Electron-Neutrino Bremsstrahlung in Electro-Weak Theory

Indranath Bhattacharyya
Department of Applied Mathematics
University of Calcutta, Kolkata-700 009, INDIA
E-mail : *i_bhattacharyya@hotmail.com*

Abstract

The electron-neutrino bremsstrahlung process has been considered in the framework of electro-weak theory. The scattering cross section has been calculated in the center of mass frame and approximated to extreme relativistic as well as non-relativistic case. The rate of energy-loss via this type of bremsstrahlung process has been obtained both in non-degenerate and degenerate region. The effect of this electron-neutrino bremsstrahlung process in different ranges of temperature and density characterizing the late stages of stellar evolution has been discussed. It is found from our study that this bremsstrahlung process is highly important in the non-degenerate region, although it might have some significant effect in the extreme relativistic degenerate region.

PACS : *12.15.-y; 13.85.Hd; 13.85.Lg; 14.60.Lm; 14.70.-e; 97.60.-s*

1 Introduction

It is known that neutrino emission processes have significant contribution in the late stages of the stellar evolution; when the core of the stars collapses, the neutrino emission occurs enormously in the temperature range $> 10^9$ K. In that range the neutrino emission process is important due to extremely large mean free path of the neutrino. It is believed that the stellar matter (even under some extreme conditions as in white dwarves or in neutron stars) is almost transparent to the neutrinos, such in contrast with its behavior with respect to photons. Once neutrinos are produced inside the star the rate of energy loss is higher resulting faster evolution. There are several processes functioning the major role in the energy loss from star in the late stage through the emission of neutrino-antineutrino pair. First time it was pointed out by Gamow and Schoenberg [1] that neutrino might be emitted from star via β decay, which is termed as Urca process. Poentecorvo [2] showed the theoretical possibility of the formation of neutrino pairs in collisions between electrons and nuclei and the process was investigated with possible application in astrophysics by Gandel'man and Pinaev

[3]. The reaction rate of this process was calculated in detail by Chiu and his collaborators [4, 5, 6]. Dicus [7] drew a brief outline about some neutrino emission processes from star, though there might exist a few more processes having remarkable effect during the later stages of the stellar evolution. Earlier most of the were studied in the framework of the V-A interaction theory [8]. Later, photon-neutrino weak coupling theory was introduced and some of those processes such as neutrino synchrotron process [9], photo-coulomb neutrino process [10] were studied in this framework. The advancement of the Standard Model added a new dimension to study several neutrino emission processes such as pair annihilation [7], photo production [7], photon-photon scattering [7, 11, 12, 13], photo-coulomb neutrino process [14, 15] etc. Recently Itoh et al. [16] reviewed a number of neutrino emission processes and discussed their significance from astrophysical point of view. A minor extension of the Standard Model was done due to the existence of neutrino mass resulted from the 'Solar Neutrino Problem' and 'Atmospheric Neutrino Anomaly' [17]. It should be noted that a new theory of weak interaction is yet to be developed by introducing the neutrino mass. In the calculations of some weak processes the effect of this neutrino mass, whatever small it may be, may play an important role, for example photon-neutrino interaction [18, 19].

In this paper we have studied the 'electron-neutrino bremsstrahlung process' given by

$$e^- + e^- \longrightarrow e^- + e^- + \nu + \bar{\nu}$$

according to the electro-weak interaction theory. Previously it was considered by Cazzola and Saggion [20] while calculating the energy-loss rate by using Monte Carlo Integration method without evaluating the scattering cross-section explicitly. But the scattering cross-section, depending on the energy of the incoming electron, can give a very clear idea about the nature of this electro-weak process. It is to be approximated for extreme relativistic as well as non relativistic limit. Cazzola and Saggion [20] considered only the non-degenerate case though many stars in the later phases such as white dwarves, neutron stars etc. are degenerate. We cannot ignore the possibility of occurring the electron-neutrino bremsstrahlung process in degenerate star. We have considered all such cases separately and discussed all possible outcomes of this bremsstrahlung process. We also like to visualize a picture under what circumstances the process will have some significant effect. It cannot be denied that due to some approximations a little bit deviation may occur from the original result, but that will not deter to predict the physical picture. The role of this process has been studied thoroughly at different temperature and density ranges that characterize the late stages of the stellar evolution. It has also been pointed out in which range this process has significant effect.

