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Exit probability in generalised kinetic Ising model

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Abstract In this paper we study generalised Ising Glauber models with inflow of information in one dimension and derive expressions for the exit probability using well established analytical methods. The analytical expressions agree very well with the results obtained from numerical simulation only when the interaction is restricted to the nearest neighbor. But as the range of interaction is increased the analytical results deviate from simulation results systematically. The reasons for the deviation as well as some related open questions are discussed.

Keywords Coarsening processes (Theory) · Kinetic Ising models · Stochastic processes (Theory)

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1 Introduction

Various physical systems and phenomena can be characterised using binary variables having values $\sigma = \pm 1$ (or $0, 1$). The classic example is the Ising model which has only two allowed spin states, taken as $\sigma = \pm 1$. Ising spins can be used to represent the states of individual components of many other systems, particularly in models related to social phenomena [1,2,3,4]. The voter model [1] and Sznajd model [5] for opinion formation in a society are typical examples.

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When the dynamics of ordering is studied in such systems, one usually starts with a fraction x of the variables having value 1 and the rest equal to -1 . $x = \frac{1}{2}$ corresponds to a completely random initial state. In one dimension, usually all models with binary variables end up in a consensus state with all particle states having values 1 (or -1). The two absorbing states occur with different probability. As a function of x , the probability that all particles end up in a state 1 is called the exit probability $E(x)$.

Recently a lot of effort has been put to identify the characteristic features of exit probability for models in one dimension. While for the Ising model with Glauber dynamics, $E(x) = x$ is an exact result, in the Sznajd model the variation of $E(x)$ is nonlinear [6,7,8]. Extending only to the next nearest neighbor, the generalised Ising model studied in [6] showed that the results were identical with the Sznajd model. This led to the idea that exit probability can have nonlinear variations for both inflow and outflow dynamics (examples are the Ising and the Sznajd models respectively). Introducing further neighbor interaction, asymmetry and fluctuations, the exit probability has been studied in Ising Glauber models [6,9]; once again nonlinear variations are observed. There is one school of thought that $E(x)$ has a step function like behaviour for all one dimensional models [10]. However this has been observed only in certain cases e.g. when one introduces a neighbouring domain size dependent dynamics [11,12] or in a mean field like approximation for the nonlinear q voter model [13]. These recent results have generated a lot of interest in the study of exit probability in dynamical spin models.

Most of the available results are obtained by numerical simulation when the range of interaction exceeds beyond nearest neighbor and/or other features are introduced in the Ising Glauber (IG) model. Universal forms for $E(x)$ for Ising Glauber models [9] and the nonlinear q voter model [14] have been proposed although there is an existing controversy regarding its validity [13,15] for the latter. For the so called generalised Ising Glauber models, we have obtained an expression of $E(x)$ using Kirkwood approximation (KA) following [16]. However, an additional assumption has to be used along with KA to obtain the results [7,16]. Kirkwood approximation has proven quite successful in a variety of application to reaction kinetics [17]. Also for Sznajd model this approach gives very good agreement with computer simulation results [7,8].

To derive $E(x)$ using Kirkwood approximation (KA) one has to consider the master equation for the probability distribution of a given spin configuration $P(\sigma_1, \dots, \sigma_N, t)$ as,

$$\begin{aligned} \frac{d}{dt}P((\sigma_1, \dots, \sigma_N); t) = & - \sum_j w(\sigma_j)P(\sigma_1, \dots, \sigma_j, \dots, \sigma_N; t) \\ & + \sum_j w(-\sigma_j)P(\sigma_1, \dots, -\sigma_j, \dots, \sigma_N; t), \end{aligned}$$

where $w(\sigma_j)$ be the probability per unit time that the j th spin flips from σ_j to $-\sigma_j$ which we refer to as flipping rate.

