

General Relativistic effect on the energy deposition rate for neutrino pair annihilation above the equatorial plane along the symmetry axis near a rotating neutron star

Ritam Mallick^{w,a}, Abhijit Bhattacharyya^{z,b}, Sanjay K. Ghosh^{x,y,c} and Sibaji Raha^{x,y,d}

^w*Institute of Physics; Sachivalaya Marg; Bhubaneswar - 751005; Orissa; INDIA*

^x*Department of Physics; Bose Institute; 93/1,*

A.P.C Road; Kolkata - 700009; INDIA

^y*Centre for Astroparticle Physics and Space Sciences; Bose Institute; 93/1,*

A.P.C Road; Kolkata - 700009; INDIA and

^z*Department of Physics; University of Calcutta; 92,*

A.P.C Road; Kolkata - 700009; INDIA

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Abstract

The estimate of the energy deposition rate (EDR) for neutrino pair annihilation has been carried out. The EDR for the neutrinos coming from the equatorial plane of a rotating neutron star is calculated along the rotation axis using the Cook-Shapiro-Teukolsky (CST) metric. The neutrino trajectories and hence the neutrino emitted from the disk is affected by the redshift due to disk rotation and gravitation. The EDR is very sensitive to the value of the temperature and its variation along the disk. The rotation of the star has a negative effect on the EDR; it decreases with increase in rotational velocity.

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^a Email : ritam.mallick5@gmail.com

^b Email : abphy@caluniv.ac.in

^c Email : sanjay@bosemain.boseinst.ac.in

^d Email : sibaji@bosemain.boseinst.ac.in

I. INTRODUCTION

Neutron stars are objects formed in the aftermath of a supernovae. The central density of these stars can be as high as 10 times that of normal nuclear matter. At such high density, any small perturbation, *e.g.* spin down of the star, may trigger a phase transition from nuclear to quark matter system. As a result, the neutron star may convert to a quark star or a hybrid star with a quark core [1, 2]. It has been shown [3] that such a phase transition [4] produces a large amount of high energy neutrinos. These neutrinos (and antineutrinos) could annihilate and give rise to electron-positron pairs through the reaction $\nu\bar{\nu} \rightarrow e^+e^-$. These e^+e^- pairs may further give rise to gamma rays which may provide a possible explanation of the observed GRB. Furthermore, the rotating neutron stars have been shown [5] to produce the observed beaming effect of the GRB. At present, it is necessary to have a better understanding of the energy deposition in the neutrino annihilation to e^+e^- process in the realistic neutron star environment.

Motivated by the delayed explosion of Type II supernovae, the EDR due to the reaction $\nu\bar{\nu} \rightarrow e^+e^-$ in the vicinity of a neutron star have been calculated [6, 7] based on Newtonian gravity, *i.e.* $(2GM/c^2R) \ll 1$. Goodman *et al.* [7] pointed out that the neutrino pair annihilation rate can be seriously altered by the gravitational effects. The effect of gravity was incorporated in refs. [8, 9], but only for a static star. Slow rotation was introduced by Prasanna and Goswami [10]. GR effect on energy deposition rate for neutrino pair annihilation near a black hole was studied in ref. [11, 12] to explore the possible engine for GRBs. There, the most probable candidate for the central engine was taken to be the accretion disk around a black hole. A detailed hydrodynamic simulation of the process was studied by Birkl *et al.* [13].

In an earlier paper [14], we have shown that the path of neutrino is important for the study of EDR near a massive object. We have done a complete GR calculation of the neutrino path for the most general metric describing a rotating star, and obtained its geodesic equation along the equatorial and polar plane. The minimum photosphere radius (MPR) was calculated for various stars along these two planes. This is the starting point of our present calculation. The MPR serves as the lower limit from which we begin our calculation of the energy deposition rate (EDR) during the neutrino-antineutrino annihilation to electron-positron pairs. Our aim in this work is to study, semianalytically, the relativistic effect on

the EDR above the equatorial plane for a star rotating along the polar axis. We calculate the EDR along the rotation axis for the neutrinos coming from the equatorial plane of the star.

The relativistic effect consists of three factors: the gravitational redshift, the bending of the neutrino trajectories and the redshift due to rotation. The EDR is enhanced by the effect of neutrino bending; however, the redshift due to disc rotation and gravitation reduces the EDR. Thus these effects further complicate the estimation of EDR. In this work we primarily focus on the effect of GR and rotation on the EDR. The neutrinos released during phase transition will have very high energy and will interact with the propagating medium. But first, let us calculate the EDR only due to the thermal neutrinos, which have low energies and therefore high mean free path. This EDR calculation will give us an estimate of the EDR, and we will see how it compares with the energy liberated during gamma ray bursts (GRB). In order to discuss quantitatively the central engine of GRBs, we need a comprehensive study of the formation, evolution, temperature dependence, geometry of the star, the mechanism of energy deposition etc., most of which are affected by the rotational and GR effect.

Our paper is arranged as follows. In the next section we discuss our equations of state (EOS) and describe the geometry of the star. In section III we formulate the algorithm of the EDR calculation above the equatorial plane along the rotation axis. Next we present our results and finally we discuss the results and summarize our work.

