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Extra-dimensional relaxation of the upper limit of the lightest supersymmetric neutral Higgs mass

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Abstract

The upper limit on the mass of the lightest CP-even neutral Higgs in the minimal supersymmetric standard model is around 135 GeV for soft supersymmetry breaking masses in the 1 TeV range. We demonstrate that this upper limit may be sizably relaxed if supersymmetry is embedded in extra dimensions. We calculate, using the effective potential technique, the radiative corrections to the lightest Higgs mass induced by the Kaluza-Klein towers of quarks and squarks with one and two compactified directions. We observe that the lightest Higgs may comfortably weigh around 200 GeV (300 GeV) with one (two) extra dimension(s).

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I Introduction

The most general symmetries of local relativistic quantum field theories include supersymmetry, a phenomenological version [1] of which is awaiting a final judgement within the next few years as the Large Hadron Collider (LHC) turns on. Indeed, one of the most coveted targets of the LHC is to capture the Higgs boson, and supersymmetry, admitting chiral fermions together with their scalar partners in the same representations, tacitly provides a rationale for treating the Higgs as an elementary object [2]. Furthermore, through the removal of the quadratic divergence that plagues the ordinary Higgs mass, phenomenological supersymmetry has emerged as a leading candidate of physics beyond the standard model (SM). A key signature of the minimal version of supersymmetry is that the lightest Higgs boson mass obeys an upper bound ($\sim$ 135 GeV, see below) – a prediction which will be put to test during the LHC run. Now, supersymmetry is an integral part of string theory which attempts to provide a quantum picture of all interactions. Since string theory is intrinsically a higher dimensional theory, a reanalysis of some 4-dimensional (4d) supersymmetric wisdom in the backdrop of extra dimensions might provide important clues to our search strategies. Considering that the Higgs is the most-wanted entity at the LHC, in this paper we address the following question which we believe is extremely timely: *What is the upper limit of the lightest CP-even neutral Higgs mass if the minimal supersymmetric standard model (MSSM) is embedded in extra dimensions*? We consider the embedding first in one and then in two extra dimensions.

Let us first discuss why this is an important issue. Recall that MSSM has two Higgs doublet superfields (H_1 and H_2), and supersymmetry does not allow the scalar potential to have independent quartic couplings. Gauge interactions generate them through supersymmetry breaking D-terms and the effective quartic interactions are written in terms of the gauge couplings. This makes the Higgs spectrum partially predictive, in the sense that at the tree level the lightest neutral Higgs (h) weighs less than M_Z ($m_h^2 < M_Z^2 \cos^2 2\beta$, where $\tan\beta$ is the ratio of two vacuum expectation values (VEVs)). However, m_h receives quantum corrections which, due to the large top quark Yukawa coupling and for heavy stop squarks, can become as large as $\Delta m_h^2 \sim (3G_F m_t^4/\sqrt{2}\pi^2)\ln(m_{\tilde{t}}^2/m_t^2)$, where $m_{\tilde{t}}$ is an average stop squark mass [3, 4]. The upper limit on m_h is then pushed to around 135 GeV for squark mass in the $\mathcal{O}$(TeV) range. Notice that the non-observation of the Higgs boson at LEP2 has already set a lower limit $m_h > 114.5$ GeV [5, 6], which is satisfied only if a sizable quantum correction elevates the Higgs mass beyond the tree level upper limit of M_Z. This implies (i) lower values of $\tan\beta$, which is usually chosen in the range $1 < \tan\beta < m_t/m_b$, are disfavoured, and (ii) the squark mass $m_{\tilde{t}}$ has to be in the TeV range, which also sets the scale of a generic soft supersymmetry breaking mass M_S. The MSSM prediction of a light Higgs is also in line with the indication coming from electroweak precision tests that the neutral Higgs should weigh below 199 GeV[1] [7]. The so called 'little hierarchy' problem then arises out of an order of magnitude mass splitting between the Higgs and the superparticles.

Adding a gauge singlet superfield (N) in the MSSM spectrum and coupling it with $H_{1,2}$ via the superpotential $\lambda N H_1 H_2$ helps to ease the tension. Not only does this next to minimal version of supersymmetry (the so called NMSSM [8]) help to address the 'μ problem', it also generates a tree level quartic coupling in the scalar potential which modifies the tree level upper limit on m_h through $m_h^2 < M_Z^2 \cos^2 2\beta[1 + 2\lambda^2 \tan^2 2\beta/(g^2 + g'^2)]$ (see [9]). Assuming λ to be in the perturbative regime, i.e., $\lambda \sim g, g'$, one basically obtains a new contribution $\sim M_Z^2 \sin^2 2\beta$ to the tree level m_h^2. This way the low $\tan\beta$ regime can be revived. Since many supersymmetric couplings depend on $\tan\beta$, search strategies alter in a significant way if the disfavoured low $\tan\beta$ region is thus resurrected[2,3].

In this paper we adopt a different approach which also revives the low $\tan\beta$ region. We stick to the MSSM particle content, but embed it in a higher dimension compactified at the inverse TeV scale [12]. Although we argued in the beginning that string theory provides a rationale for linking the two ideas, namely, supersymmetry and extra dimension, establishing any rigourous connection between the two at the level of phenomenological models is still a long shot. Here we take a 'bottom-up' approach: we first outline what has already been studied in the phenomenological context of TeV scale extra-dimensional scenarios, and then illustrate what we aim to achieve in this paper.

1. Consider first scenarios with one extra dimension (with inverse radius of compactification around a TeV) but without supersymmetry. A typical model is the universal extra-dimensional scenario (UED) where all particles access the extra dimension [13]. Constraints on this scenario from $g-2$ of the muon [14], flavour changing neutral currents [15, 16, 17], $Z \to b\bar{b}$ decay [18], the ρ parameter [13, 19], other electroweak precision tests [20], implications from hadron collider studies [21], etc. imply that $R^{-1} \gtrsim 300$ GeV. A recent inclusive $\bar{B} \to X_s\gamma$ analysis sets a stronger constraint $R^{-1} \gtrsim 600$ GeV [22]. These scenarios are motivated from many phenomenological

[1]This indirect upper limit as well as the LEP2 direct search lower limit of $m_h > 114.5$ GeV apply, strictly speaking, for the SM Higgs. However, in the 'decoupling limit' of the MSSM (large m_A leading to full-strength ZZh coupling), which is the region of interest in the present paper, the above limits continue to hold.

[2]Low $\tan\beta$ is preferred by electroweak baryogenesis as well [10].

[3]The constraint arising from perturbativity of couplings can be evaded if the Higgs is charged under an asymptotically free gauge group [11].

angles. They could lead to a new mechanism of supersymmetry breaking [23], address the fermion mass hierarchy in an alternative way [24], provide a cosmologically viable dark matter candidate [25], stimulate power law renormalization group running [12, 26], admit substantial evolution of neutrino mixing angles defined through an effective Majorana neutrino mass operator [27], etc[4].