Using this master equation the mean value of the j th spin

$$\langle \sigma_j \rangle = \sum_{\{\sigma\}} \sigma_j P(\sigma; t)$$

evolves as

$$\frac{d}{dt} \langle \sigma_j \rangle = -2 \langle w(\sigma_j) \sigma_j \rangle \quad (1)$$

and the nearest-neighbor correlation function $\langle \sigma_j \sigma_{j+1} \rangle$ evolves as,

$$\frac{d}{dt} \langle \sigma_j \sigma_{j+1} \rangle = -2 \langle \sigma_j \sigma_{j+1} \{w(\sigma_j) + w(\sigma_{j+1})\} \rangle. \quad (2)$$

Because of spatial homogeneity all $\langle \sigma_j \rangle$ are identical and we write the magnetisation as $m \equiv \langle \sigma_j \rangle$. The two-spin correlation $\langle \sigma_j \sigma_{j+1} \rangle$ is written as m_2 .

In the Kirkwood approximation the 3-spin correlation function is decoupled as

$$\langle \sigma_{j-1} \sigma_j \sigma_{j+1} \rangle \approx m m_2 \quad (3)$$

and the 4-point function is factorized as the product of 2- point functions,

$$\langle \sigma_{j-1} \sigma_j \sigma_{j+1} \sigma_{j+2} \rangle \approx m_2^2. \quad (4)$$

We have used another approximation given in [7,16]. It is assumed that $\langle \sigma_i \sigma_{i+n} \rangle$ only weakly depends on distance n . This is justified if the domains are large i.e. at later stages of the evolution.

In this paper we needed to assume

$$\langle \sigma_i \sigma_{i+n} \rangle \equiv m_2 \quad (5)$$

for $1 \leq n \leq 4$. The boundary conditions at time $t = 0$ and $t = \infty$ are

$$m(0) = 2x - 1$$

and

$$m(\infty) = 2E(x) - 1.$$

Since initially all the spins are uncorrelated, we also have $m_2(0) = m(0)^2$.

Here we have studied $E(x)$ for models with nearest and next nearest neighbor interaction in one dimension using Kirkwood approximation and the approximation given in equation (5). For all the models considered here, numerical results for $E(x)$ were reported in an earlier study [9] showing strong dependence on the model parameters. It is possible to obtain approximate results for $E(x)$ in all these models using equations (3-5) and also compare them with the numerical ones.

For each of the models considered, we evaluate $w(\sigma_i)$ at zero temperature. Solving the equations of motion and applying the above boundary conditions we obtain an expression of $E(x)$. In section II we calculate the exit probability for a generalised IG model with nearest neighbor interaction and in section III we calculate the exit probability for generalised IG models with next nearest neighbor interaction. In section IV we have made comparison with the simulation results. In section V the results are discussed and we conclude that this method works very well for models with nearest neighbor interaction but not so good for models with long ranged interaction.

2 W_0 model with nearest neighbor interaction

The generalised IG model with nearest neighbor interaction considered here is called the W_0 model [18]. In the W_0 model at zero temperature, the central spin σ_i is flipped with probability W_0 when the sum of the neighboring spins is zero, i.e. $\sigma_{i-1} + \sigma_{i+1} = 0$. Except this fact it is exactly like IG model with nearest neighbor interaction. $W_0 = 1/2$ corresponds to original Glauber dynamics and $W_0 = 1$ is the Metropolis rule. $W_0 = 0$ is not allowed since this is the case of constrained zero temperature Glauber dynamics.

Here the flipping rate of σ_i can be written as,

$$w(\sigma_i) = \frac{1}{4}[1 + 2W_0 - \sigma_i(\sigma_{i+1} + \sigma_{i-1}) + (1 - 2W_0)\sigma_{i+1}\sigma_{i-1}]. \quad (6)$$

The details of the derivation of the spin flip rate is given in Appendix A.

Using the expression for transition rate in equation (1) we have,

$$\begin{aligned} \frac{d}{dt}\langle\sigma_j\rangle = & -\frac{1}{2}[\langle\sigma_j\rangle + 2W_0\langle\sigma_j\rangle - \langle\sigma_{j+1}\rangle \\ & - \langle\sigma_{j-1}\rangle + \langle\sigma_j\sigma_{j+1}\sigma_{j-1}\rangle - 2W_0\langle\sigma_j\sigma_{j+1}\sigma_{j-1}\rangle]. \end{aligned} \quad (7)$$