II. STAR STRUCTURE

The structure of the star is described by the CST metric given by [15]

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{2\alpha} (dr^2 + r^2 d\theta^2) + e^{\gamma-\rho} r^2 \sin^2\theta (d\phi - \omega dt)^2. \quad (1)$$

The four gravitational potentials, namely α, γ, ρ and ω are functions of θ and r only. All the potentials have been solved for both static as well as rotating stars using the 'rns' code [16–18]. For a rotating star, all the potentials become functions of both r and θ .

Tabulated equations of state (EOS) are needed to run the code. In this paper we have used hadronic EOS evaluated using the nonlinear Walecka model [19]. The Lagrangian in the model includes nucleons (neutrons and protons), electrons, isoscalar scalar, isoscalar vector

and isovector vector mesons denoted by ψ_i , ψ_e , σ , ω^μ and $\rho^{a,\mu}$, respectively. The Lagrangian also includes cubic and quartic self interaction terms of the σ field. The parameters of the nonlinear Walecka model are meson-baryon coupling constants, meson masses and the coefficient of the cubic and quartic self interaction of the σ mesons. The meson fields interact with the baryons through linear coupling. The ω and ρ meson masses have been chosen to be their physical masses. The rest of the parameters, namely, nucleon-meson coupling constants and the coefficients of cubic and quartic terms of the σ meson self interaction are determined by fitting the nuclear matter saturation properties, namely, the binding energy/nucleon (-16 MeV), baryon density ($\rho_0=0.17 \text{ fm}^{-3}$), symmetry energy coefficient (32.5 MeV), Landau mass ($0.83 m_n$) and nuclear matter incompressibility (300 MeV).

Using this tabulated EOS and a fixed central density, we run the **rns** code to obtain all the gravitational potentials as a function of r and θ , which thereby define the shape, mass, rotational velocity and other parameters of the star. The shape of a fast rotating neutron star becomes oblate spheroid [15]. The star gets compressed along the z-axis, on the other hand along x and y-axes it bulges by equal amounts, thereby making the polar radius smaller than equatorial radius.

III. GR CALCULATION

In this section we study the GR effect on the neutrino pair annihilation rate. The coordinate system is oriented such that the equatorial plane lies along $\theta = \frac{\pi}{2}$ and the polar plane along $\theta = 0$. We treat the equatorial plane as a disk from which the neutrinos are coming out and depositing their energy along different radial points on the rotation axis. Here we are interested in the EDR via neutrino pair annihilation near the rotation axis, $\theta = 0$ [11, 12]. It has been shown earlier [11] that in the absence of gravitation the θ dependence of the EDR is weak for small values of θ . Although the θ dependence of the gravitational effect may not necessarily be small, here we calculate the EDR at $\theta = 0$ and assume it to be approximately the same for small values of θ .

The energy deposited per unit volume per unit time is given as [6, 7]

$$\dot{q}(r, \theta) = \int \int f_\nu(p_\nu, r) f_{\bar{\nu}}(p_{\bar{\nu}}, r) (\sigma |v_\nu - v_{\bar{\nu}}| \epsilon_\nu \epsilon_{\bar{\nu}}) \frac{\epsilon_\nu + \epsilon_{\bar{\nu}}}{\epsilon_\nu \epsilon_{\bar{\nu}}} d^3 p_\nu d^3 p_{\bar{\nu}}, \quad (2)$$

where f_ν ($f_{\bar{\nu}}$) is the number density of neutrino (antineutrino), v_ν is the neutrino velocity, and σ is the rest frame cross section for the process $\nu\bar{\nu} \rightarrow e^+e^-$. The Lorentz invariant term is given by

$$(\sigma|v_\nu - v_{\bar{\nu}}|\epsilon_\nu\epsilon_{\bar{\nu}}) = \frac{DG_F^2}{3\pi}(\epsilon_\nu\epsilon_{\bar{\nu}} - p_\nu \cdot p_{\bar{\nu}})^2 \quad (3)$$

with

$$G_F^2 = 5.29 \times 10^{-44} \text{cm}^2 \text{MeV}^{-2}; \quad (4)$$

$$D = 1 \pm 4\sin^2\theta_w + 8\sin^4\theta_w. \quad (5)$$

$\sin^2\theta_w = 0.23$. (+) sign is for electron neutrino-antineutrino pair and (-) sign for other pairs.

As the geometry of the equatorial plane is circular, we can decouple the energy and angular dependence. Thus the rate of energy deposition is

$$\dot{q}(r) = \frac{DG_F^2}{3\pi} F(r) \int \int f_\nu f_{\bar{\nu}} (\epsilon_\nu + \epsilon_{\bar{\nu}}) \epsilon_\nu^3 \epsilon_{\bar{\nu}}^3 d\epsilon_\nu d\epsilon_{\bar{\nu}} \quad (6)$$

where, $F(r)$, the angular integral, is given by [12]