2 Calculation of scattering cross-section :

The electron neutrino bremsstrahlung has some structural similarity with the bremsstrahlung process in quantum electrodynamics [21, 22, 23]. In this process a slight complication arises since the identical particles (electrons) are involved here. It is not possible to identify which of the two outgoing particles is the 'target' particle for a particular 'incident' electron. In classical physics such identification can be done by tracing out the trajectories. In quantum physics the two alternatives are completely indistinguishable and therefore, the two cases may interfere. There exist 8 possible Feynman diagrams shown in Figure-1 and Figure-2. The total scattering amplitude for all possible diagrams can be constructed according to the Feynman rules as follows:

$$\mathcal{M}^Z = -\frac{4\pi i e^2 g^2}{8\cos^2\theta_W M_Z^2}[(\mathcal{M}_1^Z + \mathcal{M}_2^Z + \mathcal{M}_3^Z + \mathcal{M}_4^Z) - (\mathcal{M}_5^Z + \mathcal{M}_6^Z + \mathcal{M}_7^Z + \mathcal{M}_8^Z)] \tag{2.1}$$

$$\mathcal{M}_1^Z = [\overline{u}(p_1')(C_V - C_A\gamma_5)\gamma_\rho \frac{(q^\tau\gamma_\tau + p_1^{'\tau}\gamma_\tau + m_e)}{(q+p_1')^2 - m_e^2 + i\epsilon}\gamma_\mu u(p_1)][\overline{u}(p_2')\frac{\gamma^\mu}{(p_2 - p_2')^2 + i\epsilon}u(p_2)][\overline{u}_\nu(q_1)(1-\gamma_5)\gamma^\rho v_\nu(q_2)] \tag{2.2}$$

$$\mathcal{M}_2^Z = [\overline{u}(p_1')\gamma_\mu \frac{(-q^\tau\gamma_\tau + p_1^\tau\gamma_\tau + m_e)}{(q-p_1)^2 - m_e^2 + i\epsilon}(C_V - C_A\gamma_5)\gamma_\rho u(p_1)][\overline{u}(p_2')\frac{\gamma^\mu}{(p_2 - p_2')^2 + i\epsilon}u(p_2)][\overline{u}_\nu(q_1)(1-\gamma_5)\gamma^\rho v_\nu(q_2)] \tag{2.3}$$

$$\mathcal{M}_3^Z = \mathcal{M}_1^Z(p_1 \leftrightarrow p_2, p_1' \leftrightarrow p_2') \qquad \mathcal{M}_4^Z = \mathcal{M}_2^Z(p_1 \leftrightarrow p_2, p_1' \leftrightarrow p_2') \tag{2.4}$$

$$\mathcal{M}_5^Z = \mathcal{M}_1^Z(p_1' \leftrightarrow p_2') \quad \mathcal{M}_6^Z = \mathcal{M}_2^Z(p_1' \leftrightarrow p_2') \quad \mathcal{M}_7^Z = \mathcal{M}_3^Z(p_1' \leftrightarrow p_2') \quad \mathcal{M}_8^Z = \mathcal{M}_4^Z(p_1' \leftrightarrow p_2') \tag{2.5}$$

where,

$$C_V = -\frac{1}{2} + 2sin^2\theta_W \qquad C_A = -\frac{1}{2}$$

The prefix Z associated with the matrix element and each of its component indicates that the neutrino anti-neutrino pair emission takes place through the exchange of Z boson. It is worth noting that there exist few more diagrams related to the electron neutrino bremsstrahlung process. For example, there are some diagrams in which the virtual photon, indicated in the given figures, might be replaced by Z and W bosons. In addition Higgs bosons might be present as the virtual lines in some diagrams. We are discussing this process in view of its effect during the late stages of the stellar evolution, where,

$$p_1^0, p_2^0 \ll M_Z, M_W$$

and so in this energy range all such additional diagrams containing more than one ofshell gauge boson lines would have negligible effect compared to the 16 diagrams considered in this article. We can thus ignore those diagrams to make our calculations relatively simpler.