2. Our object of interest is a supersymmetric theory (e.g. MSSM) but embedded in a higher dimension. Here we ask the following question: *What would be the shift in the Higgs mass due to radiative effects induced by extra dimensions*? In the kind of scenarios we consider, the SM bosons along with their superpartners access the higher dimensional bulk. Additionally, SM fermions of one or more generations together with their superpartners also do so. From a 4d perspective, all the states which access the bulk will have Kaluza-Klein (KK) towers. The zero modes, i.e., those states which do not have any momenta along the extra coordinates, are identified with the standard 4d MSSM spectra. Now, not only the top quark and the stop squarks would contribute to the radiative correction to m_h^2, their KK partners would do so as well. As it turns out, the radiative correction driven by the KK states has the same sign as the one from the zero modes. As a result, Δm_h^2 becomes larger and thus the upper limit on m_h is pushed to higher values beyond the usual 4d MSSM limit of around 135 GeV. As we shall see, in the absence of any left-right scalar mixing, the new contribution coming from KK modes is to a good approximation proportional to $R^2(m_{\tilde{t}}^2 - m_t^2)/n^2$. This fits our intuition that the KK contribution falls with higher KK modes and vanishes both when $R \to 0$ and in the limit of exact supersymmetry. We can interpret the result in two ways. Either, we take large $\tan\beta$ and $\mathcal{O}$(TeV) squark mass that yielded the 4d supersymmetry limit $\sim$ 135 GeV, in which case the new upper limit shoots up by several tens of GeV. Or, we may admit lower $\tan\beta$ and/or accommodate lighter zero mode squarks which were hitherto disfavoured in the 4d context. Either way, the Higgs phenomenology gets an interesting twist which is intuitively comprehensible and analytically tractable, owing largely due to the fact that we are here dealing with only *one additional* parameter, namely, the radius of compactification. Moreover, the top quark mass which appears with fourth power in the expression of Δm_h^2 is now known to a precision better than ever ($m_t = 170.9 \pm 1.8$ GeV [29]).

As mentioned before, we have considered the embedding of 4d supersymmetry in one as well as two extra dimensions. There are quite a few advantages of considering a 6d gauge theory [30] even in a non-supersymmetric scenario: (i) number of fermion generations is restricted to three, or a multiple of it, if the global SU(2) gauge anomaly has to cancel [31], (ii) proton decay is adequately suppressed, which is difficult to achieve in 5d UED, thanks to a discrete symmetry that survives as a subgroup of the 6d Lorentz group [32], (iii) observed neutrino masses and mixings can be nicely explained [33], and (iv) KK vector modes offer better opportunities to be explored [34]. We shall see that qualitatively the KK contributions to the radiative corrections of m_h from 5d and 6d theories are similar, the quantitative estimates differ due to the different density of KK states in the two cases. In 5d, the KK states are spaced as n/R (modulo their zero mode masses) where n, an integer, is the KK number, whereas in 6d, a similar expression holds except $n^2 \Rightarrow j^2 + k^2$, where j and k are two different sets of KK numbers corresponding to the two compactified directions.

Section II is basically a review of the standard derivation of the upper limit of the lightest neutral Higgs in conventional 4d MSSM in the effective potential approach. This paves the way, in Section III, to upgrade the above derivation for accommodating contributions from the KK modes of the top

[4]Ultraviolet cutoff sensitivity in different kinds of TeV scale extra-dimensional models has been dealt in [28].

quark and squarks in 5d and 6d scenarios. In Section IV, we shall comment on the numerical impact of the higher KK modes on the lightest neutral Higgs mass and its consequences. We shall draw our conclusion in the final section.

II MSSM neutral Higgs spectrum in 4 dimensions

II.1 Tree level mass relations

MSSM requires two Higgs doublets for three good reasons: (i) to avoid massless charged degrees of freedom, (ii) to maintain analyticity of the superpotential, and (iii) to keep the theory free from chiral anomaly, which requires two Higgs doublets with opposite hypercharges.

We denote these two complex scalar doublets as

$$H_1 = \begin{pmatrix} {H_1}^0 \\ {H_1}^- \end{pmatrix}, \qquad H_2 = \begin{pmatrix} {H_2}^+ \\ {H_2}^0 \end{pmatrix}, \tag{1}$$

whose SU(2)× U(1) quantum numbers are (2,−1) and (2,+1) respectively. H_1^0 couples with down-type quarks and charged leptons, while H_2^0 couples with up-type quarks. This guarantees natural suppression of flavour-changing neutral currents in the limit of exact supersymmetry. The tree level potential involving these two doublets is given by

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_{12}^2(H_1H_2 + \text{h.c}) + \frac{1}{8}g^2(H_2^\dagger\sigma^a H_2 + H_1^\dagger\sigma^a H_1)^2 + \frac{1}{8}g'^2(|H_2|^2 - |H_1|^2)^2, \tag{2}$$

where m_1^2, m_2^2 and m_{12}^2 are soft supersymmetry breaking mass parameters, g and g' are the SU(2) and U(1) gauge couplings, and σ^a $(a = 1, 2, 3)$ are the Pauli matrices. Note that the quartic coupling is related to the gauge couplings. The part involving the neutral fields is given by

$$V_0 = m_1^2|H_1^0|^2 + m_2^2|H_2^0|^2 - m_{12}^2(H_1^0H_2^0 + \text{h.c}) + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2. \tag{3}$$

After spontaneous symmetry breaking the minimum of V_0 involves the following two VEVs: $\langle H_1^0\rangle = v_1$ and $\langle H_2^0\rangle = v_2$. The combination $v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV sets the Fermi scale.

Now, out of the eight degrees of freedom contained in the two Higgs doublets three are absorbed as the longitudinal modes of the W and the Z bosons, while the remaining five modes appear as physical states. Of these five states, two are charged $(H^\pm)$ and three are neutral (h, H, A). Our present concern is the neutral sector of which (h, H) are CP-even, while A is CP-odd. From the separate diagonalisation of the CP-odd and CP-even neutral mass matrices two important relations emerge:

$$m_A^2 = \frac{2m_{12}^2}{\sin 2\beta}\,, \quad \text{where} \quad \tan\beta = \frac{v_2}{v_1}, \tag{4}$$

$$m_{h,H}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2M_Z^2\cos^2 2\beta}\right], \tag{5}$$

where, by definition, h is the lighter of the two CP-even Higgs. These in turn give rise to the following sum rule and inequality:

$$m_h^2 + m_H^2 = m_A^2 + M_Z^2 \tag{6}$$

$$m_h \le \min\,(m_A, M_Z)|\cos 2\beta| \le \min\,(m_A, M_Z), \tag{7}$$

i.e., at the tree level (i) the lighter of the two CP-even Higgs (h) weighs less than M_Z, and (ii) the CP-odd Higgs (A) is heavier than h but lighter than H.

II.2 Radiative corrections

We shall now discuss how the above tree level relations are affected by quantum loops [3, 4]. We shall confine our discussion on the correction to m_h only, and that too at the one-loop level. We note two important points:

1. Radiative corrections to m_h are dominated by the top quark Yukawa coupling (h_t) and the masses of the stop squarks ($\tilde{t}_1$, $\tilde{t}_2$). For large values of $\tan\beta$, the contributions from the b-quark sector also assume significance. We shall ignore loop contributions mediated by lighter quarks or the gauge bosons.

2. The tree level Higgs mass is protected by supersymmetry. In the limit of exact supersymmetry, the entire quantum correction vanishes. So radiative corrections to m_h will be controlled by M_S.