$$\begin{aligned} F(r) = & \frac{2\pi^2}{T_{eff}^9} \left(\frac{e^{\gamma+\rho}}{e^{\gamma+\rho} - r^2\omega^2 \sin^2\theta e^{\gamma-\rho}} \right)^4 \left(2 \int_{\theta_m}^{\theta_M} d\theta_\nu T_0^5 \sin\theta_\nu \int_{\theta_m}^{\theta_M} d\theta_{\bar{\nu}} T_0^4 \sin\theta_{\bar{\nu}} \right. \\ & + \int_{\theta_m}^{\theta_M} d\theta_\nu T_0^5 \sin^3\theta_\nu \int_{\theta_m}^{\theta_M} d\theta_{\bar{\nu}} T_0^4 \sin^3\theta_{\bar{\nu}} + 2 \int_{\theta_m}^{\theta_M} d\theta_\nu T_0^5 \cos^2\theta_\nu \sin\theta_\nu \int_{\theta_m}^{\theta_M} d\theta_{\bar{\nu}} T_0^4 \cos^2\theta_{\bar{\nu}} \sin\theta_{\bar{\nu}} \\ & \left. - 4 \int_{\theta_m}^{\theta_M} d\theta_\nu T_0^5 \cos\theta_\nu \sin\theta_\nu \int_{\theta_m}^{\theta_M} d\theta_{\bar{\nu}} T_0^4 \cos\theta_{\bar{\nu}} \sin\theta_{\bar{\nu}} \right) \quad (7) \end{aligned}$$

where $\theta_\nu, \theta_{\bar{\nu}}, \theta_m$ and θ_M are explained later.

We calculate the energy integral numerically, integrating it from 0 to ∞ . Both the angular and energy integrals have temperature dependence which is affected by the GR and rotational effects. T_{eff} is the effective temperature of the disc that is observed in the comoving frame. The value of T_0 is the temperature of the disc that is observed at infinity. In the locally nonrotating frame a neutrino moves normally to the direction of the disk motion. In that case, the temperature suffers redshift both due to disk rotation and gravitation. Therefore the two temperature T_0 and T_{eff} are related [12], as

$$T_0 = \frac{T_{eff}}{\Gamma} \sqrt{g_{tt} - \frac{g_{t\phi}^2}{g_{\phi\phi}}}. \quad (8)$$

$g_{tt}, g_{t\phi}$ and $g_{\phi\phi}$ are calculated from the metric. Γ is the Lorentz factor defined as $\Gamma = \frac{1}{\sqrt{1-v^2}}$ with v being $v = (\Delta - \omega)r\sin\theta e^{-\rho}$, Δ the rotational velocity of the star.

In an earlier paper [14] a detailed GR calculation of the neutrino path has been done. For the present coordinate system, the 4- momentum [12] is given by the equation,

$$g_{\mu\nu}p^\mu p^\nu + \mu_m^2 = 0, \quad (9)$$

where μ_m is the rest mass of the particle and $p^\mu = \frac{dx^\mu}{d\lambda}$, λ being the affine parameter. After a bit of algebra the final geodesic equation can be written as,

$$e^{2\alpha} \left[\frac{A(r, \theta)}{A(r, \theta) + \omega} \right]^2 \frac{e^{(\gamma-\rho)r^2 \sin^2 \theta}}{e^{2\alpha}} \tan^2 \theta_r \left[\omega(1 - \omega b) + \frac{e^{2\rho} b}{r^2 \sin^2 \theta} \right]^2 - e^{(\gamma+\rho)} (1 - \omega b)^2 + \frac{b^2}{r^2 \sin^2 \theta} e^{(\gamma+3\rho)} = 0. \quad (10)$$

$A(r, \theta)$ is defined as

$$A(r, \theta) = \frac{L}{E - \omega L} \frac{e^{2\rho}}{r^2 \sin^2 \theta}$$

where E and L are the total energy and total angular momentum of the neutrino measured from infinity.

This equation can be solved using the potentials obtained from **rns** code to obtain a minimum radius $r = R_{MPR}$, the minimum photosphere radius, below which a massless particle (neutrino) emitted tangentially to the stellar surface ($\theta_R = 0$) would be gravitationally bound. This is also the minimum radius from which the neutrinos can come out from the disk.

From an angle θ_ν at a point $(r, 0)$ (refer fig. 1), we can trace back a trajectory of the neutrino to its emission point on the disc. Thus the temperature T_0 depends on θ_ν , and therefore T_{eff} appears in the integration of θ_ν in the angular integral. The neutrinos are coming out of the disk and depositing their energy along the rotation (symmetry) axis of the disk. The minimum radius from which the neutrinos can come out is given by MPR. The outermost point from which the neutrinos come out is the surface of the disc (*i. e.* the surface of the star along the equatorial plane). Figure 1. shows the layout of our problem, where neutrinos coming out from the disk are depositing their energy along the rotation axis. Neutrinos emitted from the disk at $(r, \theta) = (R, \frac{\pi}{2})$, arrive at point $(r, 0)$ at the rotation axis. A neutrino coming from the MPR (R_{MPR}) subtend angle θ_m with the rotation axis, the minimum value of θ_ν and that coming from the outer surface of the disk (R_{Sur}) subtends angle θ_M , the maximum value of θ_ν . The angle θ_ν is defined as the angle between position

vector r and p_ν . The minimum and the maximum angles are given by

$$\sin\theta_m = \frac{(R_{MPR}e^{-\rho_{MPR}})(1 + \omega r e^{-\rho})}{(1 + \omega_{MPR}R_{MPR}e^{-\rho_{MPR}})(r e^{-\rho})} \quad (11)$$

and

$$\sin\theta_M = \frac{(R_{Sur}e^{-\rho_{Sur}})(1 + \omega r e^{-\rho})}{(1 + \omega_{Sur}R_{Sur}e^{-\rho_{Sur}})(r e^{-\rho})}. \quad (12)$$