From equation (2) we have,

$$\begin{aligned} \frac{d}{dt}\langle\sigma_j\sigma_{j+1}\rangle = & -\frac{1}{2}[2(1 + 2W_0)\langle\sigma_j\sigma_{j+1}\rangle - 2 \\ & - 2\langle\sigma_{j+1}\sigma_{j-1}\rangle + 2(1 - 2W_0)\langle\sigma_j\sigma_{j-1}\rangle] \end{aligned} \quad (8)$$

Using all the approximations mentioned in section I we have,

$$\frac{dm}{dt} = \frac{1}{2}m(1 - 2W_0)(1 - m_2) \quad (9)$$

and

$$\frac{dm_2}{dt} = 1 - m_2. \quad (10)$$

First we solve for eq. (10). This gives

$$m_2 = 1 - Ce^{-t},$$

where $C = 1 - m_2(0)$. Now inserting this into eq. (9) we have for the average magnetisation,

$$m = m(0) \exp\left(\frac{C}{2}(1 - 2W_0)\right) \exp\left(\frac{C}{2}(2W_0 - 1)e^{-t}\right). \quad (11)$$

Using the expressions for $m(\infty)$ and $m(0)$ and after some straightforward algebra we find the exit probability as,

$$E(x) = \frac{1}{2}[1 + (2x - 1)e^{2x(1-x)(1-2W_0)}]. \quad (12)$$

3 Generalised Ising Glauber models with next nearest neighbor interaction

We consider two types of generalised IG model with next nearest neighbor interaction. The first type can be described by a Hamiltonian defined as,

$$H = -J_1 \sum_i \sigma_i \sigma_{i+1} - J_2 \sum_i \sigma_i \sigma_{i+2} \quad (13)$$

where J_1 is the interaction strength with nearest neighbor and J_2 is the interaction strength with next nearest neighbor and $J_1, J_2 > 0$. This can be called ferromagnetic asymmetric next nearest model or FA model as in [9]. The dynamics are different for the three cases, $J_1 = J_2$, $J_1 > J_2$ and $J_1 < J_2$. The cases $J_1 = J_2$ and $J_1 > J_2$ correspond to $G(2)$ and C_2 models of reference [9]. Under zero temperature dynamics here one can again calculate the flipping rate using Kirkwood approximation. We have studied the three different cases separately.

The second type of model is defined by a dynamical rule; actually we consider the W_0 model with next nearest neighbor interaction. Dynamical rules here are exactly like the case $J_1 = J_2$ (13) except for the case $\sum_{j=1}^2 [\sigma_{i+j} + \sigma_{i-j}] = 0$. In this case the central spin σ_i is flipped with probability W_0 .

3.1 Case I : FA model with $J_1 = J_2$

The flipping rate of a spin at site i for this case at zero temperature can be written as,

$$\begin{aligned} w(\sigma_i) = & \frac{1}{2} \left[1 - \frac{3}{8} \sigma_i (\sigma_{i-2} + \sigma_{i-1} + \sigma_{i+2} + \sigma_{i+1}) \right. \\ & + \frac{1}{8} \sigma_i (\sigma_{i+1} \sigma_{i+2} \sigma_{i-2} + \sigma_{i-1} \sigma_{i+2} \sigma_{i-2} \\ & \left. + \sigma_{i+1} \sigma_{i-1} \sigma_{i-2} + \sigma_{i+1} \sigma_{i+2} \sigma_{i-1}) \right]. \end{aligned} \quad (14)$$

The detailed derivation of equation (14) is given in Appendix B.

Using the above expression for transition rate in equation (1) we have,

$$\begin{aligned} \frac{d}{dt} \langle \sigma_j \rangle = & - \left[\langle \sigma_j \rangle - \frac{3}{8} (\langle \sigma_{j-2} \rangle + \langle \sigma_{j-1} \rangle + \langle \sigma_{j+1} \rangle + \langle \sigma_{j+2} \rangle) \right. \\ & + \frac{1}{8} (\langle \sigma_{j+1} \sigma_{j+2} \sigma_{j-2} \rangle + \langle \sigma_{j+2} \sigma_{j-2} \sigma_{j-1} \rangle \\ & \left. + \langle \sigma_{j-2} \sigma_{j-1} \sigma_{j+1} \rangle + \langle \sigma_{j-1} \sigma_{j+1} \sigma_{j+2} \rangle) \right]. \end{aligned} \quad (15)$$