Since the collision occurs between two identical particles, both the halves of the phase are identical. In one half of the phase direct scattering dominates over the exchange graph, whereas in the other half later one dominates over the former. So it is quite reasonable to calculate the process in the half of the phase where the graphs representing the direct scattering dominates. We have carried out our calculations in CM frame in which

$$\vec{p_1} + \vec{p_2} = \vec{p}'_1 + \vec{p}'_2 + \vec{q_1} + \vec{q_2} = 0 \tag{2.6}$$

where $\vec{q_1}$ and $\vec{q_2}$ are the linear momenta of neutrino and anti-neutrino. In this frame we have to calculate the term $|\mathcal{M}^Z|^2$ over the spin sum. It is not very easy task and can be done with some choices and approximations. We can express the term $\sum|\mathcal{M}_1^Z|^2$ as

$$\sum|\mathcal{M}_1^Z|^2 = X_{\rho\sigma\alpha\beta}(p_1, p_2, p_1', p_2')Y^{\alpha\beta}(p_2, p_2')N^{\rho\sigma}(q_1, q_2) \tag{2.7}$$

$$X_{\rho\sigma\alpha\beta}(p_1, p_2, p_1', p_2') = \frac{1}{4m_e^2|(q+p_1')^2 - m_e^2 + i\epsilon|^2}[C_V^2 T_1 - C_A^2 T_2 + C_V C_A T_3 - C_V C_A T_4] \tag{2.8}$$

$$T_1 = Tr[(p_1^\tau\gamma_\tau + m_e)\gamma_\alpha(P^\tau\gamma_\tau + m_e)\gamma_\rho(p_1'^\tau\gamma_\tau + m_e)\gamma_\sigma(P^\tau\gamma_\tau + m_e)\gamma_\beta] \tag{2.8a}$$

$$T_2 = Tr[(p_1^\tau\gamma_\tau + m_e)\gamma_\alpha(P^\tau\gamma_\tau + m_e)\gamma_\rho(-p_1'^\tau\gamma_\tau + m_e)\gamma_\sigma(P^\tau\gamma_\tau + m_e)\gamma_\beta] \tag{2.8b}$$

$$T_3 = Tr[(p_1^\tau\gamma_\tau + m_e)\gamma_\alpha(P^\tau\gamma_\tau + m_e)\gamma_\rho(p_1'^\tau\gamma_\tau + m_e)\gamma_5\gamma_\sigma(P^\tau\gamma_\tau + m_e)\gamma_\beta] \tag{2.8c}$$

$$T_4 = Tr[(p_1^\tau\gamma_\tau + m_e)\gamma_\alpha(P^\tau\gamma_\tau + m_e)\gamma_\rho\gamma_5(-p_1'^\tau\gamma_\tau + m_e)\gamma_\sigma(P^\tau\gamma_\tau + m_e)\gamma_\beta] \tag{2.8d}$$

$$Y^{\alpha\beta}(p_2, p_2') = \frac{1}{m_e^2|(p_2 - p_2')^2 + i\epsilon|^2}[p_2^\alpha p_2'^\beta + p_2^\beta p_2'^\alpha + \{(p_2 p_2') - m_e^2\}g^{\alpha\beta}] \tag{2.9}$$

$$N^{\rho\sigma}(q_1, q_2) = \frac{2}{m_\nu^2}[q_1^\rho q_2^\sigma + q_1^\sigma q_2^\rho - (q_1 q_2)g^{\rho\sigma} + i q_{1\tau_1} q_{2\tau_2}\epsilon^{\tau_1\tau_2\rho\sigma}] \tag{2.10}$$

$$q = q_1 + q_2 = (p_1 + p_2) - (p_1' + p_2') \tag{2.11a}$$

$$P = q + p_1' \tag{2.11b}$$

Instead of calculating the term $\sum|\mathcal{M}_1^Z|^2$ it is more convenient to calculate the term

$$\int \sum|\mathcal{M}_1^Z|^2 \frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\delta^4(q - q_1 - q_2)$$