Three different approaches have been adopted in the literature to calculate the radiative corrections to m_h: (i) effective potential technique, (ii) direct diagrammatic calculations, and (iii) renormalisation group (RG) method, assuming $M_S \gg M_Z$ and fixing the quartic coupling proportional to $(g^2 + g'^2)$ at that scale and then evolving down to weak scale. In this paper, we shall follow the effective potential approach primarily for the sake of conveniently including the effect of new physics later.

We first start with an RG-improved tree level potential $V_0(Q)$ which contains running masses $m_i^2(Q)$ and running gauge couplings $g_i(Q)$. The full one-loop effective potential is now given by

$$V_1(Q) = V_0(Q) + \Delta V_1(Q), \tag{8}$$

where, in terms of the field dependent masses $M(H)$,

$$\Delta V_1(Q) = \frac{1}{64\pi^2} \mathrm{Str} M^4(H) \left\{ \ln \frac{M^2(H)}{Q^2} - \frac{3}{2} \right\}. \tag{9}$$

The Q-dependence of $\Delta V_1(Q)$ cancels against that of $V_0(Q)$ making $V_1(Q)$ independent of Q up to higher loop orders. The supertrace in Eq. (9), defined through

$$\mathrm{Str} f(m^2) = \sum_i (-1)^{2J_i} (2J_i + 1) f(m_i^2), \tag{10}$$

has to be taken over all members of a supermultiplet and where $m_i^2 \equiv m_i^2(H)$ is the field-dependent mass eigenvalue of the particle i with spin J_i. As an example, the contribution from the chiral multiplet containing the top quark and squarks is given by

$$\Delta V_t = \frac{3}{32\pi^2} \left\{ m_{\tilde{t}_1}^4 \left(\ln \frac{m_{\tilde{t}_1}^2}{Q^2} - \frac{3}{2} \right) + m_{\tilde{t}_2}^4 \left(\ln \frac{m_{\tilde{t}_2}^2}{Q^2} - \frac{3}{2} \right) - 2m_t^4 \left(\ln \frac{m_t^2}{Q^2} - \frac{3}{2} \right) \right\}, \tag{11}$$

where the overall factor of 3 comes from colour. Note that $m_{\tilde{t}_i}$ and m_t in Eq. (11) are field dependent masses. Even though $h_b \ll h_t$, the contribution from the bottom supermultiplet turns out to be numerically significant in the large $\tan\beta$ region. ΔV_b can be written analogously to ΔV_t with the appropriate replacements of top and stop masses by bottom and sbottom masses respectively.

We now explicitly write down the field dependent mass terms. This simply means a replacement of v_i by H_i^0 $(i = 1, 2)$ wherever v_i appear in the expression of masses. The field dependent top and bottom quark masses are given by

$$m_t^2(H) = h_t^2|H_2^0|^2 \ ; \ m_b^2(H) = h_b^2|H_1^0|^2. \tag{12}$$

The field dependent stop and sbottom squark mass matrices are written as

$$M_{\tilde{t}}^2(H) = \begin{pmatrix} m_Q^2 + h_t^2|H_2^0|^2 & h_t(A_t H_2^0 + \mu H_1^{0*}) \\ h_t(A_t H_2^{0*} + \mu H_1^0) & m_U^2 + h_t^2|H_2^0|^2 \end{pmatrix}, \tag{13}$$

and

$$M_{\tilde{b}}^2(H) = \begin{pmatrix} m_Q^2 + h_b^2|H_1^0|^2 & h_b(A_b H_1^0 + \mu H_2^{0*}) \\ h_b(A_b H_1^{0*} + \mu H_2^0) & m_D^2 + h_b^2|H_1^0|^2 \end{pmatrix}. \tag{14}$$

In Eqs. (13) and (14) m_Q, m_U and m_D are soft supersymmetry breaking masses, A_t and A_b are trilinear soft supersymmetry breaking mass dimensional couplings, and μ is the supersymmetry preserving mass dimensional parameter connecting H_1 and H_2 in the superpotential. We take both trilinear and the μ couplings to be real. We have neglected the D-term contributions which are small, being proportional to gauge couplings. The squark masses appearing in Eq. (11) are obtained from the diagonalisation of Eq. (13).

We now consider the radiative correction to the CP-odd scalar mass matrix. The one-loop corrected mass matrix square, obtained by taking double derivatives of the full potential with respect to the pseudo-scalar excitations, can be written as

$$\mathcal{M}_{(\text{odd})}^2 = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} (m_{12}^2 + \Delta). \tag{15}$$

The radiative corrections generated as a consequence of supersymmetry breaking are contained in $\Delta = \Delta^{\text{t}} + \Delta^{\text{b}}$, which is given by

$$\Delta^{\text{t(b)}} = -\frac{3}{32\pi^2} \frac{h_{t(b)}^2 \mu A_{t(b)}}{\left[m_{\tilde{t}_1(\tilde{b}_1)}^2 - m_{\tilde{t}_2(\tilde{b}_2)}^2\right]} \left[f\left(m_{\tilde{t}_1(\tilde{b}_1)}^2\right) - f\left(m_{\tilde{t}_2(\tilde{b}_2)}^2\right) \right] \tag{16}$$

where

$$f(m^2) = 2m^2 \left(\ln \frac{m^2}{Q^2} - 1 \right). \tag{17}$$

The zero eigenvalue corresponds to the massless Goldstone boson which is eaten by the Z boson. The massive state is the pseudo-scalar A whose radiatively corrected mass square is given by

$$m_A^2 = \frac{2(m_{12}^2 + \Delta)}{\sin 2\beta}. \tag{18}$$

The Q-dependence of m_A cancels in Eq. (18) up to one-loop order. In any case, we shall treat the radiatively corrected m_A as an input parameter.

Now we are all set to calculate the radiative corrections in the neutral CP-even mass eigenvalues. The one-loop corrected mass matrix square is obtained by taking double derivatives of the full potential with respect to the scalar excitations and is given by

$$\mathcal{M}^2_{(\rm even)} = \begin{pmatrix} M_Z^2\cos^2\beta + m_A^2\sin^2\beta & -(m_A^2+M_Z^2)\sin\beta\cos\beta \\ -(m_A^2+M_Z^2)\sin\beta\cos\beta & M_Z^2\sin^2\beta + m_A^2\cos^2\beta \end{pmatrix} + \frac{3}{4\pi^2 v^2}\begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix}, \tag{19}$$

where $\Delta_{ij} = \Delta^{\rm t}_{ij} + \Delta^{\rm b}_{ij}$. The individual Δ_{ij}'s are explicitly written below:

$$\begin{aligned}
\Delta^t_{11} &= \frac{m_t^4}{\sin^2\beta}\left(\frac{\mu(A_t+\mu\cot\beta)}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\right)^2 g(m^2_{\tilde t_1}, m^2_{\tilde t_2}),\\
\Delta^t_{12} &= \frac{m_t^4}{\sin^2\beta}\frac{\mu(A_t+\mu\cot\beta)}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\left[\ln\frac{m^2_{\tilde t_1}}{m^2_{\tilde t_2}} + \frac{A_t(A_t+\mu\cot\beta)}{m^2_{\tilde t_1}-m^2_{\tilde t_2}} g(m^2_{\tilde t_1}, m^2_{\tilde t_2})\right],\\
\Delta^t_{22} &= \frac{m_t^4}{\sin^2\beta}\left[\ln\frac{m^2_{\tilde t_1}m^2_{\tilde t_2}}{m_t^4} + \frac{2A_t(A_t+\mu\cot\beta)}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\ln\frac{m^2_{\tilde t_1}}{m^2_{\tilde t_2}} + \left(\frac{A_t(A_t+\mu\cot\beta)}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\right)^2 g(m^2_{\tilde t_1}, m^2_{\tilde t_2})\right],\\
\Delta^b_{11} &= \frac{m_b^4}{\cos^2\beta}\left[\ln\frac{m^2_{\tilde b_1}m^2_{\tilde b_2}}{m_b^4} + \frac{2A_b(A_b+\mu\tan\beta)}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}} + \left(\frac{A_b(A_b+\mu\tan\beta)}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\right)^2 g(m^2_{\tilde b_1}, m^2_{\tilde b_2})\right],\\
\Delta^b_{12} &= \frac{m_b^4}{\cos^2\beta}\frac{\mu(A_b+\mu\tan\beta)}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\left[\ln\frac{m^2_{\tilde b_1}}{m^2_{\tilde b_2}} + \frac{A_b(A_b+\mu\tan\beta)}{m^2_{\tilde b_1}-m^2_{\tilde b_2}} g(m^2_{\tilde b_1}, m^2_{\tilde b_2})\right],\\
\Delta^b_{22} &= \frac{m_b^4}{\cos^2\beta}\left(\frac{\mu(A_b+\mu\tan\beta)}{m^2_{\tilde b_1}-m^2_{\tilde b_2}}\right)^2 g(m^2_{\tilde b_1}, m^2_{\tilde b_2}).
\end{aligned} \tag{20}$$

where

$$g(m_1^2, m_2^2) = 2 - \frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_1^2}{m_2^2}. \tag{21}$$

Two points deserve mention at this stage:

1. While the leading log contribution appears in Δ_{22} for the top sector, the same appears in Δ_{11} for the bottom sector. This happens because the right-handed top quark couples to H_2 while the right-handed bottom quark couples to H_1. In the absence of any left-right scalar mixing, these leading logs are the only radiative contributions.

2. Ignoring the left-right scalar mixing, the radiative shift to the Higgs mass square coming from the top-stop sector turns out to be $\Delta m_h^2 = (3/4\pi^2 v^2)\Delta^t_{22}\sin^2\beta \sim (3m_t^4/2\pi^2v^2)\ln(m^2_{\tilde t}/m_t^2)$, where $m_{\tilde t} = \sqrt{m_{\tilde t_1} m_{\tilde t_2}}$ is an average stop mass. This is the expression we quoted in the Introduction.

III Radiative corrections due to extra dimensions

Let us first consider just one extra dimension which is compactified on a circle of radius R. We further consider a Z_2 orbifolding identifying $y \to -y$, where y denotes the compactified coordinate. The orbifolding is crucial for reproducing the chiral zero modes of the observed fermions. After the compactified direction is integrated out, the 4d Lagrangian can be written in terms of the zero modes and their KK partners. For illustration, we first take a non-supersymmetric scenario and look into the KK mode expansion of gauge boson, scalar and fermion fields from a 4d perspective. Each component of a 5d field is either even or odd under Z_2. The KK expansions are given by,

$$\begin{aligned}
A_\mu(x,y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} A_\mu^{(0)}(x) + \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \cos\frac{ny}{R}, \qquad A_5(x,y) = \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} A_5^{(n)}(x) \sin\frac{ny}{R}, \\
\phi(x,y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} \phi^{(0)}(x) + \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \phi^{(n)}(x) \cos\frac{ny}{R}, \\
\mathcal{Q}(x,y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} \left[\begin{pmatrix} t \\ b \end{pmatrix}_L (x) + \sqrt{2} \sum_{n=1}^{\infty} \left\{ \mathcal{Q}_L^{(n)}(x) \cos\frac{ny}{R} + \mathcal{Q}_R^{(n)}(x) \sin\frac{ny}{R} \right\} \right], \\
\mathcal{T}(x,y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} \left[t_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left\{ \mathcal{T}_R^{(n)}(x) \cos\frac{ny}{R} + \mathcal{T}_L^{(n)}(x) \sin\frac{ny}{R} \right\} \right], \\
\mathcal{B}(x,y) &= \frac{\sqrt{2}}{\sqrt{2\pi R}} \left[b_R(x) + \sqrt{2} \sum_{n=1}^{\infty} \left\{ \mathcal{B}_R^{(n)}(x) \cos\frac{ny}{R} + \mathcal{B}_L^{(n)}(x) \sin\frac{ny}{R} \right\} \right].
\end{aligned} \tag{22}$$

The complex scalar field $\phi(x, y)$ and the gauge boson $A_\mu(x, y)$ are Z_2-even fields with their zero modes identified with the SM scalar doublet and a SM gauge boson respectively. The field $A_5(x, y)$ is a real pseudoscalar field transforming in the adjoint representation of the gauge group, and does not have any zero mode. The fields $\mathcal{Q}$, $\mathcal{T}$, and $\mathcal{B}$ denote the 5d quark SU(2) doublet and SU(2) singlet states of a given generation (e.g. third generation) whose zero modes are identified with the 4d SM quark states. We draw attention to two points at this stage: (i) even though the Z_2 orbifolding renders the zero mode fermions to be chiral, the KK fermions are vector-like; (ii) the KK number n is conserved at all tree level vertices, while what actually remains conserved at all order is the KK parity, defined as $(-1)^n$. As is well known, the kinetic terms in the extra dimension give rise to the KK masses in 4d. For a flat extra dimension the KK masses are added in quadrature with the corresponding zero mode masses both for fermions and bosons. A generic expression for the n-th mode mass, where m_0 is the zero mode mass, is given by

$$m_n^2 = m_0^2 + \frac{n^2}{R^2}, \qquad n = 0, 1, 2, \ldots \tag{23}$$

We now discuss the supersymmetric version of the theory. A 5d $N = 1$ supersymmetry from a 4d perspective appears as two $N = 1$ supersymmetries forming an $N = 2$ theory. For the details of the hypermultiplet structures of this theory, we refer the readers to [12]. Our concern in this paper is to calculate the radiative contribution to m_h coming from the KK partners of particles and superparticles. We now proceed through the following steps.

1. Let us first recall that the $N = 2$ supersymmetry prohibits any bulk Yukawa interaction involving three chiral multiplets. The Yukawa interaction is considered to be localised at a brane, like $-(h_{t5}/\Lambda^{3/2}) \int d^4x \int dy\ \delta(y) \int d^2\theta\ (\mathcal{H}_2 \mathcal{Q} \mathcal{T}\ +\ \text{h.c.})$, where the residual supersymmetry is that of $N = 1$, h_{t5} is a dimensionless Yukawa coupling in 5d and Λ the cutoff scale. This localisation has a consequence in the counting of KK degrees of freedom that contribute to the Higgs mass radiative correction. The delta function ensures that those fields which accompany the sine function after Fourier decomposition do not sense the Yukawa interaction.