From equation (2) we have,

$$\begin{aligned} \frac{d}{dt} \langle \sigma_j \sigma_{j+1} \rangle = & - \left[2 \langle \sigma_j \sigma_{j+1} \rangle - \frac{3}{4} (\langle \sigma_{j+1} \sigma_{j-2} \rangle + \langle \sigma_{j+1} \sigma_{j-1} \rangle + 1 \right. \\ & + \langle \sigma_{j+1} \sigma_{j+2} \rangle) + \frac{1}{4} (\langle \sigma_{j+2} \sigma_{j-2} \rangle + \langle \sigma_{j+2} \sigma_{j-2} \sigma_{i+1} \sigma_{i-1} \rangle \\ & \left. + \langle \sigma_{j-2} \sigma_{j-1} \rangle + \langle \sigma_{j-1} \sigma_{j+2} \rangle) \right]. \end{aligned} \quad (16)$$

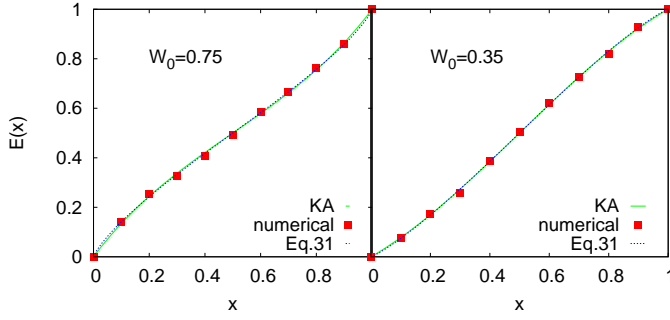


Fig. 1 Exit probability for W_0 model with nearest neighbor interaction with $W_0 = 0.75$ (left) and $W_0 = 0.35$ (right). KA indicates Kirkwood approximating in all the diagrams.

Proceeding as before we have the rate equations for m and m_2 as,

$$\frac{dm}{dt} = \frac{m}{2}(1 - m_2) \quad (17)$$

and

$$\frac{dm_2}{dt} = -\frac{1}{4}[(1 + m_2)^2 - 4]. \quad (18)$$

Solving eq. (18) and using the initial conditions we have,

$$m_2 = \frac{3e^{-t} + C}{C - e^{-t}},$$

where $C = \frac{m_2(0)+3}{m_2(0)-1}$. Now inserting this into eq. (17) we get for the average magnetisation,

$$m = \frac{16m(0)}{(m(0)^2 - 1)^2} \frac{1}{(C - e^{-t})^2}. \quad (19)$$

The exit probability is obtained following some simple algebraic steps as,

$$E(x) = \frac{x^4 - 2x^3 + 3x^2}{2(x^2 - x + 1)^2}. \quad (20)$$

3.2 Case II : FA model with $J_1 > J_2$

In this case the flipping rate for the i th spin can be written as,

$$w(\sigma_i) = \frac{1}{2} \left[1 - \frac{1}{2}\sigma_i(\sigma_{i+1} + \sigma_{i-1}) - \frac{1}{4}\sigma_i(\sigma_{i+2} + \sigma_{i-2}) + \frac{1}{4}\sigma_i\sigma_{i+1}\sigma_{i-1}(\sigma_{i+2} + \sigma_{i-2}) \right].$$

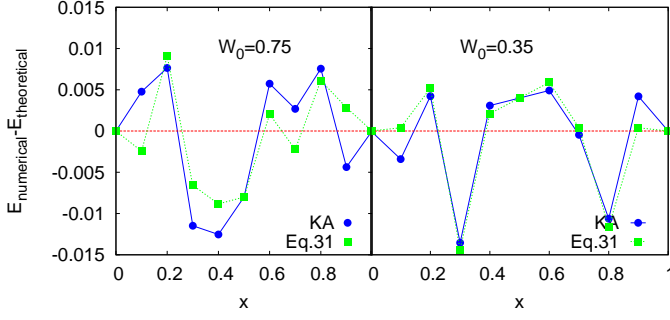


Fig. 2 Difference between the exit probability obtained from numerical simulation and the one predicted by eq. (12) and eq. (31) for the W_0 model with nearest neighbor interaction.