Let us consider

$$I^{\rho\sigma}(q) = \frac{2}{m_\nu^2}\int[q_1^\rho q_2^\sigma + q_1^\sigma q_2^\rho - (q_1 q_2)g^{\rho\sigma} + i q_{1\tau_1} q_{2\tau_2}\epsilon^{\tau_1\tau_2\rho\sigma}]\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\delta^4(q - q_1 - q_2) \tag{2.12}$$

$$= \frac{1}{m_\nu^2}(Aq^2 g^{\rho\sigma} + Bq^\rho q^\sigma)$$

It is to be remembered that the neutrino mass is very small compared to the magnitude of its linear momentum. This is valid throughout our calculations, even in the non-relativistic case. Even if the neutrino mass is comparable to the magnitude of the linear momentum of the neutrino, then no such neutrino anti-neutrino pair will be emitted and the process becomes superfluous. This is very much consistent with the Standard Model which is based on the concept of mass less neutrino. In principle neutrino may have a little mass, but it is too little to violate the basic assumption of the well known existing theory. Thus taking $m_\nu \ll q^0$ we can evaluate the integral $I^{\rho\sigma}(q)$ and find the value of A and B. It is found to be

$$A = -B = -\frac{\pi}{3}$$

and also we get

$$\int \sum |\mathcal{M}_1^Z|^2 \frac{d^3 q_1}{2q_1^0} \frac{d^3 q_2}{2q_2^0} \delta^4(q - q_1 - q_2) = X_{\rho\sigma\alpha\beta}(p_1, p_2, p_1', p_2') Y^{\alpha\beta}(p_2, p_2') I^{\rho\sigma}(q) \tag{2.13}$$

In the same manner the term $\Sigma|\mathcal{M}_2^Z|^2$ can be evaluated to obtain an expression similar to equation (2.13). In this case P will be replaced by Q, where

$$Q = p_1 - q$$

Evaluating the various trace terms rigorously and simplifying those expressions we obtain

$$\int \sum |\mathcal{M}^Z|^2 \frac{d^3 q_1}{2q_1^0} \frac{d^3 q_2}{2q_2^0} \delta^4(q - q_1 - q_2) = F(p_1, p_2, p_1', p_2') \tag{2.14}$$

The right hand side of this equation is a scalar obtained by the various combinations of the scalar product of initial and final momenta of the electrons. Thus ultimately F becomes the function of either energies or momenta of incoming and outgoing electrons. The scattering cross section of this process can be calculated by using the formula

$$\sigma = \frac{\mathcal{S}}{4\sqrt{(p_1 p_2)^2 - m_e^4}} N_{p_1} N_{p_2} \frac{1}{(2\pi)^2} \int \frac{N_{p_1'} d^3 p_1'}{2p_1'^0 (2\pi)^3} \frac{N_{p_2'} d^3 p_2'}{2p_2'^0 (2\pi)^3} N_{q_1} N_{q_2} F(p_1, p_2, p_1', p_2') \tag{2.15}$$

Here all incoming and outgoing particles are spin-$\frac{1}{2}$ fermions. For that reason N_i ($i = p_1, p_2, p_1', p_2', q_1, q_2$) is twice the mass of the corresponding fermion. The square root term present in the denominator comes from the incoming flux that is directly proportional to the relative velocity of the incoming electrons and written in Lorentz invariant way. As the final state contains the incoming particle there must be a non-unit statistical degeneracy factor $\mathcal{S}$ given by

$$\mathcal{S} = \prod_l \frac{1}{g_l!}$$

if there are g_l particles of the kind l in the final state. This factor arises since for g_l identical final particles there are exactly $g_l!$ possibilities of arranging those particles; but only one such arrangement is measured experimentally. We Calculate the expression for $F(p_1, p_2, p_1', p_2')$ in the equation (2.15) and then integrating that expression the scattering cross section is obtained. We are interested to obtain a clear analytical expression and so we do not use any numerical technique. Instead, with some special choice of approximations we have calculated the integral present in (2.15). Let us consider the following four vector

$$p' = p_1' + p_2' = p_1 + p_2 - q$$

It is clear that p' is timelike i.e. $(p')^2 > 0$ and so we can take a proper Lorentz transform such that $p' = (p'^0, 0)$. To obtain the integral over d^3p_1' we should write

$$\int \frac{F}{p_2'^0} d^3p_2' = \frac{4\pi}{3} \frac{|\overrightarrow{p}'_1|^3}{p_1'^0} F(|\overrightarrow{p}'_2| = |\overrightarrow{p}'_1|, ...) + \epsilon$$