The EDR is now calculated by integrating \dot{q} over the proper volume. We calculate the amount of energy deposited along the rotation axis within an angular opening of about ten degrees [11]. The point on the rotation axis from where we start our calculation is $r = R_{MPR}$. Therefore the EDR is given by

$$EDR = 2\pi \int_{r_n}^{r_{n+1}} dr \int_{\theta_1}^{\theta_2} \dot{q}(r) r^2 \sin\theta \frac{e^{(2\alpha+(\gamma-\rho)/2)}}{\sqrt{1-v^2}} d\theta. \quad (13)$$

The integration is done over radial bin of $100m$, **i.e.** $r_{n+1} - r_n = 100m$, and an angular width of ten degree, **i.e.** $\theta_1 = 0^\circ$ and $\theta_2 = 10^\circ$.

The observed luminosity at infinity L_∞ of the neutrino pairs annihilation is given by

$$L_\infty = \left[\frac{g^2_{t\phi}(R)}{g_{\phi\phi}(R)} - g_{tt}(R) \right] L(R) \quad (14)$$

The neutrino luminosity at the MPR is

$$L(R) = L_\nu + L_{\bar{\nu}} = \frac{7}{16} 4\pi R^2 a T_{eff}^4(R) \quad (15)$$

where a is the radiation constant.

In general the total neutrino luminosity will depend on the neutrino emissivity and can be evaluated with the volume integral of angle averaged neutrino emissivity [20]. Since some of the neutrinos will be depositing their energy in e^+e^- pairs, the total emissivity and hence neutrino luminosity will decrease accordingly. One should also consider the actual number of neutrinos trapped, the number of which will depend on the neutrino energy, matter density and temperature. In eqn.[14] we have considered only the total luminosity as observed due to the red-shift in temperature. Moreover, eqn. [15] is valid for the case where neutrino chemical potential inside MPR is small enough compared to the temperature. Hence, the above values of luminosity, given by eqns. (14) and (15) will provide with a lower limit.

We consider a NS which is of the order of a minute old. The average neutrino energy is of the order of few MeV , and the neutrino mean free path is of the order of the radius of the

star [21]. We start our calculation with three different central temperature, $1MeV$, $5MeV$ and $10MeV$. We consider the neutrinos comes out from the disk beyond the MPR (which is $4Km$). At this distance the star is much less dense than the central part, actually if the central density is 7 times the nuclear saturation density, the density at MPR is less than 4 times nuclear matter saturation density. In our problem we are considering only the thermal neutrinos. Following the prescription given in ref. [22], the mean free path of the neutrinos with such energy and the given density is of the order of kilometers. Other calculations [23, 24] do not alter the result much and the mean free path of the neutrinos for density and temperature of the star considered in the present study remains of the order of kilometers. Therefore the neutrinos can travel quite a distance within the star and deposit their energy along the rotation axis.

IV. RESULTS

We choose the central density of the star to be $1 \times 10^{15} gm/cm^3$, for which the Keplerian velocity is $0.61 \times 10^4 s^{-1}$. For comparison we have also done the calculation for a slow star ($0.4 \times 10^4 s^{-1}$). For the Keplerian star the equatorial radius is $16km$, which serves as the outermost point of the disk. The innermost point from which the neutrinos can contribute in the EDR is the MPR, which is $4km$ from the center. The same two points for the slowly rotating star are $12km$ and $4.7km$ respectively [14].

Before discussing our results, we would like to mention that our present approach is a simplification of the actual scenario and the effect of neutrino inside NS may be more complicated. The trapping of neutrino inside the MPR may result in a change in the temperature profile as well as structure of the NS. For example, during the Neutrino cooling period, the external layer near the MPR, where the neutrino trapping reaches highest efficiency, temperature may become higher than the temperature in the inner layers. This effect will then lead to an inflow of heat to the interior through some processes [25]. This could then influence the internal structure of the NS.

It has also been found [26] that the presence of neutrino changes the composition of matter significantly with respect to the neutrino free case; matter becomes more proton rich and hyperons appear at higher densities. Since the presence of hyperons softens the EOS, in presence of neutrino trapping EOS will become stiffer.

In general the neutrino mean free path decreases with temperature (at fixed density) and density (for fixed temperature)[21, 27]. So the high neutrino trapping and hence the increase in the temperature near the MPR will result in the decrease in the mean free path thereby changing the temperature profile of the star [20].

The emissivity (energy emitted per unit volume per unit time) may be taken to be proportional to T^6 and $n_B^{1/3}$ [28], n_B being the baryon density. This is the amount of energy (per unit volume per unit time) deposited in the matter, if all the neutrinos are trapped. For a nonrotating spherical NS, made up of nucleons only, the density n_B may be taken to be proportional to r^{-3} , r being the radius of the star. Hence the emissivity varies as $1/r$. If we assume all the energy deposited goes into thermal energy, then temperature due to this deposition will also vary as $1/r$. So there will be a temperature gradient (TG) of $1/r$ along the disk in the present problem. Since a rotating star changes its shape and becomes an oblate spheroid, the density and hence the temperature variation may change accordingly.

