

Gauge-invariant matter field actions from iterative Nöther coupling

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Abstract

Generalizing Deser's work on pure $SU(2)$ gauge theory, we consider scalar, spinor and vector matter fields transforming under arbitrary representations of a non-Abelian, compact, semisimple internal Lie group which is a global symmetry of their actions. These matter fields are coupled to Abelian gauge fields through the process of *iterative Nöther coupling*. This procedure is shown to yield precisely the same locally gauge invariant theory (with the non-Abelian group as the gauge group) as obtained by the usual *minimal coupling* prescription originating from the Gauge Principle. Prospects of this non-geometrical formulation, towards better understanding of physical aspects of gauge theories, are briefly discussed.

1 Introduction

To formulate theories of fundamental interactions invariant under local, non-Abelian gauge invariance, the standard practice is to appeal to the Gauge Principle ([1]-[3]) which is inspired in its turn by the principle of General Covariance underlying Einstein's general relativity. According to this principle, physical quantities must be gauge invariant, i.e., invariant under local gauge transformations. Implementation of the principle leads one to the *minimal coupling* prescription, under which any partial derivative of a field transforming non-trivially under the action of the gauge group, must be augmented by a *connection*

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term. This term compensates for the difference in gauge transformation property of the field at two different (neighbouring) points. In general relativity, this prescription is understood in the following way [4]: Naïve parallel transport on a curved surface (which is embedded in a higher dimensional flat space) of a tangent vector from its initial location, does not yield a vector which is tangent to the surface at the new location. We need to make a projection, of the naïvely parallel-transported vector, to the tangent space at the new location. This is effected by the connection term added on to the partial derivative of the vector. In gauge theory, the connection term is specified uniquely by the gauge transformation properties of the fields. Once the augmented (or ‘covariant’) derivatives are constructed and *curvatures* or field strengths of the gauge field are obtained through the Ricci identity, gauge-invariant actions for all fields can be written down. The Gauge Principle is, thus, a very *geometrical* principle.

Starting from the mid-1940s however, many physicists have sought more physical alternatives to this geometrical principle ([5]-[17]). Physicists have also questioned whether the Gauge Principle is truly a physical principle, since all dynamical variables in the theory must of necessity be gauge invariant. It is thus not clear precisely what new physical information is obtained from the gauge principle, apart from a statement of redundancy of some of the field degrees of freedom used to construct the theory [18].

Further, while the standard formulation has yielded a plethora of physical results all consistent with experimental data [19], certain very special physical aspects of non-Abelian gauge interactions, like the anti-screening property and asymptotic freedom, can only be understood after detailed calculation of renormalization-group beta function. *We aim to understand the more complicated physics of non-Abelian gauge theory as a result of simpler constituent dynamics based essentially on Abelian gauge invariance and non-Abelian global symmetries.* This is the aim of a programme initiated with the present paper.

The alternative approach that we are most motivated by has been proposed by Deser [11] (some more field theoretic works based on [11] are [20]-[24]). The starting point in Deser’s work is a Lagrangian with three copies of free Abelian gauge field, with the Lagrangian also possessing a global $SU(2)$ invariance. The global symmetry gives rise to a Nöther current for each species of the Abelian gauge field. These currents are then coupled to the Abelian gauge fields to generate an additional term in the Lagrangian which again is invariant under the global symmetry. From this new Lagrangian, one again constructs Nöther currents, and iterates this process until such currents cease to be generated. At the point of termination, one ends up with a Lagrangian with full $SU(2)$ *local* gauge invariance.

In this paper, we show that the procedure discussed above can be generalized to include *arbitrary matter fields*: starting with a globally $U(1)$ invariant action for charged scalar, spinor and vector fields, and then proceeding with the iterative Nöther coupling, yields the same *locally* $U(1)$ gauge invariant matter action as obtained from the minimal coupling prescription pertaining to local $U(1)$ gauge invariance. This procedure is then generalized to matter actions with global $SU(N)$ symmetry, with arbitrary representations of the matter fields under $SU(N)$, yielding at the end a locally $SU(N)$ gauge invariant

theory for the corresponding matter field. The number of iterations is always the same as the number of spacetime derivatives needed to describe the globally symmetric theory. The procedure thus amply illustrates that *the minimal coupling prescription need not be invoked ab initio to construct non-Abelian gauge-invariant matter field actions; iterative Nöther coupling achieves the same result.*

Various field theoretic actions have been derived in Refs. [20], [21], [23] and [24] by iterative Nöther coupling (also referred to as self-interaction). But, to our knowledge, *this procedure has not been applied before to derive $U(1)$ and $SU(N)$ gauge-invariant actions for matter fields of various spins.*