The error term ϵ will be present since we consider the system in CM frame, defined by the equation (2.6), simultaneously. Here neglecting this error term we can use the following approximation.

$$\int \frac{F}{p_2'^0} d^3p_2' \approx \frac{4\pi}{3} \frac{|\overrightarrow{p}'_1|^3}{p_1'^0} F(|\overrightarrow{p}'_2| = |\overrightarrow{p}'_1|, ...) \tag{2.16}$$

Next we integrate over d^3p_1' without any more approximation and obtain the expression for the scattering cross-section as follows:

$$\sigma = \frac{(C_V^2 + C_A^2)}{9\pi^2} \left(\frac{eg}{M_Z cos\theta_W}\right)^4 \frac{(p^0)^2}{\sqrt{1 - (\frac{m_e}{p^0})^2}} [\ln(\frac{p^0}{m_e}) + f(p^0, r)] \tag{2.17}$$

where,

$$f(p^0, r) = \ln r - [r - \tfrac{m_e}{p^0}][14 - \frac{16}{(1+\frac{C_V^2}{C_A^2})}(\tfrac{m_e}{p^0})^2 + \frac{3(1+\frac{3C_V^2}{2C_A^2})}{(1+\frac{C_V^2}{C_A^2})}(\tfrac{m_e}{p^0})^4] + \tfrac{1}{2}[r^2 - (\tfrac{m_e}{p^0})^2][31 - \frac{12(1+\frac{27C_V^2}{24C_A^2})}{(1+\frac{C_V^2}{C_A^2})}(\tfrac{m_e}{p^0})^2 - 3(\tfrac{m_e}{p^0})^4] - \tfrac{2}{3}[r^3 - (\tfrac{m_e}{p^0})^3][7 - 3(\tfrac{m_e}{p^0})^2] + \tfrac{1}{4}[r^4 - (\tfrac{m_e}{p^0})^4] - 3(\tfrac{m_e}{p^0})^2 \ln(\tfrac{rp^0}{m_e})[\frac{4(1+\frac{27C_V^2}{24C_A^2})}{(1+\frac{C_V^2}{C_A^2})} - (\tfrac{m_e}{p^0})^2 - \frac{1}{(1+\frac{C_V^2}{C_A^2})}(\tfrac{m_e}{p^0})^4] + 3(\tfrac{m_e}{p^0})^2[\tfrac{p^0}{m_e} - \tfrac{1}{r}][2 - \frac{(1+\frac{2C_V^2}{3C_A^2})}{(1+\frac{C_V^2}{C_A^2})}(\tfrac{m_e}{p^0})^2] - \tfrac{3}{2}(\tfrac{m_e}{p^0})^4[(\tfrac{p^0}{m_e})^2 - (\tfrac{1}{r})^2][1 - \frac{1}{(1+\frac{C_V^2}{C_A^2})}(\tfrac{m_e}{p^0})^2]$$

and

$$\frac{m_e}{p^0} < r = \frac{max(p_1'^0, p_2'^0)}{p^0} < 1$$

p^0 represents the center of mass energy i.e.

$$p_1^0 = p_2^0 = p^0$$

whereas $p_1^{'0}$ and $p_2^{'0}$ stand for energies of the outgoing electrons.

