hyperon contributions to the source terms for all the meson fields are taken into consideration, thus allowing for rearrangement of the normal nucleonic core.

Pairing is introduced under the BCS approximation using a non zero range pairing force of strength 300 MeV-fm for both protons and neutrons. The RMF-BCS equations are solved under the usual assumptions of classical mean fields, time reversal symmetry, no-sea contribution etc. Solutions of the Dirac and Klein-Gordon equations has been obtained directly in coordinate space. Spherical symmetry is assumed for all the nuclei considered here.

2.2 Meson Parameters for the ΛN and $\Lambda\Lambda$ interactions

In the naive quark model, it is assumed that the omega and the rho fields couple only to the u and d quarks, and the strange quark in the baryon acts as a spectator when coupling to the vector mesons. The vector meson-hyperon coupling constant ($g_{\omega h}$), is related to the vector meson-nucleon coupling constant $g_{\omega n}$ as

$$\frac{1}{3}g_{\omega n} = \frac{1}{2}g_{\omega\Lambda} \quad (4)$$

for a Λ hypernuclei.[28] The scalar meson-hyperon coupling constant has often been determined by the requirement to reproduce the potential depth of the hyperon in normal nuclear matter according to

$$U_h^n = g_{\sigma h}\sigma + g_{\omega h}\omega_0 \quad (5)$$

The potential depth for a Λ in nuclear matter has a well known value $U_\Lambda^n = -30$ MeV,[29] which can be used to obtain $g_{\sigma\Lambda}$.

However, the usual procedure to extract the meson-nucleon coupling constants and the meson masses in RMF approach involves reproducing not only the saturation properties of nuclear matter but also various properties of finite nuclei. Hence, the nuclear matter properties may not be exactly reproduced. For example, the FSU Gold force predicts a binding energy per nucleon of 16.30 MeV for symmetric nuclear matter while the commonly accepted value is 16.0 MeV.

With the above idea in mind, the hyperon-meson coupling constants have been fitted in the present work to reproduce the experimental hyperon binding energy in case of single- Λ hypernuclei. One needs to remember that the mean field approximation may not work very well in very light nuclei. Thus, only hypernuclei with $A \geq 16$ were chosen for the fitting procedure. The contribution of σ^* and ϕ mesons are taken into consideration for both single- Λ and double- Λ systems; these parameters are discussed later in this section.

We assume that the non-strange sector of the Lagrangian is completely determined and the corresponding parameters are adopted from the work by Todd-Rutel *et al.*[25] and the only task that remains is fixing the hyperon-meson coupling constants. The mass of the Λ has been fixed at 1115.6 MeV. The initial values for the meson-hyperon coupling constants were taken from the naive quark model as described above. The two parameters were then varied in order to fit the experimental hyperon separation energies. Thus a best fit procedure was adopted to obtain the values of the two coupling constants. However, it was

observed that the vector meson-hyperon coupling constant determined from the naive quark model is sufficient for fit. Consequently, only the value of the $g_{\sigma\Lambda}$ needed to be varied. The best fit parameters and the fitted separation energies are presented in Tables 1 and 2, respectively.

It is, of course, imperative that the constants determined in the above procedure give quite a reasonable agreement with the nuclear matter properties. The present values of the parameters give rise to a lambda potential depth of -28.7 MeV, close to the value of -30.0 MeV adopted by Schaffner *et al.*[29]

Determining the $\Lambda\Lambda$ contribution to the experimental binding energy $B_{\Lambda\Lambda}$ is subject to large uncertainty because of the scarcity of data available on double- Λ nuclei. Nuclear emulsion experiments reported the observation of three double- Λ hypernuclei: ${}^6_{\Lambda\Lambda}\text{He}$, ${}^{10}_{\Lambda\Lambda}\text{Be}$ and ${}^{13}_{\Lambda\Lambda}\text{B}$. From these events, an effective $\Lambda\Lambda$ matrix element $-V_{\Lambda\Lambda} = \Delta B_{\Lambda\Lambda} = |B_{\Lambda\Lambda}| - 2|B_{\Lambda}| \cong 4-5$ MeV was determined,[30] $|B_{\Lambda\Lambda}|$ being the separation energy of the Λ pair from the ${}^A_{\Lambda\Lambda}Z$ hypernucleus, given by

$$B_{\Lambda\Lambda}({}^A_{\Lambda\Lambda}Z) = B({}^A_{\Lambda\Lambda}Z) - B({}^{A-2}Z) \quad (6)$$

and $|B_{\Lambda}|$, the hyperon separation energy from the AZ hypernucleus.