2. As in the case of 4d (zero mode) supersymmetry, here too the dominant effect arises solely from the third generation quark superfields, only that now we have to include the contributions from their KK towers. We shall continue to ignore contributions from the gauge interactions or those from the first two quark families, as they are not numerically significant. We might as well formulate a scheme in which the first two generation of matter superfields are brane-localised and *only* the third generation superfields access the bulk[5]. Keeping this in mind, we displayed the Fourier decomposition of only the third generation superfields in Eq. (22).

3. In our scheme M_S and R are independent parameters, although we take them to be of the same order[6]. Towards the end of Section IV, we briefly remark on the numerical implications of any possible connection between M_S and R.

4. The KK equivalent of Eq. (11), which captures the KK contribution arising from the top quark chiral hypermultiplet, is then given by

$$\Delta V_t^n = \frac{3}{32\pi^2}\left[m_{\tilde{t}_1^n}^4\left(\ln\frac{m_{\tilde{t}_1^n}^2}{Q^2}-\frac{3}{2}\right)+m_{\tilde{t}_2^n}^4\left(\ln\frac{m_{\tilde{t}_2^n}^2}{Q^2}-\frac{3}{2}\right)-2m_{t^n}^4\left(\ln\frac{m_{t^n}^2}{Q^2}-\frac{3}{2}\right)\right], \qquad (24)$$

where the field dependent KK masses are given by $m_{\tilde{t}_1^n}^2 = m_{\tilde{t}_1}^2 + n^2/R^2$, $m_{\tilde{t}_2^n}^2 = m_{\tilde{t}_2}^2 + n^2/R^2$, and $m_{t^n}^2 = m_t^2 + n^2/R^2$. The field dependence is hidden inside the zero mode masses, as illustrated in Eqs. (12), (13) and (14). The corresponding contribution triggered by the bottom quark hypermultiplet, ΔV_b^n, can be written *mutatis mutandis*.

5. We now calculate the KK loop contribution to the neutral scalar mass matrix. The procedure will be exactly the same as that followed for the 4d MSSM scenario in the previous section. Since we are going to treat the radiatively corrected physical m_A as an input parameter, we concentrate only on the CP-even mass matrix. We first take another look at the expressions of the different Δ_{ij}, assembled in Eq. (20), calculated in the context of the 4d MSSM. The prefactors like m_t^4 or m_b^4 originated by the action of double differentiation on the field dependent squark or quark masses. Recall that the squark and quark masses are (quadratically) separated by the soft supersymmetry breaking mass-squares which are *not* field dependent. So, irrespective of whether we double-differentiate the squark or quark masses we get either the top or bottom quark Yukawa coupling[7]. In the same way, the KK mass-squares are separated from the zero mode mass-squares by a field independent quantity n^2/R^2. Therefore, the expressions for $(\Delta_{ij})^n$, the radiative corrections from the nth KK level, continue to have the zero mode quark masses m_t^4 or m_b^4 as prefactors, but now the arguments of the other functions contain the corresponding KK masses.

[5]If all the three matter generations are bulk fields, then the theory become non-perturbative too soon, unless $1/R > 5.0 \times 10^{10}$ GeV [26]. If only one generation accesses the bulk and the other two are confined to a brane, then the validity of the theory extends further, allowing even a perturbative gauge coupling unification, we checked, around $E \sim 40/R$.

[6]This is in contrast to other higher dimensional supersymmetric scenarios in which both the superpartner masses and the scale of electroweak symmetry breaking arising from quantum loops are set by $1/R$, where R is the distance between the brane at which top quark Yukawa coupling is localised and the brane where supersymmetry is broken [35]. Higher order finiteness of the Higgs mass, where supersymmetry is broken in the bulk by Scherk-Schwarz boundary conditions [36], has been discussed in [37].

[7]This also indicates that by fixing the first and second generation matter superfields at the brane we have not made any numerically serious compromise as otherwise their contributions would have been adequately suppressed on account of their small Yukawa couplings.

We are now all set to write down the expressions for different $(\Delta_{ij})^n$ for $n \neq 0$. They are given by

$$
\begin{aligned}
(\Delta^t_{11})^n &= \frac{m_t^4}{\sin^2\beta}\left(\frac{\mu(A_t+\mu\cot\beta)}{m^2_{\tilde{t}^n_1}-m^2_{\tilde{t}^n_2}}\right)^2 g(m^2_{\tilde{t}^n_1}, m^2_{\tilde{t}^n_2}), \\
(\Delta^t_{12})^n &= \frac{m_t^4}{\sin^2\beta}\frac{\mu(A_t+\mu\cot\beta)}{m^2_{\tilde{t}^n_1}-m^2_{\tilde{t}^n_2}}\left[\ln\frac{m^2_{\tilde{t}^n_1}}{m^2_{\tilde{t}^n_2}} + \frac{A_t(A_t+\mu\cot\beta)}{m^2_{\tilde{t}^n_1}-m^2_{\tilde{t}^n_2}} g(m^2_{\tilde{t}^n_1}, m^2_{\tilde{t}^n_2})\right], \\
(\Delta^t_{22})^n &= \frac{m_t^4}{\sin^2\beta}\left[\ln\frac{m^2_{\tilde{t}^n_1}m^2_{\tilde{t}^n_2}}{m^4_{t^n}} + \frac{2A_t(A_t+\mu\cot\beta)}{m^2_{\tilde{t}^n_1}-m^2_{\tilde{t}^n_2}}\ln\frac{m^2_{\tilde{t}^n_1}}{m^2_{\tilde{t}^n_2}} + \left(\frac{A_t(A_t+\mu\cot\beta)}{m^2_{\tilde{t}^n_1}-m^2_{\tilde{t}^n_2}}\right)^2 g(m^2_{\tilde{t}^n_1}, m^2_{\tilde{t}^n_2})\right], \\
(\Delta^b_{11})^n &= \frac{m_b^4}{\cos^2\beta}\left[\ln\frac{m^2_{\tilde{b}^n_1}m^2_{\tilde{b}^n_2}}{m^4_{b^n}} + \frac{2A_b(A_b+\mu\tan\beta)}{m^2_{\tilde{b}^n_1}-m^2_{\tilde{b}^n_2}}\ln\frac{m^2_{\tilde{b}^n_1}}{m^2_{\tilde{b}^n_2}} + \left(\frac{A_b(A_b+\mu\tan\beta)}{m^2_{\tilde{b}^n_1}-m^2_{\tilde{b}^n_2}}\right)^2 g(m^2_{\tilde{b}^n_1}, m^2_{\tilde{b}^n_2})\right], \\
(\Delta^b_{12})^n &= \frac{m_b^4}{\cos^2\beta}\frac{\mu(A_b+\mu\tan\beta)}{m^2_{\tilde{b}^n_1}-m^2_{\tilde{b}^n_2}}\left[\ln\frac{m^2_{\tilde{b}^n_1}}{m^2_{\tilde{b}^n_2}} + \frac{A_b(A_b+\mu\tan\beta)}{m^2_{\tilde{b}^n_1}-m^2_{\tilde{b}^n_2}} g(m^2_{\tilde{b}^n_1}, m^2_{\tilde{b}^n_2})\right], \qquad (25) \\
(\Delta^b_{22})^n &= \frac{m_b^4}{\cos^2\beta}\left(\frac{\mu(A_b+\mu\tan\beta)}{m^2_{\tilde{b}^n_1}-m^2_{\tilde{b}^n_2}}\right)^2 g(m^2_{\tilde{b}^n_1}, m^2_{\tilde{b}^n_2}).
\end{aligned}
$$