It is worth noting that all three type of neutrinos may involve in this process, since there is no lepton number violation for it. So far we have used the technique applicable for both muon and tau neutrino; but for electron type of neutrino another 8 Feynman diagrams having $e - W^- - \nu_e$ effect may contribute in this process. Four of them are for direct processes (Figure-3) and the rest four represent exchange diagrams (Figure-4). These extra diagrams contribute in the calculations of scattering cross-section, but only for electron type of neutrino emission. In that case the matrix element will be modified as

$$\mathcal{M} = \mathcal{M}^Z + \mathcal{M}^W \tag{2.18}$$

where,

$$\mathcal{M}^W = -\frac{4\pi i e^2 g^2}{8M_W^2}[(\mathcal{M}_1^W + \mathcal{M}_2^W + \mathcal{M}_3^W + \mathcal{M}_4^W) - (\mathcal{M}_5^W + \mathcal{M}_6^W + \mathcal{M}_7^W + \mathcal{M}_8^W)] \tag{2.19}$$

$$\mathcal{M}_1^W = [\overline{u}(p_1')(1-\gamma_5)\gamma_\rho \frac{(q^\tau\gamma_\tau + p_1^{'\tau}\gamma_\tau + m_e)}{(q+p_1')^2 - m_e^2 + i\epsilon}\gamma_\mu v_\nu(q_2)][\overline{u}(p_2')\frac{\gamma^\mu}{(p_2-p_2')^2 + i\epsilon}u(p_2)][\overline{u}_\nu(q_1)(1-\gamma_5)\gamma^\rho u(p_1)] \tag{2.20a}$$

$$\mathcal{M}_2^W = [\overline{u}(p_1')\gamma_\mu \frac{(-q^\tau\gamma_\tau + p_1^\tau\gamma_\tau + m_e)}{(q-p_1)^2 - m_e^2 + i\epsilon}(1-\gamma_5)\gamma_\rho v_\nu(q_2)][\overline{u}(p_2')\frac{\gamma^\mu}{(p_2-p_2')^2 + i\epsilon}u(p_2)][\overline{u}_\nu(q_1)(1-\gamma_5)\gamma^\rho u(p_1)] \tag{2.20b}$$

Other $\mathcal{M}_i^W$'s (i=3,..8) have the similar expressions as defined in the equations (2.4) and (2.5). We use Fierz rearrangement to obtain the full expression for $\mathcal{M}$ containing the contributions for both Z and W bosons exchanged diagrams. If we introduce Fierz rearrangement on $\mathcal{M}_1^W$ in (2.20a) and add it to (2.2) we obtain

$$\mathcal{M}_1 = [\overline{u}(p_1')(C_V' - C_A'\gamma_5)\gamma_\rho \frac{(q^\tau\gamma_\tau + p_1^{'\tau}\gamma_\tau + m_e)}{(q+p_1')^2 - m_e^2 + i\epsilon}\gamma_\mu u(p_1)][\overline{u}(p_2')\frac{\gamma^\mu}{(p_2-p_2')^2 + i\epsilon}u(p_2)][\overline{u}_\nu(q_1)(1-\gamma_5)\gamma^\rho v_\nu(q_2)] \tag{2.21a}$$

and thus the total scattering matrix becomes

$$\mathcal{M} = -\frac{4\pi i e^2 G_F}{\sqrt{2}}[(\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4) - (\mathcal{M}_5 + \mathcal{M}_6 + \mathcal{M}_7 + \mathcal{M}_8)] \tag{2.21b}$$

where,

$$C_V' = \frac{1}{2} + 2sin^2\theta_W \qquad\qquad C_A' = -\frac{1}{2}$$

and

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8M_Z^2 cos^2\theta_W}$$

Each $\mathcal{M}_i$ (i=1,2..8) is formed by adding $\mathcal{M}_i^Z$ and rearranged $\mathcal{M}_i^W$. Then we proceed in the same way as before and calculate the scattering cross section as

$$\sigma_{\nu_e} = \frac{4(C_V'^2 + C_A'^2)}{9\pi^2}\alpha^2 G_F^2 \frac{(p^0)^2}{\sqrt{1-(\frac{m_e}{p^0})^2}}[\ln(\frac{p^0}{m_e}) + f_{\nu_e}(p^0, r)] \tag{2.22}$$

The expression $f_{\nu_e}(p^0, r)$ present in the equation (2.22) is almost similar to the expression $f(p^0, r)$. In fact if we replace C_V and C_A present in $f(p^0, r)$ by C'_V and C'_A respectively the expression $f_{\nu_e}(p^0, r)$ can be found out.