The motivation for our work is as follows. On the one hand, there is a non-Abelian gauge theory. On the other hand, there is a theory constructed by starting with Abelian gauge fields and a global non-Abelian symmetry, and then iteratively coupling the Nöther current of that symmetry with the Abelian gauge fields. When the iteration stops, one needs to do an *identification* of fields between the two theories to complete the equivalence. In this way, any non-Abelian gauge theory (with matter fields in our paper) has been shown here to be classically equivalent to a theory *without any explicit non-Abelian gauge invariance (redundancy)*, but having instead a non-Abelian physical (global) symmetry, and including all types of realistic matter. This way of interpreting Deser's original program leads to the remarkable possibility that one can deal with the dynamics of non-Abelian gauge theories avoiding the mathematical complexity inherent in such theories.

The paper is organized as follows. In Sec. 2, we obtain the $U(1)$ gauge-invariant Lagrangians involving matter fields with spin 0, spin 1/2 and spin 1. In Sec. 3, we obtain the pure gauge Lagrangian for the $SU(N)$ gauge group, extending Deser's $SU(2)$ gauge group calculation. Sec. 4 contains our main results leading to the generalization of the iterative Nöther coupling procedure to the case of matter fields of spin 0, 1/2 and 1, transforming under arbitrary representations of $SU(N)$ as the global symmetry group, and demonstrating that the resultant action is identical to the one obtained from 'gauging' the appropriate matter actions through the minimal coupling prescription. In Sec. 5, we present our conclusions and outlook.

2 $U(1)$ gauge invariance for matter field

2.1 Scalar field

We start with the Lagrangian of a free complex scalar field and a free Abelian gauge field

$$\mathcal{L}_0 = (\partial_\mu \phi)^* (\partial^\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (1)$$

which is invariant under a global $U(1)$ transformation $\phi \rightarrow \phi e^{i\omega}$ (in addition to invariance under $U(1)$ gauge transformation of A_μ). For an infinitesimal transformation, $\delta\phi = i\omega\phi$ and $\delta\phi^* = -i\omega\phi^*$. To construct the Nöther current $j^{1\mu}$ (where the superscript 1 stands

for the first iteration), we use

$$\frac{\partial \mathcal{L}_0}{\partial(\partial_\mu \phi)} \delta \phi + \frac{\partial \mathcal{L}_0}{\partial(\partial_\mu \phi^*)} \delta \phi^* = \omega j^{1\mu} \quad (2)$$

which gives

$$j^{1\mu} = ie(\phi \partial^\mu \phi^* - \phi^* \partial^\mu \phi). \quad (3)$$

We add to the Lagrangian the new term

$$\mathcal{L}_1 = j^{1\mu} A_\mu. \quad (4)$$

Then from \mathcal{L}_1 we get a further contribution to the Nöther current, making the replacements $\mathcal{L}_0 \rightarrow \mathcal{L}_1$ and $j^{1\mu} \rightarrow j^{2\mu}$ in (2):

$$j^{2\mu} = 2e^2 \phi^* \phi A^\mu. \quad (5)$$

So we further add

$$\mathcal{L}_2 = \frac{1}{2} j^{2\mu} A_\mu \quad (6)$$

to the Lagrangian. Then $\mathcal{L}_2 = e^2 \phi^* \phi A^\mu A_\mu$, and we see that the factor of $\frac{1}{2}$ in (6) is needed to ensure that

$$\frac{\delta}{\delta A_\mu} \int d^4x \mathcal{L}_2 = j^{2\mu}. \quad (7)$$

As \mathcal{L}_2 contains does not contain any derivative of ϕ , no further contribution to the Nöther current is generated, and the iteration stops here. Thus the final Lagrangian is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2. \quad (8)$$

It can be easily checked that this final Lagrangian equals (1) with ∂_μ replaced by

$$D_\mu = \partial_\mu + ieA_\mu \quad (9)$$

in the scalar part. Thus iterative Nöther coupling has converted the Lagrangian in (1), which had only global $U(1)$ invariance in the matter part, to the Lagrangian in (8), in which the matter part has $U(1)$ gauge invariance.

We also note that \mathcal{L}_0 , \mathcal{L}_1 and \mathcal{L}_2 split up the Lagrangian with full $U(1)$ invariance into the propagator, the vertex of order e and the vertex of order e^2 respectively. This will happen in the other cases also.