Using this transition rate and following the same procedure the rate equations for the average magnetisation m and nearest-neighbor spin correlation m_2 are obtained as,

$$\frac{dm}{dt} = \frac{1}{2}m(1 - m_2) \quad (21)$$

and

$$\frac{dm_2}{dt} = 1 - m_2. \quad (22)$$

Solving eq. (22) we have $m_2 = 1 - Ce^{-t}$, where $C = 1 - m_2(0)$. Hence we get from eq. (21)

$$m = m(0) \exp\left(\frac{1 - m(0)^2}{2}\right) \frac{1}{\exp(\frac{C}{2}e^{-t})}.$$

After some straightforward steps we have,

$$E(x) = \frac{1}{2} \left[1 + (2x - 1)e^{2x(1-x)} \right]. \quad (23)$$

3.3 Case III : FA model with $J_1 < J_2$

For FA model with $J_1 < J_2$, the flipping rate for the i th spin at zero temperature is given by,

$$w(\sigma_i) = \frac{1}{2} \left[1 - \frac{1}{4}\sigma_i(\sigma_{i+1} + \sigma_{i-1}) - \frac{1}{2}\sigma_i(\sigma_{i+2} + \sigma_{i-2}) \right. \\ \left. + \frac{1}{4}\sigma_i\sigma_{i+2}\sigma_{i-2}(\sigma_{i+1} + \sigma_{i-1}) \right].$$

The rate equations for this case are as follows,

$$\frac{dm}{dt} = \frac{1}{2}m(1 - m_2) \quad (24)$$

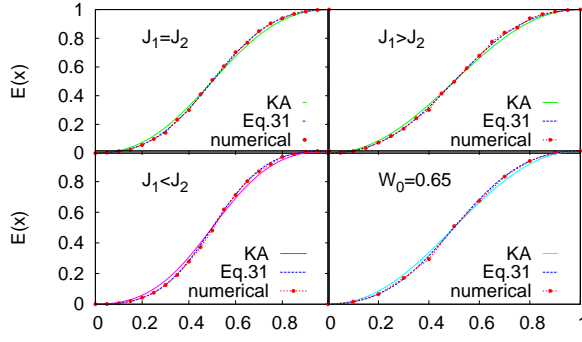


Fig. 3 Exit probability for models with next nearest neighbor interaction.

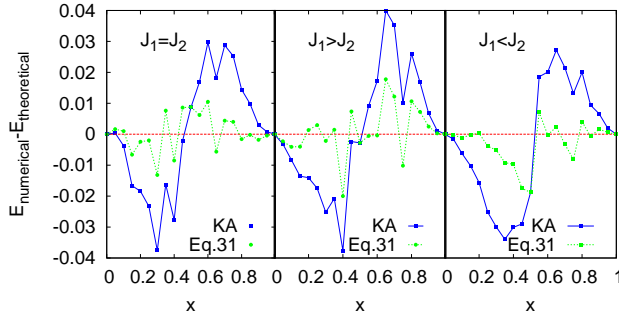


Fig. 4 Difference between the exit probability obtained from numerical simulation and the one predicted by Kirkwood approximation (KA) and universal form (eq. 31) for FA model with $J_1 = J_2$, $J_1 > J_2$ and $J_1 < J_2$.

and

$$\frac{dm_2}{dt} = \frac{1}{2}(1 - m_2^2). \quad (25)$$

Solving eq. (25) we have $m_2 = \frac{e^t - C}{e^t + C}$, where $C = \frac{1 - m_2(0)}{1 + m_2(0)}$. Inserting m_2 in eq. (24) we have

$$m = m(0) \frac{1 + C}{1 + C e^{-t}},$$

and we finally get

$$E(x) = \frac{x^2}{x^2 + (1 - x)^2}. \quad (26)$$

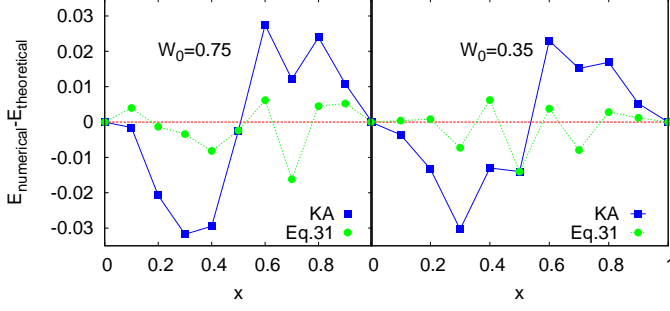


Fig. 5 Difference between the exit probability obtained from numerical simulation and the one predicted by Kirkwood approximation (KA) and universal form (eq. 31) for W_0 model with next nearest neighbor interaction.