On the other hand, a very recent counter-emulsion hybrid experiment, performed at KEK,[31] favours a quite weaker $\Lambda\Lambda$ interaction: $\Delta B_{\Lambda\Lambda}({}^6_{\Lambda\Lambda}\text{He}) = 0.67 \pm 0.17$ MeV from a recent reanalysis of the data[32].

We take the meson masses equal to $m_{\sigma^*} = 980$ MeV and $m_{\phi} = 1020$ MeV. For the ϕ coupling we take the naive quark model value[4] obtained from the relation

$$\frac{1}{3}g_{\omega n} = -\frac{1}{\sqrt{2}}g_{\phi\Lambda} \quad (7)$$

while the σ^* coupling strength is determined by reproducing the value of $\Delta B_{\Lambda\Lambda}$ for the nucleus ${}^6_{\Lambda\Lambda}\text{He}$ within error.

2.3 Parameters for the ΞN interaction

In the naive-quark model, the relations between the vector meson-nucleon and the vector meson-hyperon coupling constants are given as,

$$\frac{1}{3}g_{\omega n} = g_{\omega\Xi^-} = g_{\omega\Xi^0} \quad (8)$$

$$g_{\rho\Xi^-} = g_{\rho\Xi^0} = g_{\rho n} \quad (9)$$

for a Ξ hypernucleus.[28] Dover and Gal[33] analyzed old emulsion data of Ξ^- hypernuclei and obtained a nuclear potential well depth of $U_{\Xi} = -21$ to -24 MeV. Fukuda *et al.*[34] fitted the very low energy part of Ξ^- hypernuclear spectrum in the ${}^{12}\text{C}(K^-, K^+)X$ reaction and estimated the value of U_{Ξ} to be between -16 to -20 MeV. E885 at the AGS[35] have indicated a potential depth of $U_{\Xi} = -14$ MeV or less. Here, we choose $U_{\Xi^-} = U_{\Xi^0} = -16$ MeV initially to determine the parameters of effective ΞN couplings. However, the corresponding parameters does not reproduce the empirical Ξ^- separation energies. Therefore,

we tune the parameters to match the experimental results slightly better (as there are not enough experimental data available to formally fit the data). The parameters thus determined give a Ξ potential depth of -20.59 MeV, which is closer to the value -21.0 MeV determined from the old emulsion data.[33]

The adopted values for all these parameters are presented in Table 1. We have calculated separation energies of the single- Λ systems using the NLSH force to make a comparison. The ΛN coupling constants for the NLSH parameter set were fitted in the same procedure. Table 1 also lists the parameters used for calculations of single- Λ separation energies with the force NLSH.

Table 1: Model parameters used in this work. The hyperon masses are in MeV.

| Model | M_Λ | $g_{\sigma\Lambda}$ | $g_{\omega\Lambda}$ | $g_{\sigma^*\Lambda}$ | $g_{\phi\Lambda}$ | M_Ξ | $g_{\sigma\Xi}$ | $g_{\omega\Xi}$ | $g_{\rho\Xi}$ |
|----------|-------------|---------------------|---------------------|-----------------------|-------------------|---------|-----------------|-----------------|---------------|
| FSU Gold | 1115.6 | 6.519 | 9.530 | 6.515 | -6.740 | 1670 | 3.471 | 4.767 | 5.884 |
| NLSH | 1115.6 | 6.465 | 8.630 | - | - | - | - | - | - |