Now we have to add the $(\Delta^t)^n$ and $(\Delta^b)^n$ matrices to the one-loop corrected (from zero modes only) mass matrix in Eq. (19), sum over n, and then diagonalise to obtain the eigenvalues m_h^2 and m_H^2. The KK radiative corrections decouple in powers of (R^2/n^2). To provide intuition to the expressions in Eq. (25), we display below the approximate formulae for $(\Delta^t)^n$ in leading powers of (R^2/n^2):

$$
\begin{aligned}
(\Delta^t_{11})^n &= -\frac{1}{6}\left(\frac{R^4}{n^4}\right)\frac{m_t^4}{\sin^2\beta}\left[\mu(A_t+\mu\cot\beta)\right]^2, \\
(\Delta^t_{12})^n &= \left(\frac{R^2}{n^2}\right)\frac{m_t^4}{\sin^2\beta}\mu(A_t+\mu\cot\beta), \qquad (26) \\
(\Delta^t_{22})^n &= \left(\frac{R^2}{n^2}\right)\frac{m_t^4}{\sin^2\beta}\left[(m^2_{\tilde{t}_1}+m^2_{\tilde{t}_2}-2m_t^2)+2A_t(A_t+\mu\cot\beta)\right].
\end{aligned}
$$

Similar expressions for $(\Delta^b)^n$ can be written, with appropriate replacements like $m_t \leftrightarrow m_b$, $\cot\beta \leftrightarrow \tan\beta$, etc. So, in the absence of any left-right scalar mixing, the KK contribution to Δm_h^2 is controlled by $R^2(m^2_{\tilde{t}} - m_t^2)/n^2$ and its higher powers.

Six dimensional scenario: For the 6d scenario we follow the compactification on a chiral square, as done in [30], which admits zero mode chiral fermions. The two extra spatial coordinates (y_1, y_2) are compactified on a square of side length L, such that $0 < y^1,\ y^2 < \pi R (\equiv L)$. The boundary condition is the identification of the two pairs of adjacent sides of the squares such that the values of a field at two identified points differ by a phase (θ). Nontrivial solutions exist when θ takes four discrete values ($n\pi/2$) for $n = 0, 1, 2, 3$ and the zero modes appear when $n = 0$. What matters to our calculation in this paper is the structure of the KK masses, a generic pattern of which is given by

$$
m^2_{j,k} = m_0^2 + \frac{j^2+k^2}{R^2}, \qquad (27)
$$

(j,k)	1, 0	(1,1)	(2,0)	(2,1) or (1,2)	(2,2)	(3,0)	(3,1) or (1,3)	(3,2) or (2,3)	(4,0)
$m_{j,k}$	1	$\sqrt{2}$	2	$\sqrt{5}$	$2\sqrt{2}$	3	$\sqrt{10}$	$\sqrt{13}$	4

Table 1: *6d scenario mass spectrum in* $(1/R)$ *units, neglecting the zero mode mass.*

where j, k are integers such that $j \geq 0$ and $k \geq 0$. We display in Table 1 the KK mass spectrum (neglecting the zero mode mass m_0 for simplicity of presentation while in the actual calculation we do keep it). The formalism we developed for 5d will simply go through for 6d. More concretely, the structure of Eqs. (24) and (25) would remain the same in 6d, only that one should now read $n \Rightarrow (j,k)$. The numerical impact in the two cases obviously differ, as we shall witness in the next section[8].

IV Results

In this section we explore the consequences of the extra-dimensional contributions to the Higgs mass encoded in the exact one-loop expressions in Eq. (25). But to start with, to get a feel for the numerical impact of the extra dimensions, consider the scenario pared down to its bare minimum by assuming that left-right scalar mixing ingredients are vanishing, i.e., $\mu = A_t = A_b = 0$. This leads to two degenerate stop squarks: $m_{\tilde{t}}^2 = M_S^2 + m_t^2$. Then, for a moderate $\tan\beta$,

$$\Delta m_h^2 \ (n=0) \sim \frac{3m_t^4}{2\pi^2 v^2} \ln\left(1 + \frac{M_S^2}{m_t^2}\right); \quad \Delta m_h^2 \ (n \neq 0) \sim \frac{3m_t^4}{2\pi^2 v^2} \frac{(M_S R)^2}{n^2}. \tag{28}$$

Indeed, non-zero trilinear and μ terms would complicate the expressions, yet Eq. (28) provides a good intuitive feel for our results displayed through the different plots. The expected decoupling of extra-dimensional effects in the $1/R \to \infty$ limit is transparent in Eq. (28), leaving the logarithmic dependence on the supersymmetry scale, M_S.

As stressed already, the primary emphasis in this work is to examine the effect of extra dimensions on the upper bound of m_h. In 4d supersymmetry it is usual to choose the pseudoscalar Higgs mass, m_A, as a free parameter and exhibit m_h as its function. This has been done for the extra-dimensional MSSM models in Figs. 1 (5d case) and 2 (6d case). Let us discuss them in turn.

In these and the subsequent figures, the parameters involved are chosen as follows:
(a) $m_Q = m_U = m_D \equiv M_S$, which is a common soft supersymmetry breaking mass. Several values of M_S have been chosen in the figures to depict its impact.
(b) The trilinear scalar couplings A_t and A_b are varied in the range $[0.8 - 1.2]\ M_S$. This results in bands in the figures. We have found that the results are not particularly sensitive to μ and we hold it fixed at 200 GeV. Also, sign flips in the trilinear couplings do not change the results.
(c) The stop and sbottom (zero mode) mass eigenvalues are calculated from the diagonalisation of matrices in Eqs. (13) and (14) after setting the Higgs fields to their VEVs. For a chosen value of $\tan\beta$ and M_S, those eigenvalues will vary in a range in accord with the variation of A_t and A_b stated above.
(d) Since we are interested in probing the upper limit of the lightest Higgs, we maximize its *tree* level

[8]Admittedly, the 6d sum is logarithmically sensitive to the cutoff. The low-lying KK states we include reflect the dominant contribution to the Higgs mass shift. We thank Anindya Datta for raising the 6d divergence issue.

mass as much as possible. For displaying our results we have fixed $\tan\beta = 10$, a moderate value for which the tree level m_h is almost close to M_Z.

In Fig. 1 we have displayed the result in the m_h-m_A plane for only one extra dimension. The dependence of m_h on m_A in the MSSM case is mimicked in the extra-dimensional case and m_h settles at its upper limit for m_A greater than about 150 GeV. In the left panel, M_S has been fixed at 500 GeV. As anticipated, larger the value of $1/R$ smaller is the extra dimensional impact. The 4d MSSM case corresponds to $1/R \to \infty$. The width of each band reflects the variation of the trilinear parameters in the zone mentioned above. For the chosen supersymmetry parameters, the maximum value of m_h is a little below 125 GeV for the 4d MSSM case while for the extra-dimensional situation it is enhanced to above 135 (130) GeV for $1/R = 600$ GeV (1 TeV). In the right panel, the dependence on M_S is exhibited holding $1/R$ at 1 TeV. Clearly, a larger M_S results in bigger radiative corrections – recall Eq. (28) – both from the zero mode as well as from the KK modes.