3.4 W_0 model with next nearest neighbor interaction

In this case the flipping rate can be written as,

$$\begin{aligned}
w(\sigma_i) = & \frac{1}{8}(3W_0 + \frac{5}{2}) - \frac{3}{16}\sigma_i(\sigma_{i+1} + \sigma_{i+2} + \sigma_{i-1} + \sigma_{i-2}) \\
& + \frac{1}{16}\sigma_i(\sigma_{i+1}\sigma_{i+2}\sigma_{i-2} + \sigma_{i-1}\sigma_{i+2}\sigma_{i-2} \\
& + \sigma_{i+1}\sigma_{i-1}\sigma_{i-2} + \sigma_{i+1}\sigma_{i+2}\sigma_{i-1}) \\
& + \frac{1}{8}(\frac{1}{2} - W_0)(\sigma_{i+1}\sigma_{i+2} + \sigma_{i+1}\sigma_{i-2} + \sigma_{i+1}\sigma_{i-1} \\
& + \sigma_{i+2}\sigma_{i-2} + \sigma_{i+2}\sigma_{i-1} + \sigma_{i-1}\sigma_{i-2}) \\
& + \frac{3}{8}(W_0 - \frac{1}{2})\sigma_{i+1}\sigma_{i+2}\sigma_{i-1}\sigma_{i-2}.
\end{aligned}$$

We have the rate equations for m and m_2 using this flipping rate as,

$$\frac{dm}{dt} = m[(\frac{7}{8} - \frac{3}{4}W_0) + (\frac{3}{2}W_0 - \frac{5}{4})m_2 + \frac{3}{4}(\frac{1}{2} - W_0)m_2^2] \quad (27)$$

and

$$\frac{dm_2}{dt} = \frac{1}{4}[4 - (1 + m_2)^2]. \quad (28)$$

Here we have factorized the 5-point correlation function $\langle \sigma_i \sigma_{i+1} \sigma_{i+2} \sigma_{i-1} \sigma_{i-2} \rangle$ into mm_2^2 while writing down $\frac{dm}{dt}$.

Now solving the eq. (28) we have $m_2 = \frac{e^t - 3C}{e^t + C}$, where $C = \frac{1 - m_2(0)}{3 + m_2(0)}$ and using this expression for m_2 we have from eq. (27),

$$m = C_1(1 + Ce^{-t})^{(12W_0 - 8)} e^{\frac{C(6 - 12W_0)}{e^t + C}}, \quad (29)$$

where $C_1 = \frac{m(0)}{(1+C)^{(12W_0 - 8)} \exp(\frac{C(6 - 12W_0)}{1+C})}$.

Then following the same procedure as above we find $E(x)$ as,

$$E(x) = \frac{1}{2} [1 + (2x - 1) (x^2 - x + 1)^{(12W_0 - 8)} e^{6x(1-x)(2W_0 - 1)}]. \quad (30)$$

4 Simulation results and comparison

Earlier simulations [9] made for the five cases discussed in this paper suggested that the exit probability has a general form given by

$$E(x) = x^\alpha / [x^\alpha + (1 - x)^\alpha]. \quad (31)$$

In [6] only the $J_1 = J_2$ case (eq. 13) was considered which also yielded the same form with $\alpha = 2$, agreeing with the result obtained later in [9]. We find that for the W_0 model with nearest neighbor interaction, both eq. (31) and the Kirkwood approach eq. (12) fit the data quite well (fig. 1) and the differences between these two forms and the simulation data plotted against x (Fig. 2) do not show any systematic variation. Also the magnitude of the variations for both are of the same order (≤ 0.015).