3 Results

3.1 Single Λ hypernuclei: ground states

Table 2: Binding energy per nucleon (BE/A) and separation energy (S_Y) of Λ hyperon calculated for the hypernuclei included in the fitting procedure. All energy values are in MeV. Experimental values, unless otherwise indicated, are from Hashimoto *et al.*[1]

| Nuclei | FSU Gold | | NLSH | | Exp. |
|---------------------------|----------|-------|--------|-------|----------------|
| | BE/A | S_Y | BE/A | S_Y | S_Y |
| $^{16}_\Lambda\text{O}$ | 7.79 | 12.21 | 7.95 | 12.29 | 12.42(05)[36] |
| $^{17}_\Lambda\text{O}$ | 8.23 | 12.51 | 8.33 | 12.52 | 13.39(55) |
| $^{28}_\Lambda\text{Si}$ | 8.19 | 17.49 | 8.41 | 18.05 | 16.60(20) |
| $^{32}_\Lambda\text{S}$ | 8.29 | 18.26 | 8.44 | 18.55 | 17.50(50)[37] |
| $^{33}_\Lambda\text{S}$ | 8.42 | 18.55 | 8.53 | 18.59 | 17.96(000)[38] |
| $^{40}_\Lambda\text{Ca}$ | 8.62 | 18.45 | 8.64 | 18.59 | 18.70(110)[39] |
| $^{41}_\Lambda\text{Ca}$ | 8.79 | 18.57 | 8.79 | 18.68 | 19.24(100) |
| $^{51}_\Lambda\text{V}$ | 8.85 | 20.37 | 8.96 | 20.90 | 19.97(100)[40] |
| $^{56}_\Lambda\text{Fe}$ | 8.83 | 21.18 | 8.97 | 21.69 | 21.00(100) |
| $^{89}_\Lambda\text{Y}$ | 8.84 | 22.62 | 8.89 | 23.19 | 23.10(50) |
| $^{139}_\Lambda\text{La}$ | 8.54 | 24.59 | 8.60 | 24.68 | 24.50(120) |
| $^{208}_\Lambda\text{Pb}$ | 7.98 | 25.12 | 8.03 | 26.02 | 26.30(86) |

In Table 2, the results of our calculation for the single- Λ hypernuclei, in-

cluded in the fitting procedure, are presented. We tabulate the total energy per nucleon as well as the separation energy (S_Y), the latter being compared with experimental values. The results for the NLSH Lagrangian density are also tabulated. The hyperon is placed in the $1s_{1/2}$ state in all the cases. One can see that the values come very close to the experimental measurements. Except in the case of ${}_{\Lambda}^{208}\text{Pb}$, the values differ by less than 1 MeV. It is also easy to see that FSU Gold gives a better agreement than NLSH, since the rms deviation for FSU Gold comes out to be 0.64 MeV, whereas the same for NLSH is 0.72 MeV.

With the success of this model in $A \geq 16$, we have also extended our calculations for lighter hypernuclei. The results are presented in Table 3. We find that though the errors are slightly larger, the present approach can reproduce the hyperon separation energy to a reasonable degree.

Table 3: Binding energy per nucleon (BE/A) and separation energy of Λ hyperon (S_Y) for hypernuclei with $A < 16$. All energy values are in MeV. Hyperons are placed in $1s_{1/2}$ state. Experimental values are taken from Bando *et al.*[43].

| Nuclei | Present work | | Exp. |
|------------------------------|--------------|-------|------------|
| | BE/A | S_Y | S_Y |
| ${}_{\Lambda}^8\text{Be}$ | 5.38 | 5.53 | 6.84(05) |
| ${}_{\Lambda}^{10}\text{Be}$ | 6.16 | 8.11 | 9.11(22) |
| ${}_{\Lambda}^{11}\text{B}$ | 6.55 | 9.29 | 10.24(05) |
| ${}_{\Lambda}^{12}\text{B}$ | 7.04 | 10.49 | 11.37(06) |
| ${}_{\Lambda}^{12}\text{C}$ | 6.79 | 10.44 | 10.76(19) |
| ${}_{\Lambda}^{14}\text{C}$ | 7.62 | 11.78 | 12.17(33) |
| ${}_{\Lambda}^{14}\text{N}$ | 7.39 | 11.76 | 12.17(000) |
| ${}_{\Lambda}^{15}\text{N}$ | 7.74 | 11.95 | 13.59(15) |