2.2 Spinor field

$$\mathcal{L}_0 = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (10)$$

is invariant under a global $U(1)$ rotation $\delta\psi = ie\omega\psi$, which gives the Nöther current

$$j^{1\mu} = -e\bar{\psi}\gamma^\mu\psi. \quad (11)$$

We add \mathcal{L}_1 as in (4) to \mathcal{L}_0 . Since \mathcal{L}_0 has only a single spacetime derivative of the field, $j^{1\mu}$ has no field derivative and so, unlike in the previous case, \mathcal{L}_1 gives no further Nöther current. Our final Lagrangian is therefore $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$. This equals (10) with ∂_μ replaced by D_μ , as given by (9), in the spinor part.

2.3 Vector field

$$\mathcal{L}_0 = -\frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^*(\partial^\mu W^\nu - \partial^\nu W^\mu) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (12)$$

is invariant under $\delta W_\mu = ie\omega W_\mu$ and $\delta W_\mu^* = -ie\omega W_\mu^*$. This gives

$$j^{1\mu} = ie(W^{\nu*}(\partial_\mu W_\nu - \partial_\nu W_\mu) - W^\nu(\partial_\mu W_\nu^* - \partial_\nu W_\mu^*)). \quad (13)$$

Then (4) gives

$$j^{2\mu} = -e^2(2A^\mu W^{\nu*}W_\nu - A^\nu(W_\mu^*W_\nu + W_\nu^*W_\mu)). \quad (14)$$

Again (6) and (8) lead us to (12) with ∂_μ replaced by D_μ in the part involving the vector field W_μ [25].

3 $SU(N)$ gauge invariance for pure gauge field

The calculation of this Section extends Deser's [11] calculation, which was done for the $SU(2)$ group, to the case of the $SU(N)$ group. This is also intended to set the stage for the inclusion of matter fields. We start with

$$\mathcal{L}_0 = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) \quad (15)$$

where each A_μ^a (a running from 1 to $N^2 - 1$) is an Abelian gauge field. This Lagrangian has $U(1)$ gauge invariance $A_\mu^a \rightarrow A_\mu^a + \partial_\mu\omega^a$ for each species of A_μ^a . It is also invariant under the global $SU(N)$ transformation

$$\delta A_\mu^a = gf^{abc}A_\mu^b\alpha^c. \quad (16)$$

To see this, note that (upto a constant factor) \mathcal{L}_0 in (15) equals $\text{Tr}[(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)]$ where the matrix $A_\mu = A_\mu^a T^a$. (We use the result that $\text{Tr}[T^a T^b]$ is proportional to δ^{ab} , T^a being the generators of $SU(N)$). So \mathcal{L}_0 is invariant under $A_\mu \rightarrow UA_\mu U^\dagger$ where U is a constant $SU(N)$ matrix. Eq. (16) is the infinitesimal version of this transformation.

The Nöther current in the first iteration satisfies the relation

$$\frac{\partial \mathcal{L}_0}{\partial(\partial_\mu A_\nu^a)} \delta A_\nu^a = j^{1c\mu} \omega^c. \quad (17)$$

This gives us

$$j^{1c\mu} = -g f^{abc} (\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) A_\nu^b. \quad (18)$$

We set up the next term in the Lagrangian as $\mathcal{L}_1 = \frac{1}{2} j^{1c\mu} A_\mu^c$. Now (upto a constant factor), \mathcal{L}_1 equals $\text{Tr}[(\partial^\mu A^\nu - \partial^\nu A^\mu) A_\mu A_\nu]$. (This can be shown from the results that $\text{Tr}[T^a [T^b, T^c]]$ is proportional to f^{abc} and that f^{abc} is completely antisymmetric.) So \mathcal{L}_1 also is invariant under $A_\mu \rightarrow U A_\mu U^\dagger$. Therefore we iterate the process once more to obtain

$$j^{2e\mu} = g^2 f^{abc} f^{ade} A^{b\mu} A^{c\nu} A_\nu^d. \quad (19)$$

Then we add

$$\mathcal{L}_2 = \frac{1}{4} j^{2e\mu} A_\mu^e \quad (20)$$

so that we satisfy

$$\frac{\delta}{\delta A_\mu^a} \int d^4x \mathcal{L}_2 = j^{2a\mu}. \quad (21)$$

\mathcal{L}_2 is also global $SU(N)$ invariant, as it equals (upto a constant factor) $\text{Tr}[[A^\mu, A^\nu][A_\mu, A_\nu]]$. But as \mathcal{L}_2 does not involve derivatives, the iteration stops, and the final Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$ is the familiar $SU(N)$ gauge invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c. \quad (22)$$

It can also be written as $\mathcal{L} = -\frac{1}{4} \text{Tr}[(\partial^\mu A^\nu - \partial^\nu A^\mu + ig[A_\mu, A_\nu])^2]$. The terms of order g and order g^2 are respectively \mathcal{L}_1 and \mathcal{L}_2 deduced above.