Comparing the simulation results in the cases when interaction upto second neighbor is considered with equations [20, 23, 26, 30] obtained using Kirkwood approximation, we note that there is a considerable difference between the two results and also the fact that the fitting with eq. (31) is definitely better (Fig. 3). The difference between the simulation result and the analytical formula obtained using Kirkwood approximation plotted against x shows systematic variations (Figures 4 and 5) as was observed in [13, 15] for the nonlinear q voter model. Here the magnitude of variations for Kirkwood approach (≤ 0.04) is much larger compared to those for the form given in eq. (31) which have maximum value ~ 0.02 . Although for the $J_1 < J_2$ case we find a form for $E(x)$ (eq. 26) which has the same form as eq. (31), the exponent α is quite different. For $J_1 > J_2$ and $J_1 < J_2$ the exponents α estimated were 1.85 ± 0.03 and 2.24 ± 0.04 respectively [9].

Simulation data shown in all the figures are from reference [9]. The simulation data do not show any appreciable system size dependence.

5 Summary and discussions

In this paper, we have derived analytical expressions for the exit probability for generalised Ising Glauber models in one dimension. We have used Kirkwood approximation together with an additional approximation following [7, 16] to derive the results. The models considered here involve either nearest neighbour interaction or both nearest and next nearest neighbour interactions. The analytical results are compared with earlier results obtained by numerical simulations reported in [6, 9].

The Kirkwood approximation scheme presented here appears to be very efficient for models with nearest neighbour interaction. Although mathematically eq. (31) and eq. (12) cannot be reduced to the same form, it is

interesting to note that their behaviour is almost indistinguishable for any W_0 in general (figures 1 and 2) and it is difficult to conclude which expression fits the data better. For $W_0 > \frac{1}{2}$, one can simplify eq. (12) as the exponential function is less than 1. We take the example of $W_0 = 1$. Here the numerical simulation fits eq. (31) with $\alpha = 0.7$. Eq. (12) for $W_0 = 1$ can be written as

$$\begin{aligned} E(x) &= \frac{1}{2}[1 + (2x - 1)e^{-2x(1-x)}] \\ &= \frac{1}{2}\left[1 + \frac{2x - 1}{1 + 2x(1-x) + 2x^2(1-x)^2}\right]. \end{aligned} \quad (32)$$

The right hand side of eq. (32) can be reduced to the form

$$\frac{x^{0.7}}{x^{0.7} + (1-x)^{0.7}\left[\left(\frac{1-x}{x}\right)^{0.3}\left(\frac{1+x+x^2-x^3}{x^3-2x^2+2}\right)\right]}.$$

The term within the third bracket is found to be very close to 1 except for $x = 0$ and $x = 1$ such that we find that effectively one gets a form as in eq. (31) with $\alpha = 0.7$. However in general the two forms cannot be shown to be identical.

One may try to analyse the reasons behind the failure of KA (along with eq. (5)) for cases with second neighbor interaction. The first step, that of decomposing three spin correlations has been employed in both nearest neighbor and next nearest neighbor cases. In fact in the W_0 model with next nearest neighbor interaction, we have extended this for 5 spin correlations as well. The other assumption of approximating any 2-point correlation as m_2 (eq. 5) has also been employed in both; however $n \leq 2$ for the nearest neighbor case (eq. (8)) while for the next nearest neighbor cases, we use this approximation for $n = 3$ and 4 as well (eq. (16)). Using this approximation (eq. 5) for values of $n > 2$ as well as using equations (3) and (4) coming from KA may appear to be conflicting unless a mean field scenario is valid. The fact is the combination of approximations (3), (4) and (5) yield results which deviate considerably in the next nearest neighbor cases. Apparently this is consistent with the fact that applying equation (5) for $n > 2$ is responsible for the error and the mean field scenario cannot be true as the models are short range in nature (even when the range of interaction is extended to next nearest neighbor). A remedy for this could be to introduce new variables $m_n = \langle \sigma_i \sigma_{i+n} \rangle$ for $n > 2$, however, that would complicate the method to a large extent. At present, we believe this is the best possible way to study $E(x)$ analytically.

If we look at figures 4 and 5, we see that deviations ($|E_{numerical} - E_{theoretical}|$) for the FA model are more compared to the W_0 model with next nearest neighbor interaction. The difference in these two models is that W_0 model is more deterministic in nature. Although a further decoupling scheme is employed for the W_0 model, the fact that the deviations are still less than that in FA indicates that this is not affecting the results to a large extent. The stochasticity in FA therefore appears to be responsible for the larger errors.