In Fig. 1 we present the results of our calculations for the Λ separation energies in various shells for a number of hypernuclei with mass scale and compare them with the experimental values. The relevant graph for the present discussion corresponds to the s_{Λ} shell. We note that the maximum deviation from experimental data occurs in case of ${}_{\Lambda}^{208}\text{Pb}$. However, in this case also, the under-estimation is smaller in our calculations than SHF calculations.[21, 41] RMF calculations by Mi-Xiang *et al.*[42] seems to underestimate the binding energy to varying magnitudes in all heavier systems, leading Guleria *et al.*[21] to conclude that RMF under-predicts the binding energy in hypernuclei above $A = 87$ as compared to SHF calculations. Our calculation, however, strongly disagrees with this conclusion, as the agreement between experimental data and the results of this calculation is consistent throughout the periodic table. Compared to our calculation, RMF calculation[6] using NLSH parameters either under-predicts or over-predicts the binding energy except for the case of ${}_{\Lambda}^{17}\text{O}$. However, even with the few values reported there, it is possible to note that the

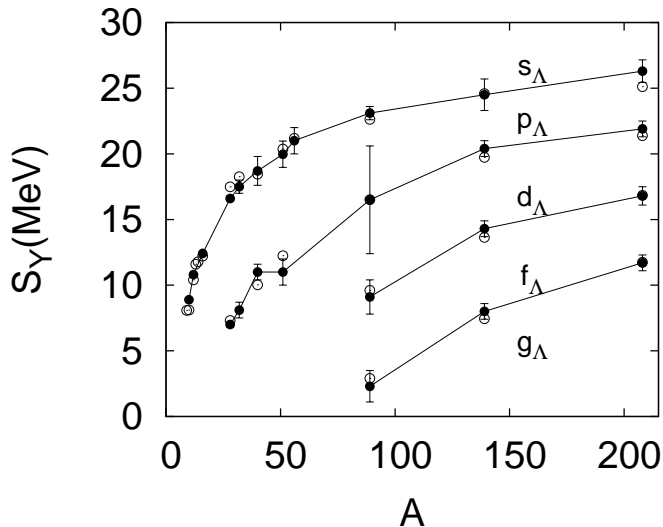


Figure 1: The separation energies of Λ in different single- Λ hypernuclei as a function of the baryon number A . Filled (empty) circles indicate experimental (theoretical) values. The lines are only for ease of visualization.

calculations we report are in much better agreement with experiment[6].

In Table 4 we present the results of our calculations (Pres.) of binding energy per nucleon for a number of hypernuclei over a wide range of mass number. We see that for hypernuclei with baryon number in the range of 35-95 the BE/A is around 8.7 MeV and it decreases gradually with further increase in the mass number. The present calculation agrees with the SHF calculations [21] (SHF) to a great extent while the RMF calculations by Mi-Xiang *et al.*[42] (RMF) seems to fail beyond $A = 87$, as already noted.

3.2 Single Λ hypernuclei: excited states

We also study the excited states that can be occupied by the Λ in hypernuclei. For this, we calculate self-consistent solutions for the ground state and the excited states separately and subtract to obtain the excitation energies. Our first interest lies in the nuclei which have closed nucleon core and one Λ -hyperon. The results for excitation energy in ${}_{\Lambda}^{17}\text{O}$, ${}_{\Lambda}^{41}\text{Ca}$ and ${}_{\Lambda}^{91}\text{Zr}$ with respect to the hypernuclear ground state are presented in Fig. 2.