4 $SU(N)$ gauge invariance for matter fields

4.1 Scalar field

We start with the action

$$\mathcal{L}_0 = (\partial_\mu \phi_i)^* (\partial^\mu \phi_i) - \frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 \quad (23)$$

where the summation over i is from 1 to the dimension of any representation of $SU(N)$ under which we want the complex scalar field to transform. This is invariant under the global $SU(N)$ transformations

$$\phi^i \rightarrow \exp(ig\alpha^a T_{ij}^a \phi_j). \quad (24)$$

Using $\delta\phi^i = ig\alpha^a T_{ij}^a \phi_j$ and $\delta\phi^{i*} = -ig\alpha^a \bar{T}_{ij}^a \phi_j^*$ (where the bar over T denotes complex conjugation), we obtain the Nöther current

$$j^{1a\mu} = ig (T_{ij}^a (\partial^\mu \phi_i^*) \phi_j - \bar{T}_{ij}^a (\partial^\mu \phi_i) \phi_j^*) . \quad (25)$$

We define the next term in the Lagrangian as $\mathcal{L}_1 = j^{1a\mu} A_\mu^a$. In terms of the matrix A^μ and the column vector Φ constructed out of ϕ_i , $\mathcal{L}_1 = ig((\partial^\mu \Phi^\dagger) A_\mu \Phi - \Phi^\dagger A_\mu (\partial^\mu \Phi))$, which is invariant under the global $SU(N)$ transformations $\Phi \rightarrow U\Phi$ and $A_\mu \rightarrow UA_\mu U^\dagger$. Therefore we generate the next contribution to the Nöther current

$$j^{2b\mu} = g^2 (\bar{T}_{ij}^a T_{ik}^b \phi_j^* \phi_k + T_{ij}^a \bar{T}_{ik}^b \phi_j \phi_k^*) A^{a\mu} . \quad (26)$$

This current is of the form $j^{2b\mu} = S^{ab} A^{a\mu}$ where $S^{ab} = S^{ba}$. Therefore (21) is satisfied when we set the next term in the Lagrangian as

$$\mathcal{L}_2 = \frac{1}{2} j^{2b\mu} A_\mu^b . \quad (27)$$

This, like \mathcal{L}_0 and \mathcal{L}_1 , is global $SU(N)$ invariant as it equals $g^2 \Phi^\dagger A^\mu A_\mu \Phi$. But the iteration stops, and it can be checked that in the final Lagrangian, the matter part of the starting Lagrangian (23) has been modified into $(D_\mu \phi)_i^* (D^\mu \phi)_i$ where

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu + ig A_\mu^a (T^a)_{ij} , \quad (28)$$

so that we have arrived at $SU(N)$ gauge invariance. Writing in the matrix form $(D_\mu \Phi)^\dagger (D^\mu \Phi)$, where $D_\mu = \partial_\mu + ig A_\mu$, we find that the terms of order g and order g^2 are respectively \mathcal{L}_1 and \mathcal{L}_2 deduced above.

An important point is that the pure gauge part of \mathcal{L}_0 in (23) also generates Nöther current due to invariance under (16). But since the corrections \mathcal{L}_1 and \mathcal{L}_2 to the matter field Lagrangian do not involve derivatives of A_μ^a , the iterative Nöther coupling from the pure gauge part proceeds simultaneously with (as the same parameters α^a are involved), but independent of, the iterative Nöther coupling from the matter part. So together with $SU(N)$ gauge invariance in the matter part, we end up with (22) as in Section 3. This will happen for the spinor and the vector fields also.

For completeness, we note that the calculations of this section are easily modified when the matter field transforms under the adjoint representation of $SU(N)$. As this representation is real, we start with $N^2 - 1$ species of real scalar field ϕ^a . The global $SU(N)$ invariant Lagrangian is

$$\mathcal{L}_0 = \frac{1}{2} (\partial_\mu \phi^a) (\partial^\mu \phi^a) - \frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 . \quad (29)$$

The generators have the elements $(T^b)_{ac} = if_{abc}$ and so the scalar field transforms as $\delta\phi^a = gf^{abc} \phi^b \alpha^c$. The currents from \mathcal{L}_0 and $\mathcal{L}_1 = j^{1a\mu} A_\mu^a$ are

$$j^{1c\mu} = gf^{abc} (\partial^\mu \phi^a) \phi^b , \quad (30)$$

$$j^{2e\mu} = g^2 f^{abc} f^{ade} \phi^b A^{c\mu} \phi^d . \quad (31)$$

Then adding $\mathcal{L}_2 = \frac{1}{2} j^{2e\mu} A_\mu^e$ gives the $SU(N)$ gauge invariant Lagrangian which contains the covariant derivative $(D_\mu)_{ac} = \delta_{ac} \partial_\mu + ig A_\mu^b (T^b)_{ac}$ in the matter part.