The equation (2.22) gives the scattering cross-section for electron type of neutrino whereas the scattering cross section for both muon and tau neutrino can be obtained by using equation (2.17). We like to find the scattering cross section in the extreme relativistic as well as non-relativistic limit. In those two limit the total scattering cross section, in c.g.s unit, for all three type of neutrinos are approximated as

$$\sigma_{\nu_e,\nu_\mu,\nu_\tau} \approx 5.8 \times 10^{-50} \times (\frac{E_{ER}}{m_e c^2})^2 \ln(\frac{E_{ER}}{m_e c^2}) cm^2 \quad [extreme - relativistic] \tag{2.23}$$

$$\approx 3.44 \times 10^{-49} \times (\frac{E_{NR}}{m_e c^2})^{\frac{1}{2}} \quad cm^2 \quad [non - relativistic] \tag{2.24}$$

To be noted that E_{ER} and E_{NR} represent the energy of the single electron related to extreme relativistic and non-relativistic limit respectively.

We are going to check the goodness of our approximated analytical method. To verify it let us obtain the scattering cross-section for electron type of neutrino in the relativistic case from the equation (2.22), which gives

$$\sigma_{\nu_e} \simeq 4.16 \times 10^{-51} \times (\frac{E}{m_e c^2})^2 \ln(\frac{E}{2 m_e c^2}) \quad cm^2 \tag{2.25}$$

where E is the CM energy.

In the Tabe-1 this result is compared to the scattering cross section for electron type of neutrino, obtained by using CalcHep software (version-2.3.7).

3 Calculation of energy loss rate :

A number of different cases are to be considered to calculate the energy loss rate via electron neutrino bremsstrahlung process. We have already obtained the scattering cross section in extreme relativistic and non-relativistic limit. The later stage of the stellar evolution may be degenerate as well as non-degenerate characterized by the chemical potential. In the evolution of some stars the electron gas exerts degenerate pressure which prevents the star from contraction due to the gravitational force. The complete degenerate gas is such in which all the lower states below the Fermi energy become occupied. There may be some ranges of temperature and density where electron energy would not be bounded by Fermi-energy. Such non-degeneracy may be evident for both relativistic and non-relativistic limit i.e. for $\kappa T < m_e c^2$ and $\kappa T > m_e c^2$ respectively, where κ represents Boltzmann's constant. We calculate the energy loss rate by using the formula

$$\rho \mathcal{E}_\nu = \frac{4}{(2\pi)^6 \hbar^6} \int_0^\infty \int_0^\infty \frac{d^3p_1}{[e^{\frac{E_1}{\kappa T}-\psi}+1]} \frac{d^3p_2}{[e^{\frac{E_2}{\kappa T}-\psi}+1]} (E_1+E_2) \int d\sigma(E_1', E_2') g(E_1', E_2') |\vec{v_1} - \vec{v_2}| \tag{3.1}$$

where ρ is the mass density of the electron gas and $g(E_1', E_2')$ stands for Pauli's blocking factor, given by

$$g(E_1', E_2') = [1 - \frac{1}{e^{\frac{E_1'}{\kappa T}-\psi}+1}][1 - \frac{1}{e^{\frac{E_2'}{\kappa T}-\psi}+1}] \tag{3.2a}$$

$\psi = \frac{\mu}{\kappa T}$ (μ represents the chemical potential of the electron gas) is related to the number density of the electron by the following relation.

$$n = \frac{2(\kappa T)^3}{\pi^2 (c\hbar)^3} \int_0^\infty \frac{x[x^2 - (\frac{m_e c^2}{\kappa T})^2]^{\frac{1}{2}}}{[e^{x-\psi}+1]} dx \tag{3.2b}$$

To be noted that in equations (3.1) and (3.2b) the upper limit of the momentum has been taken up to infinity; this may give an impression that the CM energy of the electron may be very high and comparable to M_Z or M_W. It is not true since the temperature and density of the stellar core in the later stage do not allow the electron to gain that very high energy. Thus in the equations (3.1) and (3.2b) the upper limit of the momentum depends on the temperature and density of the electron gas. We go through the different cases in the followings.