Unfortunately, no results are available in any of the above nuclei. In fact, most of the nuclei, where the energy values corresponding to excited states of Λ hyperon are known, are of even mass number. In such nuclei, the Λ hyperon couples to the odd nucleon (neutron or proton). The residual interaction between the hyperon and the nucleon is not considered at the mean field level. However, the energy separation between states arising out of the above coupling is much smaller compared to the excitation energy. In fact, due to the limitations of

Table 4: Binding energy per nucleon (BE/A) (in MeV) predicted for a number of hypernuclei. Hyperons are placed in $1s_{1/2}$ state. All energy values are in MeV. See text for details.

| ${}^A_{\Lambda}Z$ | BE/A | | | ${}^A_{\Lambda}Z$ | BE/A | | |
|------------------------------|-------|---------|---------|-------------------------------|-------|---------|---------|
| | Pres. | RMF[42] | SHF[21] | | Pres. | RMF[42] | SHF[21] |
| ${}^{15}_{\Lambda}\text{N}$ | 7.746 | - | 8.623 | ${}^{88}_{\Lambda}\text{Sr}$ | 8.849 | 7.657 | 8.762 |
| ${}^{20}_{\Lambda}\text{Ne}$ | 7.772 | 7.635 | 7.445 | ${}^{89}_{\Lambda}\text{Sr}$ | 8.862 | 7.564 | 8.742 |
| ${}^{24}_{\Lambda}\text{Mg}$ | 7.832 | 7.723 | 7.671 | ${}^{90}_{\Lambda}\text{Y}$ | 8.854 | 7.510 | 8.746 |
| ${}^{27}_{\Lambda}\text{Al}$ | 8.204 | - | 8.225 | ${}^{112}_{\Lambda}\text{Sn}$ | 8.628 | 6.925 | 8.477 |
| ${}^{33}_{\Lambda}\text{S}$ | 8.422 | - | 8.706 | ${}^{117}_{\Lambda}\text{Sn}$ | 8.624 | 7.028 | 8.515 |
| ${}^{36}_{\Lambda}\text{S}$ | 8.666 | 8.637 | 8.613 | ${}^{120}_{\Lambda}\text{Sn}$ | 8.599 | 7.026 | 8.535 |
| ${}^{49}_{\Lambda}\text{Ca}$ | 8.827 | 8.880 | 8.767 | ${}^{136}_{\Lambda}\text{Xe}$ | 8.492 | 6.949 | 8.397 |
| ${}^{55}_{\Lambda}\text{Fe}$ | 8.840 | 8.797 | 8.731 | ${}^{140}_{\Lambda}\text{La}$ | 8.403 | 6.893 | 8.366 |
| ${}^{60}_{\Lambda}\text{Ni}$ | 8.782 | 8.841 | 8.751 | ${}^{143}_{\Lambda}\text{Pm}$ | 8.390 | - | - |
| ${}^{86}_{\Lambda}\text{Kr}$ | 8.844 | 8.778 | 8.735 | ${}^{145}_{\Lambda}\text{Sm}$ | 8.360 | 6.790 | 8.294 |
| ${}^{87}_{\Lambda}\text{Kr}$ | 8.848 | 8.796 | 8.702 | ${}^{209}_{\Lambda}\text{Pb}$ | 7.982 | 6.726 | 7.888 |
| ${}^{88}_{\Lambda}\text{Rb}$ | 8.859 | 7.587 | 8.727 | ${}^{210}_{\Lambda}\text{Bi}$ | 7.970 | 6.711 | 8.857 |

experimental resolution, in most cases the individual states have not at all been observed. In Table 5, we summarize the available experimental information on excitation energy and compare them with our calculation. One can see that the theory describes the excitation energies reasonably well, considering the fact that the residual interaction has been completely ignored. We also present a number of nuclear states where experimental information is not yet available. Fig. 1 also presents the hyperon separation energies for the excited states.

It should be noted that the origin of the states in ${}^{16}\text{O}$ has not been discussed by Agnello *et al.*[46]. However, it is clear that the lowest observed excited state at 6.1 MeV does not correspond to the excited state of Λ . This has also been supported by previous measurements. We should also point out that Hasegawa *et al.*[47] have identified the Λ -excitation energy with the major shell p only. However, comparison with later experiments and with theoretical results derived in the present work, allow us to identify it with the $p_{3/2}$ state. For example, in ${}^{89}_{\Lambda}\text{Y}$, the energy of the major shell p has been measured in Hasegawa *et al.*[47] as 5.9(6) MeV while Hotch *et al.*[40] have measured the energies of $p_{3/2}$ and $p_{1/2}$ as 6.01 and 7.38 MeV respectively. In some other cases also, experimental data are available for l excitations. In such situations we have always compared them with the lower of the two j states originating from the angular momentum state.