Here we first assume an isothermal disk, *i.e.* there is no TG along the disk. Figure 2 shows the variation of the EDR along the rotation axis of the star for three different temperatures ($1MeV$, $5MeV$ and $10MeV$). The neutrino mean energy is in the range between $1-10MeV$. Initially the EDR increases with distance and it reaches a maximum value at a distance of $16km$ and after that it falls off gradually. The EDR is maximum for central temperature $T_C = 10MeV$ and minimum for $T_C = 1MeV$, which shows that the EDR increases with increase in temperature of the disk. For $T_C = 10MeV$ the maximum EDR is at a point about $16km$ and the value of EDR is $3.3 \times 10^{47}erg/s$, and the total energy deposited per second in the vicinity of the star (calculated by integrating EDR) is of the order of $10^{49}erg/s$. If we further increase the temperature to $20MeV$ the total energy deposited per second near the star is of the order of $10^{51} - 10^{52}erg/s$, close to that of the energy liberated during GRBs. At this temperature the observed luminosity L_∞ is of the order of the EDR. This implies that the neutrino becomes optically thick for the pairs annihilation. For $T_C = 10MeV$ the EDR is 0.1 percent of the neutrino luminosity (L_∞ is $9 \times 10^{51}erg/s$).

Let us try to understand the peculiar nature of initial increase in EDR with distance. From figure 3, for the isothermal disk the temperature T_0 , seen by an observer at infinity, increases as the distance from the star increases. In this case T_0 becomes hotter with distance and therefore it approaches T_{eff} (equal to T_C for isothermal disk) at large distance. As the disk is isothermal everywhere along the disk its temperature is constant. At close distance

from the center of the disk along the rotation axis the contribution from the the inner part of the disk is greater and at larger distance the contribution is mostly from the outer part of the disk. This does not make any difference for the isothermal disk as all along the disk the temperature is constant. As we go to a larger distance the GR effect reduces and T_0 approaches T_{eff} . But for the disk with TG, T_0 will decrease with distance, unlike the above isothermal case. This is because as we go to a larger distance along the rotation axis the contribution from the outer part of the disk will dominate. For the disk with TG as we move outward along the disk its temperature decreases rapidly and is much smaller than the central temperature. Although at a large distance GR effect will become less, T_0 will still decrease as here the temperature contribution comes mainly from the outer surface of the disk which is at a much lower temperature.

Looking at figure 4, for the isothermal disk, we see that $F(r)$ (eqn. 7) decreases with distance and falls to zero after $25km$. For the disk with temperature gradient we see that $F(r)$ initially increases and after about $8km$ it starts to fall, unlike the isothermal case where it decreases throughout.

For an isothermal disk, initially the temperature dominates and EDR increases but after some distance (where $F(r)$ decreases but temperature increases) $F(r)$ becomes the dominating factor and the EDR decreases. The disk temperature observed at infinity is higher at outer region and the EDR increases with temperature. On the other hand the Doppler effect due to disk rotation reduces the EDR. The EDR calculated non relativistically at the MPR is half that of the relativistic case [6, 7].

The EDR via neutrino pair annihilation is strongly dependent on disk temperature. Therefore let us calculate the pair annihilation of neutrinos emitted from the disk with TG. As discussed previously one of the other extreme case will be the disk with $TG = 1/r_d$ (r_d is the radial distance along the disk). For comparison we also plot curves with $TG = 1/r_d^{1/2}$. Due to variation of disk temperature, the temperature at the MPR (initial temperature from which our calculation starts) is much less than that of the central temperature. For $T_C = 10MeV$ and $TG = 1/r_d$ the T_0 at MPR is less than $2MeV$ and the temperature falls very rapidly and reaches negligible values beyond $50km$. The case of $TG = 1/r_d^{1/2}$ the situation is not so pessimistic. For the hotter star ($T_C = 10MeV$), T_0 at the MPR is above $3MeV$ and does not fall to zero at large distances.

Figure 5 shows the variation of EDR, for disk with TG, along the rotation axis of the

star for two different central temperatures $5MeV$ and $10MeV$. For $T_C = 10MeV$ with $TG = 1/r_d$ (surface temperature is $0.4MeV$), the initial value of EDR is $10^{40}erg/s$ and it falls very sharply to zero at a distance of $50km$. The total EDR is near to $10^{41}erg/s$. The disadvantage of the TG disk is that the EDR is 10 percent of the neutrino luminosity ($L_\infty = 9 \times 10^{41}erg/s$), thereby again imposing the problem of neutrino opacity and also the baryon contamination problem [12]. For $T_C = 5MeV$ the fall is even sharper. For $TG = 1/r_d^{1/2}$ the fall in EDR is much slower and near the MPR the EDR, for central temperature $T_C = 10MeV$ (surface temperature is $2MeV$), is of the order of $10^{43}erg/s$. For the colder disk the initial EDR is much smaller.