In conclusion, we find that although the KA is not very well understood, it is possible to arrive at analytical expressions for exit probability using it in various short range models. The method works very well for models with nearest neighbor interaction as shown by the agreement with numerical results. Also, we show that the question of existence of a universal expression for exit probability [9], even for nearest neighbor models, is reopened. For long ranged models, however, one needs to improve the approximation which may be a topic for future research.

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Appendix

A Detailed derivation of spin flip rate for W_0 model with nearest neighbor interaction

The most general spin flip rate for models with only nearest neighbor interaction satisfies the following natural requirements: [19]

1. Locality: Since it involves only nearest neighbor interaction, the spin flip rates should also depend only on the nearest neighbor of each spin. Thus, $w(\sigma_i) = w(\sigma_{i-1}, \sigma_i, \sigma_{i+1})$.
2. Left/right symmetry: Invariance under the interchange $i - 1 \rightarrow i + 1$.

The most general flip rate that satisfies these conditions has the form,

$$w(\sigma_i) = A + B\sigma_i(\sigma_{i-1} + \sigma_{i+1}) + C\sigma_{i-1}\sigma_{i+1}. \quad (\text{A.1})$$

Imposing the conditions of spin flipping for W_0 model with nearest neighbor interaction we have the following three equations:

$$A - 2B + C = 1, \quad (\text{A.2})$$

$$A + 2B + C = 0 \quad (\text{A.3})$$

and

$$A - C = W_0. \quad (\text{A.4})$$

Solving the above three equations the values of the constants are found to be $A = \frac{1}{4}(1 + 2W_0)$, $B = -\frac{1}{4}$, $C = \frac{1}{4}(1 - 2W_0)$. Thus the spin flip rate for W_0 model with nearest neighbor interaction has a form given in equation (6).

B Detailed derivation of spin flip rate for model with next nearest neighbor interaction

Invoking locality and left/right symmetry the most general spin flip rate for models with next nearest neighbor interaction has the form,

$$\begin{aligned} w(\sigma_i) = & A + B\sigma_i(\sigma_{i+1} + \sigma_{i-1}) + C\sigma_i(\sigma_{i+2} + \sigma_{i-2}) \\ & + D\sigma_i(\sigma_{i+1}\sigma_{i+2}\sigma_{i-2} + \sigma_{i+1}\sigma_{i+2}\sigma_{i-1} + \sigma_{i+2}\sigma_{i-1}\sigma_{i-2} \\ & + \sigma_{i+1}\sigma_{i-1}\sigma_{i-2}) + E(\sigma_{i+1}\sigma_{i+2} + \sigma_{i+1}\sigma_{i-2} + \sigma_{i+1}\sigma_{i-1} \\ & + \sigma_{i+2}\sigma_{i-2} + \sigma_{i+2}\sigma_{i-1} + \sigma_{i-2}\sigma_{i-1}) + F\sigma_{i+1}\sigma_{i+2}\sigma_{i-1}\sigma_{i-2}. \end{aligned} \quad (\text{B.1})$$

Using the conditions of spin flipping for FA model with $J_1 = J_2$, we have the following equations,

$$A + 2B + 2C + 4D + 6E + F = 0, \quad (\text{B.2})$$

$$A - 2B - 2C - 4D + 6E + F = 0, \quad (\text{B.3})$$

$$A + 2B - 2C - 2E + F = \frac{1}{2}, \quad (\text{B.4})$$

$$A - 2B + 2C - 2E + F = \frac{1}{2}, \quad (\text{B.5})$$

$$A - 2B + 2D - F = 1 \quad (\text{B.6})$$

and

$$A + 2B - 2D - F = 0. \quad (\text{B.7})$$

Solving the above six equations we have, $A = \frac{1}{2}$, $B = C = -\frac{3}{16}$, $D = \frac{1}{16}$, $E = F = 0$, which leads to the equation (14) for the spin flip rate of FA model with $J_1 = J_2$. Similarly the spin flip rates for other models with next nearest neighbor interaction can also be obtained.

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