One should remember that the various J levels arising out of the coupling of the neutron hole to the Λ are degenerate in the mean field level. If the single particle state occupied by the ordinary nucleon is experimentally known, we have used the tagging method to put the last nucleon in that state in our calculation. Also notable is the small spin-orbit coupling for the Λ states. Although

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Table 5: Excitation energy (in MeV) of different Λ states. Energy values marked with ‘*’ are given for l excitations in literature. The superscript to the right of the hypernucleus in column 1 indicates the reference from which the experimental values have been obtained.

| Nucleus | Λ -state | Excitation Energy | | Λ -state | Excitation Energy | |
|------------------------------------|------------------|-------------------|-------|------------------|-------------------|-------|
| | | Exp. | Theo. | | Exp. | Theo. |
| ${}_{\Lambda}^{12}\text{B}$ [44] | $p_{3/2}$ | 10.93 | 10.40 | | | |
| ${}_{\Lambda}^{12}\text{C}$ [40] | $p_{3/2}$ | 10.66 | 10.34 | | | |
| ${}_{\Lambda}^{13}\text{C}$ [45] | $p_{3/2}$ | 9.93 | 11.02 | | | |
| ${}_{\Lambda}^{16}\text{O}$ [46] | $p_{3/2}$ | 9.1 | 9.97 | $p_{1/2}$ | 11.0 | 11.21 |
| ${}_{\Lambda}^{28}\text{Si}$ [47] | $p_{3/2}$ | 9.6 | 10.18 | | | |
| ${}_{\Lambda}^{51}\text{V}$ [40] | $p_{3/2}$ | 8.07 | 7.92 | $p_{1/2}$ | 9.40 | 8.73 |
| ${}_{\Lambda}^{89}\text{Y}$ [40] | $d_{5/2}$ | 16.42 | 15.93 | $d_{3/2}$ | 18.42 | 17.51 |
| | $p_{3/2}$ | 6.01 | 6.10 | $p_{1/2}$ | 7.38 | 6.59 |
| | $d_{5/2}$ | 12.79 | 12.81 | $d_{3/2}$ | 14.42 | 13.93 |
| ${}_{\Lambda}^{139}\text{La}$ [1] | $f_{7/2}$ | 19.98 | 19.56 | $f_{5/2}$ | 21.68 | 21.19 |
| | $p_{3/2}$ | 4.1* | 4.86 | $p_{1/2}$ | | 5.64 |
| | $d_{5/2}$ | 10.2* | 10.95 | $d_{3/2}$ | | 11.01 |
| ${}_{\Lambda}^{208}\text{Pb}$ [47] | $f_{7/2}$ | 16.5* | 17.15 | $f_{5/2}$ | | 19.38 |
| | $p_{3/2}$ | 5.2 | 3.74 | $p_{1/2}$ | | 3.91 |
| | $d_{5/2}$ | | 8.28 | $d_{3/2}$ | | 8.71 |
| | $f_{7/2}$ | | 13.33 | $f_{5/2}$ | | 14.19 |

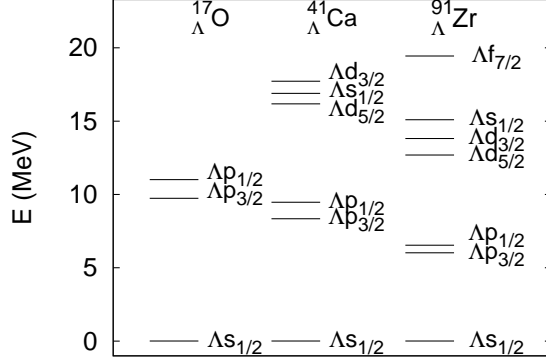


Figure 2: Excitation energy in some odd mass Λ hypernuclei.