These curves show that the EDR is very sensitive to temperature as well as to TG. At smaller r , temperature is the governing factor in determining the nature of EDR, therefore EDR decreases. As r increases, $F(r)$, which strongly decreases with r , becomes the dominant factor and EDR decreases further. Hence, here the EDR falls off much faster than the isothermal case.

The value of EDR increases if we go inwards from the MPR. If we go close towards the center, the EDR increases due to the increase in temperature of the disk. Near the center the value of EDR for non relativistic calculation and that with GR effect along with TG becomes the same. But as we go outwards the value of the EDR for the GR calculation becomes less than the non-relativistic case. This is because, for GR calculation, the disk temperature falls as we go outwards along the disk.

The slowly rotating star has a much smaller disk (smaller equatorial plane) and its MPR is $4.7km$, larger than the keplerian star. The gross nature of the EDR is more or less same, but the value of maximum EDR is slightly larger. The temperature and $F(r)$ plot is about the same, only the maximum value of $F(r)$ being slightly larger. Due to this reason the EDR for the slow star is greater (twice) than that for the keplerian star. Therefore the rotational velocity seems to have some negative effect on the EDR and as the rotational velocity increases the EDR decreases. The other important conclusion from the above discussion is that the isothermal disk is more advantageous while considering the opacity and baryon contamination problem (as for isothermal disk the ratio of EDR on L_∞ is minimum).

V. SUMMARY AND DISCUSSION

In this article we have investigated the GR effect on the EDR of $\nu\bar{\nu} \rightarrow e^+e^-$ reaction in a rotating neutron star described by CST metric. Here we have calculated the EDR for the neutrinos coming out of the equatorial disk and depositing energy along the rotation axis of the star, above the equatorial plane. The bending of the neutrino trajectories, and the redshift due to disk rotation along with gravitation has been taken into consideration.

We find that for an isothermal disk, initially the EDR increases with distance and reaches a maximum value and then decreases with distance. This is due to the fact that there is a competition between the temperature and the Doppler effect due to disk rotation (characterized by $F(r)$). The temperature observed by the observer at infinity (T_0) increases and at large distance becomes same as that of the effective temperature (T_{eff}) whereas $F(r)$ decreases with increase in distance from the disk. In the case of disk with TG, the EDR falls off very quickly with distance. For $TG = 1/r_d$ the slope with which EDR falls is much greater than that for $TG = 1/r_d^{1/2}$. The $F(r)$ increases initially but it also decreases after some distance and falls very near to zero at about $50km$. Both the isothermal and TG picture shows that the EDR is a very sensitive function of temperature, and as temperature decreases the EDR also decreases. We also conclude that the isothermal disk is more advantageous in avoiding opacity and baryon contamination problem. The slow star shows more or less the same nature except for the fact that EDR for the slow star is about twice that for the keplerian star. The rotational effect reduces the EDR.

The method in which the angular integral is calculated is somewhat similar to that mentioned in ref. [11, 12]. In that paper, the authors had calculated the energy deposition due to neutrino-antineutrino annihilation above a black hole. The Kerr metric describes the black hole, which is flat disk, and the neutrinos emitted from the disk contribute to the deposited energy. The calculation of the energy integral is different in our case and the star is at much lower temperature. In our discussion the neutrino are emitted from the equatorial plane of the star, which is a circular disk, and deposit their energy along the rotation axis of the star (described in fig. 1). The innermost point from which the neutrino contribute is the MPR and the outermost point is the star surface. We have calculated the EDR for the neutrinos coming out from the equatorial plane. If we consider the star to be spherical, the total EDR for the entire star can be obtained multiplying the EDR calculated for the

equatorial plane by 2π , for which the EDR comes out to be of the order of 10^{50} ergs/s . If we consider long duration GRB this EDR is quite close to that of the energy liberated during the GRB ($\sim 10^{52} \text{ ergs}$).

To summarize, we find that the EDR is very sensitive both to the temperature and TG of the disk. The deposition energy is contributed mainly by the neutrinos arriving from the central region where the temperature is higher. The maximum energy is deposited near the surface. As we move outwards the deposition energy reduces and is smaller than that of the non relativistic calculation. The maximum amount of energy deposited for an isothermal disk is few times 10^{49} ergs/s , within a small angle. The isothermal disk is the most advantageous while avoiding opacity and baryon contamination problem. Taking the disk to be isothermal a rough estimate of a spherical star provides EDR quite close to that of the energy liberated during a long duration GRB. The total energy deposition from a rotating neutron star would also depend on the off-axis contributions, but expected that most of the contribution will be along the rotation axis (if it coincides with the magnetic poles), as the particles comes out of the star along the magnetic poles. The high energy neutrinos created during phase transition may further change the EDR. To get an estimate of the total energy deposition, one needs to do a detailed numerical simulation taking into account all possible contributions. Such a calculation is in progress and we hope to report it in the near future.