4.2 Spinor field

$$\mathcal{L}_0 = i\bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 \quad (32)$$

is invariant under the global $SU(N)$ rotation $d\psi_i = ig\alpha^a T_{ij}^a \psi_j$, giving $j^{1a\mu} = -g\bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j$ and $\mathcal{L} = \mathcal{L}_0 + j^{1a\mu} A_\mu^a$. This modifies the matter part in \mathcal{L}_0 with ∂_μ replaced by D_μ as in (28).

4.3 Vector field

$$\mathcal{L}_0 = -\frac{1}{2}(\partial_\mu W_{i\nu} - \partial_\nu W_{i\mu})^*(\partial^\mu W_i^\nu - \partial^\nu W_i^\mu) - \frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 \quad (33)$$

is invariant under $\delta W_i^\mu = ig\alpha^a T_{ij}^a W_j^\mu$ and $\delta W_i^{\mu*} = -ig\alpha^a T_{ij}^a W_j^{\mu*}$. This gives the current

$$j^{1a\mu} = ig \left(\bar{T}_{ij}^a W_{j\nu}^* (\partial^\mu W_i^\nu - \partial^\nu W_i^\mu) - T_{ij}^a W_{j\nu} (\partial^\mu W_i^{\nu*} - \partial^\nu W_i^{\mu*}) \right). \quad (34)$$

Then setting $\mathcal{L}_1 = j^{1a\mu} A_\mu^a$ gives

$$\begin{aligned} j^{2b\mu} = & -g^2 \left(\bar{T}_{ij}^a T_{ik}^b W_j^{\nu*} W_{k\nu} + T_{ij}^a \bar{T}_{ik}^b W_j^\nu W_{k\nu}^* \right) A^{a\mu} \\ & + g^2 \left(\bar{T}_{ij}^a T_{ik}^b W_j^{\mu*} W_{k\nu} + T_{ij}^a \bar{T}_{ik}^b W_j^\mu W_{k\nu}^* \right) A^{a\nu}. \end{aligned} \quad (35)$$

This current is of the form $j^{2b\mu} = S^{ab} A^{a\mu} + S'^{ab\mu\nu} A^{a\nu}$ where $S^{ab} = S^{ba}$ and $S'^{ab\mu\nu} = S'^{bav\mu}$. This ensures that (21) is satisfied when we set $\mathcal{L}_2 = \frac{1}{2} j^{2b\mu} A_\mu^b$. Again one can check that $\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$ has the covariant derivative (28) in the matter part of the Lagrangian.

5 Conclusions and outlook

Our paper is more than an explicit completion of the Deser program for non-Abelian gauge theories to include matter field sources. We have an alternative motivation for this apparent completion, as follows. The foregoing sections establish that non-Abelian gauge invariance of very general classes of field theories is a *dynamical consequence* of the iterative Nöther coupling procedure, where one has put in only the non-Abelian global symmetries and Abelian gauge invariance. If one is able to derive all dynamical results – classical and quantum – of non-Abelian gauge theories based on these invariances alone, one may not need to consider the full non-Abelian gauge invariance with all its complications. In that case, the property of asymptotic freedom (attributed to non-Abelian gauge field self-interactions) may be traced to a somewhat different physical origin. It can then become clearer why it is only the self-interactions of non-Abelian gauge fields that possess this property, in contrast to the entire gamut of fundamental interactions which are not asymptotically free. Also, within such a formulation, *infrared* strong-coupling phenomena like the phase transition to a quark-gluon plasma, quark confinement, and low energy hadron physics in general, might become easier to handle. One can even hope for a scenario in which Abelian gauge fields are used on a lattice for extracting non-perturbative dynamical information like hadron masses and decay widths.

Another motivation for this programme arises from the fact that Abelian gauge theories have already been formulated in terms of gauge-*invariant* fields, within the so-called ‘gauge-free’ approach [26], using the unique and natural projection operator given in the $U(1)$ gauge field action itself. Our current underpinning in this paper of non-Abelian gauge theories on Abelian gauge fields, may afford us a way to do this for non-Abelian gauge fields as well.

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