our calculations slightly underestimates this difference, the trend is generally reproduced in agreement with the experimental data (see Table 5) as well as the previous theoretical works.[2, 3]

3.3 Double Λ hypernuclei

We calculate binding energies for several double- Λ hypernuclei including light, medium, and heavy systems within our framework using the parameter set FSU Gold. Our results (Pres.) are presented in Table 6 for the $\Lambda\Lambda$ binding energy $B_{\Lambda\Lambda}$ and the quantity $\Delta B_{\Lambda\Lambda}$. The experimental data are also listed, where available, for comparison. Results from another RMF calculation[4] using NLSH are also presented.

Table 6: $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ of double- Λ hypernuclei. The available experimental data[31, 48, 49, 50, 51] are also presented. All energy values are in MeV. See text for details.

| Nuclei | $B_{\Lambda\Lambda}$ | | | $\Delta B_{\Lambda\Lambda}$ | | |
|--------------------------------------|----------------------|-------|-------|-----------------------------|-------|------|
| | Exp. | Pres. | NLSH | Exp. | Pres. | NLSH |
| ${}^6_{\Lambda\Lambda}\text{He}$ | 7.25 ± 0.2 | 3.98 | 4.68 | 0.67 ± 0.17 | 0.62 | 1.01 |
| ${}^{10}_{\Lambda\Lambda}\text{Be}$ | 17.7 ± 0.7 | 14.03 | 15.94 | 4.3 ± 0.4 | 0.27 | 0.29 |
| ${}^{13}_{\Lambda\Lambda}\text{B}$ | 27.5 ± 0.7 | 21.17 | 22.52 | 4.8 ± 0.7 | 0.19 | 0.21 |
| ${}^{18}_{\Lambda\Lambda}\text{O}$ | | 26.96 | 25.12 | | 0.11 | 0.07 |
| ${}^{42}_{\Lambda\Lambda}\text{Ca}$ | | 37.75 | 37.17 | | 0.03 | 0.00 |
| ${}^{210}_{\Lambda\Lambda}\text{Pb}$ | | 50.34 | 53.02 | | 0.02 | 0.02 |

It is seen that $B_{\Lambda\Lambda}$ increases whereas $\Delta B_{\Lambda\Lambda}$ decreases with increase in mass number. This observation is in agreement with previous works.[5, 52] Our result for ${}^6_{\Lambda\Lambda}\text{He}$ matches with the NAGARA event[31] result from a recent reanalysis

of the data[32] within experimental error. The results for $\Delta B_{\Lambda\Lambda}$ of $^{10}_{\Lambda\Lambda}\text{Be}$ and $^{13}_{\Lambda\Lambda}\text{B}$ do not, however, match with the empirical data. This is reasonable as the empirical data signifies a strong $\Lambda\Lambda$ interaction, which is now-a-days believed to be wrong in view of the NAGARA event results. The $\Delta B_{\Lambda\Lambda}$ for all the nuclei presented here agrees well with the results of Shen *et al.*[4]. However, we should point out that the quantity $\Delta B_{\Lambda\Lambda}$ decreases very rapidly with increase in mass. Thus, in heavier nuclei, this quantity becomes so small that the error related to the convergence of the mean field solutions may become comparable to it.

As pointed out by Marcos *et al.*[5] RMF theory cannot compete with more elaborate three-body calculations for $\Delta B_{\Lambda\Lambda}$. In particular for such a light system as $^6_{\Lambda\Lambda}\text{He}$, its application is questionable. In view of extensions to multi- Λ systems, however, it is important to check the constraints it brings on the coupling of the Λ to the various meson fields. In this respect, it would be very desirable to obtain more and better experimental data for heavier hypernuclei.

The influence of Λ hyperons on the nuclear core, which is known as the core polarization effect[53], is an interesting aspect of hypernuclei as the nucleons in the core are affected by the additional Λ hyperons in hypernuclei. The so-called rearrangement energy quantifies the core polarization effect, which represents the change of nuclear core binding energies caused by the presence of Λ . We present in Table 7 the rearrangement energy in several single and double- Λ hypernuclei.