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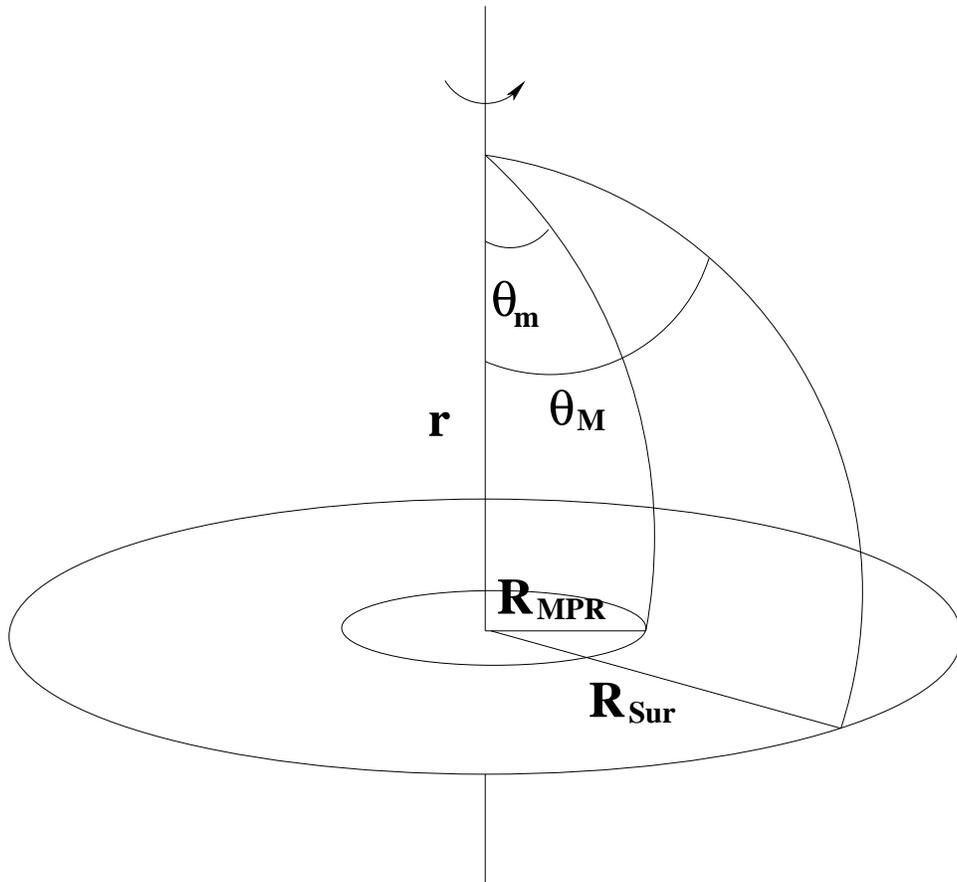


FIG. 1. Figure showing the layout of our problem. The neutrinos (antineutrinos) coming out of the equatorial plane, at an angle $(\theta_M - \theta_m)$ and depositing their energy along the rotation axis.

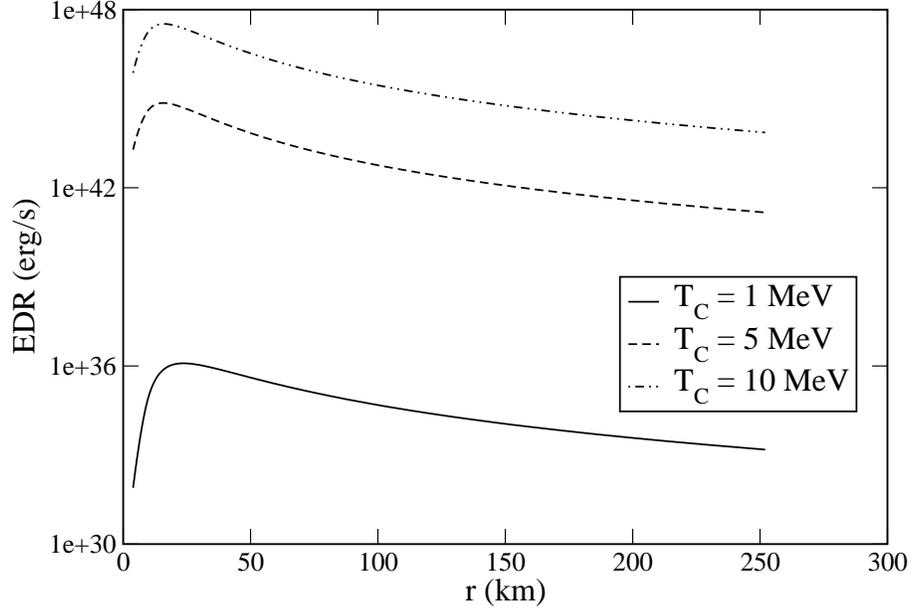


FIG. 2. Variation of EDR with distance along the rotation axis for the keplerian star having isothermal disk with three different central temperature.