Fig. 2 is a 6d version of Fig. 1. While the pure 4d MSSM band remains the same, the KK radiative effects are larger now due to the denser KK spectrum in the 6d case, specified by two sets of integers j and k, as shown in Table 1. Quantitatively, for an $1/R$ of 600 GeV (1 TeV), m_h can now be as heavy as 195 (155) GeV, to be compared with 125 GeV in 4d MSSM for these parameter values.

As mentioned earlier, the current lower bound on m_h of 114.5 GeV excludes low values of $\tan\beta$ in the 4d MSSM. It is expected that in the extra-dimensional scenarios some of this excluded range of $\tan\beta$ will make it into the allowed zone. In Fig. 3, we have shown the variation of m_h with respect to $\tan\beta$ (for low values) to illustrate this effect. For the 5d case (left panel), $1/R$ of even 1.2 TeV eases the tension somewhat while for $1/R$ of 600 GeV the effect is very prominent. For 6d (right panel), the extra-dimensional contributions are further enhanced and the restriction on $\tan\beta$ is essentially entirely lifted. We should recall that $\tan\beta$ enters in the Higgs couplings to other particles and so the above result has significant bearing on collider searches of supersymmetry.

So far, we have exhibited results for a few choices of the compactification scale, $1/R$. Fig. 4 demonstrates how the KK-induced radiative correction depends on $1/R$ for the 5d (left panel) and 6d (right panel) scenarios. If the Higgs boson is detected at the LHC then using these figures one can gain a handle on $1/R$ dependent on the supersymmetry parameters like M_S. The decoupling behaviour as $1/R$ increases is in agreement with expectation.

We have also studied, in passing, the possibility that the soft supersymmetry breaking scale arises from compactification (e.g. through the Scherk-Schwarz mechanism [36]). Let us suppose $M_S = C/R$, where C is an order one dimensionless constant. Since we are interested in weak scale supersymmetry breaking, we keep $1/R$ around a few hundred GeV to a TeV. In this region, the radiative correction roughly depends on M_S and R only through their product ($\equiv C$), and for a choice of $C \in [0.5 - 2.0]$, the upper limit on the lightest Higgs mass turns out to be in the range $m_h \in (150 - 230)$ GeV (5d) and $(200 - 450)$ GeV (6d).

It may bear mentioning again that in these calculations we have retained the loop conntributions from the t and b quarks only. The other quarks and gauge bosons make negligible impact. Also, we have dealt only with real MSSM parameters and limited our studies up to one-loop KK contributions. We have not, therefore, included either the two-loop improvements of the 4d MSSM calculations or the numerical effects of the phases associated with complex MSSM parameters in our discussions (for a recent survey, see [38]).

V Conclusions

One of the virtues for which supersymmetry stands out as a leading candidate of physics beyond the SM is that it sets an upper bound on the Higgs mass. The lightest neutral Higgs mass, m_h, could at most be M_Z at the tree level, but is pushed further obeying a definite relation, obtained from quantum corrections, involving m_h, m_t and the stop squark mass, $m_{\tilde{t}}$. The sensitivity of this correction to $m_{\tilde{t}}$ is only logarithmic. Consequently, a firm prediction results, namely, that $m_h \lesssim 135$ GeV in MSSM for $m_{\tilde{t}} \lesssim \mathcal{O}(1 \text{ TeV})$. This is regarded as a critical test of supersymmetry and is naturally high on the agenda of the upcoming LHC experiments. In this paper, we have probed how much this upper limit could be relaxed, should the MSSM be embedded in one (S^1/Z_2) or two (T^2/Z_4) extra dimensions. We highlight our main findings:

1. The KK towers of the top quark and stop squarks provide a positive contribution to m_h^2 raising it by several tens of GeV. If we ignore left-right scalar mixing and assume moderate $\tan\beta \sim (5-10)$, then using Eq. (28) and summing over all the KK modes, we obtain $\Delta m_h^2(\text{KK}) \sim (60 \text{ GeV})^2 \times (M_S R)^2$. This is a 5d result. Including the left-right scalar mixings, i.e., non-zero μ and trilinear parameters, somewhat enhances the magnitude of the correction (see Fig. 1). As in the case of 4d MSSM, here too the size of the correction is controlled by the large top Yukawa coupling.

2. If we consider a 6d theory with two extra dimensions compatcified on a chiral square, whose motivations have been mentioned earlier, the correction gets sizably enhanced (see Fig. 2), compared to 5d, due to a denser packing of KK states, which are now fixed by two independent KK numbers.

3. Non-observation of a Higgs boson weighing below 114.5 GeV disfavours low $\tan\beta$ in 4d MSSM. Some part of this region can be revived by extra dimensional embedding (see Fig. 3).

4. The 4d MSSM relationship between the lightest neutral Higgs mass and the stop squark mass is extremely profound in the sense that its specific form does not depend on the supersymmetry breaking mechanism. If supersymmetry is embedded in extra dimension(s) and, with some cooperation from Nature, the KK states happen to be light enough to mark their imprints on the LHC data recorder, then the relationship between the stop mass and the Higgs mass alters in a numerically significant way (see Fig. 4).

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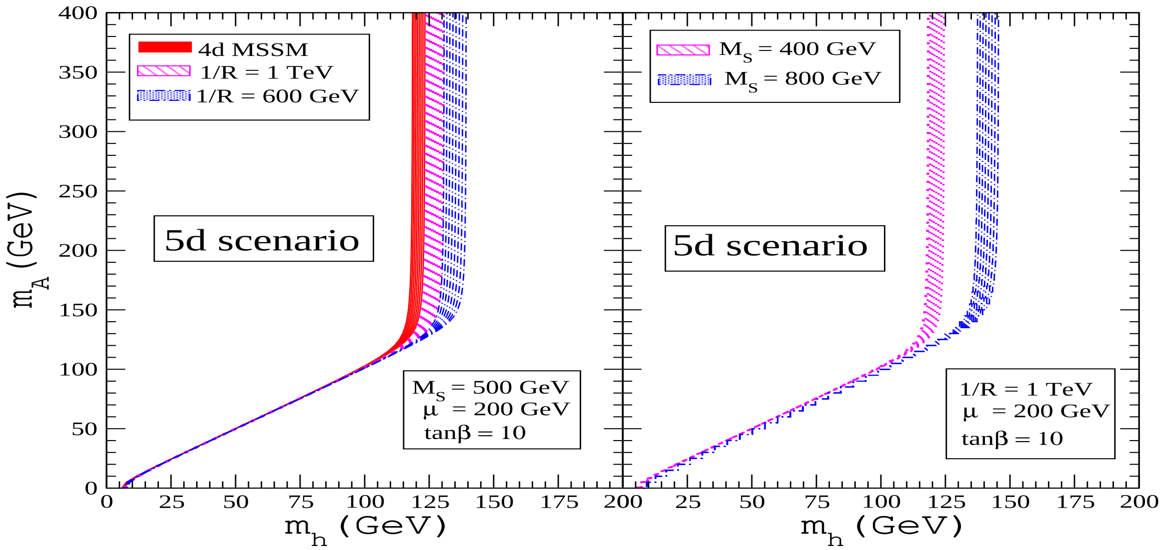