Table 7: Rearrangement energies in MeV (E_R) in several single and double- Λ hypernuclei. We also present the absolute value of the single particle energy of the Λ in the $1s_{1/2}$ state.

| Nucleus | E_R | $\epsilon_{\Lambda}(1s)$ |
|------------------------------------|-------|--------------------------|
| $^{17}_{\Lambda}\text{O}$ | 0.320 | 12.827 |
| $^{18}_{\Lambda\Lambda}\text{O}$ | 0.701 | 12.912 |
| $^{41}_{\Lambda}\text{Ca}$ | 0.127 | 18.699 |
| $^{42}_{\Lambda\Lambda}\text{Ca}$ | 0.306 | 18.738 |
| $^{209}_{\Lambda}\text{Pb}$ | 0.048 | 25.211 |
| $^{210}_{\Lambda\Lambda}\text{Pb}$ | 0.100 | 25.222 |

It is seen that the rearrangement energy decreases rapidly with increasing mass number, and is usually negligible in comparison with the binding energy except for very light systems. The rearrangement energy does not appear to be a linear function of the number of hyperons.

3.4 Single Ξ hypernuclei

The results of calculations for the Ξ hypernuclear systems are presented in the Fig. 3. We find that, except in one case, the energy value has been predicted accurately within experimental errors. The trend is also generally reproduced. Of course, the experimental errors are rather large and better values may be

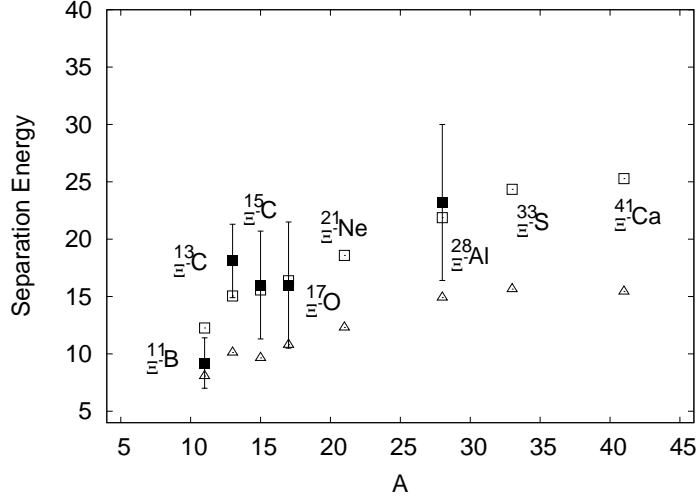


Figure 3: Separation energy(in MeV) of Ξ hyperon in hypernuclei. Filled (empty) boxes represent experimental (theoretical) Ξ^- separation energies while empty triangles represent theoretical Ξ^0 separation energies.

obtained when more accurate measurements are available. However, it is clear from the figure that our calculations reproduce the experimental results quite satisfactorily, including the kink at ${}_{\Xi}^{13}\text{C}$.

4 Summary and conclusion

To summarize, the FSU Gold Lagrangian density has been extended to include hypernuclei. The meson-hyperon coupling constants have been varied to reproduce the hyperon binding energy whenever sufficient experimental data are available. Otherwise, the naive quark model has been invoked to fix the parameters. The new parameters can reproduce the potential depth of the hyperons in nuclear matter. The hyperon separation energies are also reasonably predicted, even in very light hypernuclei. Ground states in single and doubly strange hypernuclei for a wide range of mass number and excited states in singly strange hypernuclei have been studied in the new extended Lagrangian density with reasonable success. The present calculation works well in all the observed hypernuclear systems. New and improved experimental data are desirable in order to further verify the predictions of this model. With the success of this model for the $S = -1$ and -2 systems, we would like to extend it to the study of multi-strange exotic systems. This work is in progress.

Acknowledgements

This work was carried out with financial assistance of the UGC (RFSMS, DRS, UPE). We sincerely thank the referee for the extremely useful comments and suggestions in improving the manuscript.

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