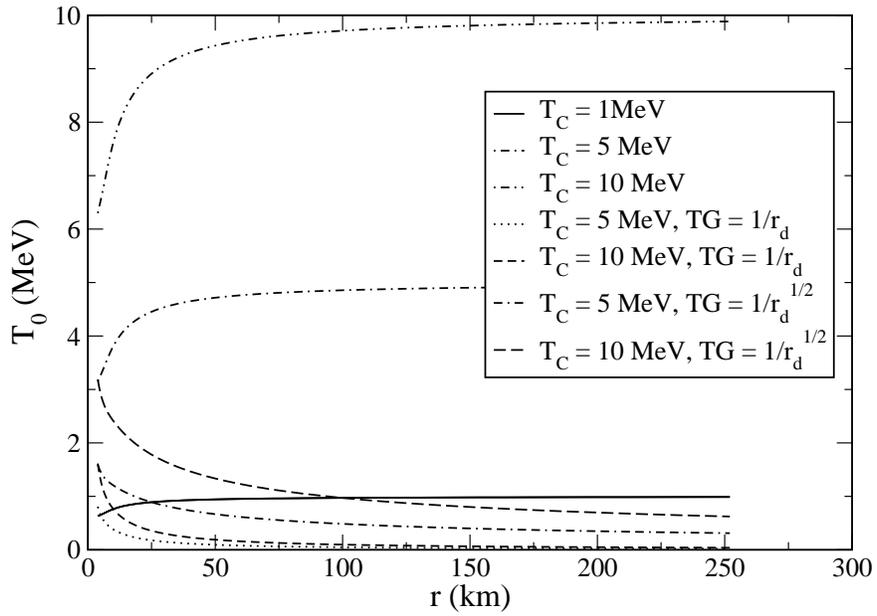


FIG. 3. Variation of temperature T_0 with distance along the rotation axis of the keplerian star. T_0 is plotted for both isothermal disk and for disk having two different temperature gradient.

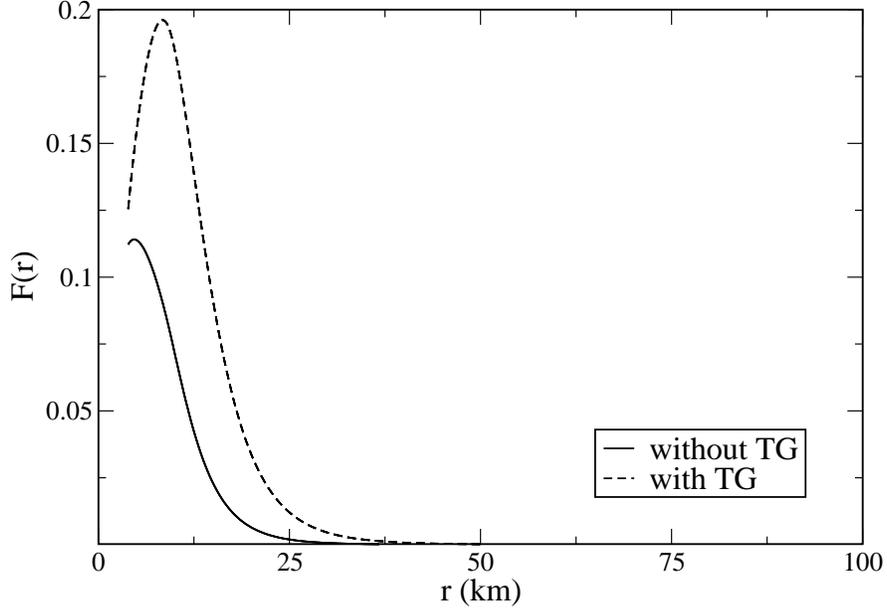


FIG. 4. Variation of $F(r)$ with distance along the rotation axis of the keplerian star. $F(r)$ is plotted both for isothermal disk and disk with temperature gradient.

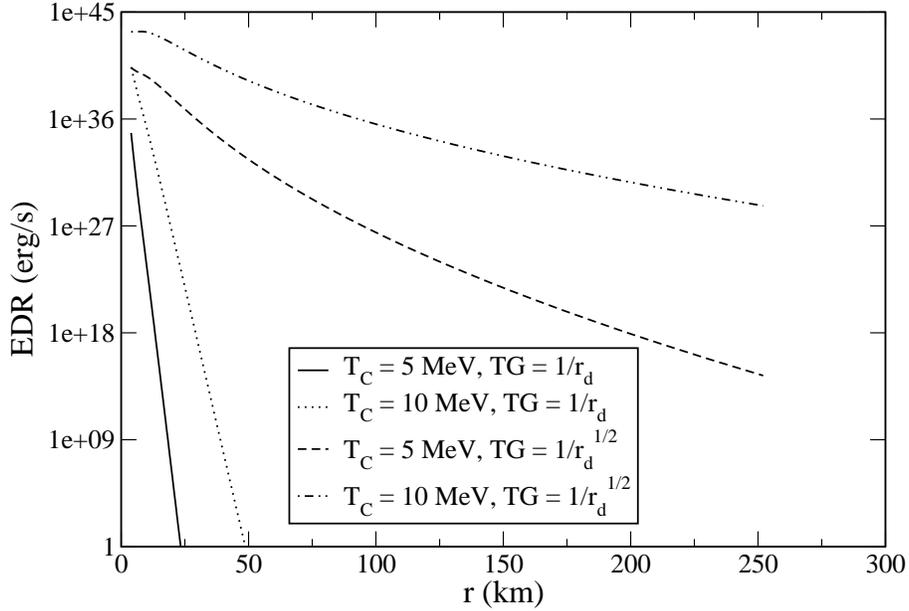


FIG. 5. Variation of EDR with distance along the rotation axis for the keplerian star. The plot have been done with disk having two different TG and the curves are plotted with two different central temperature.