Figure 1: *The variation of* m_h *with* m_A *in the 5d MSSM for different choices of the supersymmetry breaking scale* (M_S) *and the compactification radius* (R)*. The width of each band corresponds to the variation of* A_t *and* A_b *in the range* $(0.8 - 1.2)M_S$ *(see text).*

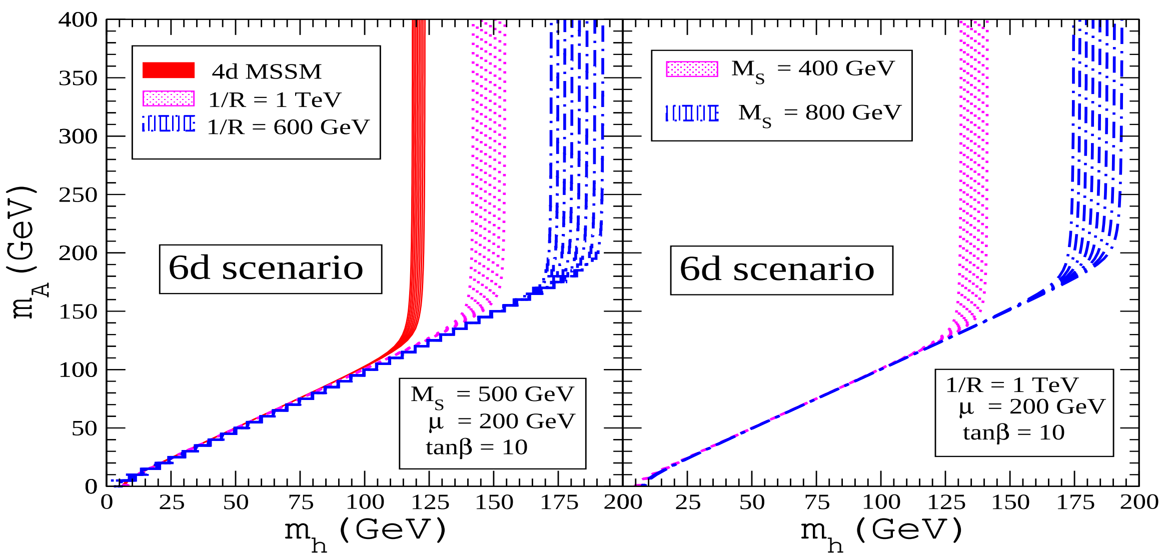

Figure 2: *As in Fig. 1 but for 6d MSSM.*

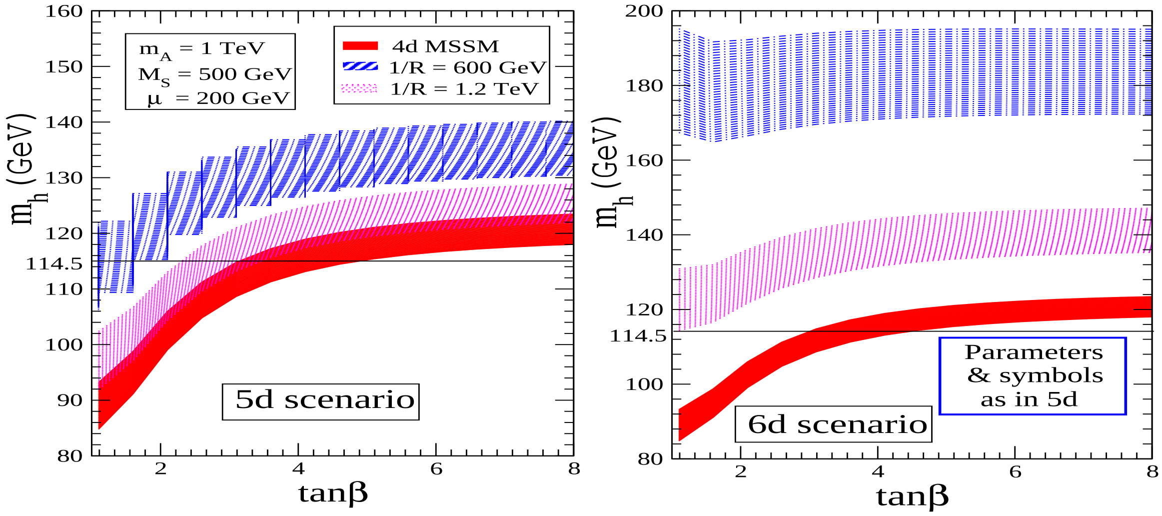

Figure 3: *The dependence of m_h on $\tan\beta$ (zoomed for the low values) in 5d (left panel) and 6d (right panel) MSSM. The width of each band corresponds to the variation of A_t and A_b in the range $(0.8-1.2)M_S$.*

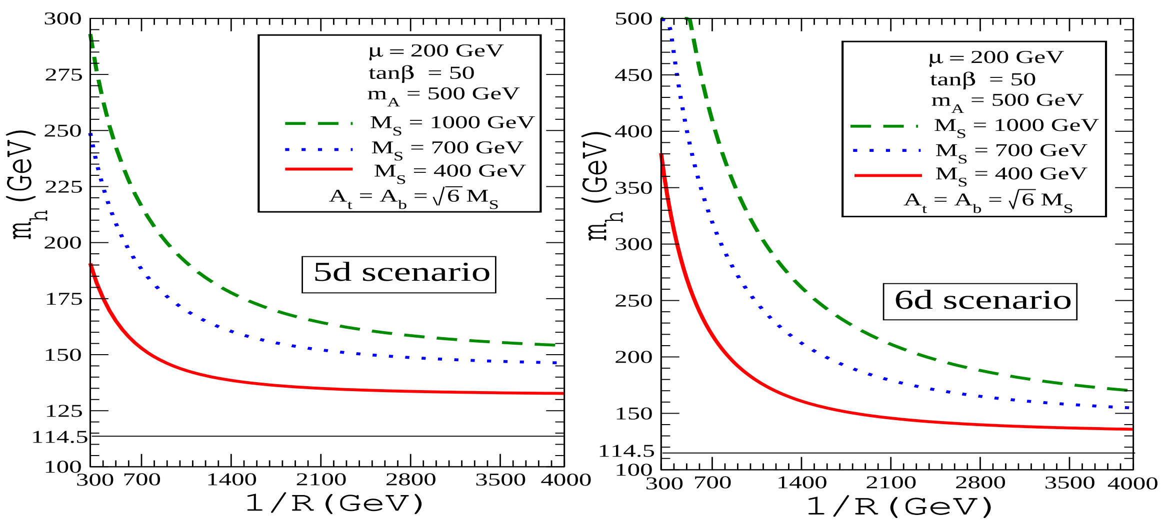

Figure 4: *The dependence of m_h on $1/R$ for different choices of M_S for 5d (left panel) and 6d (right panel) cases. The ratio $\sqrt{6}$ between $A_t(=A_b)$ and M_S maximises the trilinear contribution (see, Drees, Godbole, Roy in [1]).*