Case I: In the extreme relativistic non-degenerate case, i.e. when $T \gg 5.9 \times 10^9$ K and $\rho < 2 \times 10^6$ gm/cc, the chemical potential becomes very small compared to E_{ER}. In this case the energy loss rate is calculated as

$$\mathcal{E}_\nu \approx 5.04 \times 10^{12} \times T_{10}^6 [1 + 0.82 \ln(1.7\, T_{10})] \qquad erg - gm^{-1} sec^{-1} \tag{3.3}$$

where

$$T_{10} = T \times 10^{-10}$$

From this analytical expression it is found that the energy loss rate does not depend on the density of the electron gas so far the non-degeneracy remains effective.

Case-II: In the extreme relativistic degenerate region the density would be very high; it is to be noted that for core temperature $T \gg 5.9 \times 10^9$ K and the density would be much higher than 10^7 gm/cc. Pauli's blocking factor plays an important role to calculate the energy loss rate for degenerate electron. For extreme relativistic case it can be approximated as

$$\int d\sigma(E_1', E_2') g(E_1', E_2') \approx e^{2(1-x_F)} \sigma \tag{3.4a}$$

where x_F represents the ratio of the Fermi temperature to the maximum temperature of the degenerate electron gas at the maximum density ($\sim 10^{15}\ gm/cc$). The energy loss rate in the extreme degenerate case is obtained as

$$\mathcal{E}_\nu \approx 6.56 \times 10^{10} \times T_{10}^6[1 + 0.56 \ln(1.7\, T_{10})] \qquad erg - gm^{-1}sec^{-1} \tag{3.4b}$$

To be noted that the above expression for energy loss rate depends on density only, not on the temperature.

Case-III: The non-relativistic effect becomes important when the central temperature of the star is below the 5.9×10^9 K and the electron would be non-degenerate if $(\frac{\rho}{2})^{\frac{2}{3}} \leq (\frac{T}{2.97\times10^5 K}$). In this case $\mu < m_e c^2$, but ψ cannot be neglected as in the extreme relativistic case. We calculate the energy-loss rate as follows:

$$\mathcal{E}_\nu \approx 0.88 \times 10^{-3} \times T_8 \rho \qquad erg - gm^{-1}sec^{-1} \tag{3.5}$$

Where T_8 is defined in the same manner as T_{10}. This energy loss rate is not low in the region having the temperature $10^8 - 10^9$ K and density less than 10^6 gm/cc, which indicates the importance of this process in the non-relativistic non-degenerate region.

It is worth noting that the energy loss rate in the non-relativistic but degenerate case is very low, hence this case is not considered here.

4 Discussion :

In the calculations of scattering cross-section we have used an approximation in the equation (2.16), but the Table-1 shows our result is very close to that generated by the software. It strongly supports the approximation we have used in the equation (2.16). The scattering cross section obtained under the frame-work of electro-weak theory is very small, especially in the non-relativistic case, but as the mean free path of the neutrinos is much longer than the scale of stellar radius the electron-neutrino bremsstrahlung process may have some effect to release energy from star at high temperature and density. The relativistic effect comes into play when the temperature exceeds 6×10^9 K. It is evident from our work that the electron-neutrino bremsstrahlung process yields a large amount of energy loss from the stellar core when core temperature $\geq 10^{10}$ K, both in non-degenerate as well as degenerate region. In that temperature range the radiation pressure is so dominating that the gas pressure has negligible effect [24]. In this extreme relativistic region the process contributes significantly when the electron gas is non-degenerate. That was also shown by Cazzola and Saggion

[20]. But they did not calculate the energy loss rate in the degenerate region; though they indicated that the electron neutrino bremsstrahlung process might be highly significant in that region. We have calculated the energy-loss rate to obtain an analytical expression for the energy loss rate when the electrons are strongly degenerate. A typical example of the extreme-relativistic degenerate stellar object is the newly born neutron star, which is the result of type-II Supernova. Our study reveals that the energy loss rate in the non-degenerate region is higher than that calculated in the degenerate region. This clearly indicates that though during the neutron star cooling electron-neutrino bremsstrahlung plays a significant role, but the process becomes more important to carry away the energy from the core of pre-Supernova star, which is a relativistic non-degenerate stellar object.

Non-relativistically the process becomes insignificant unless the temperature is sufficiently high, at least the temperature should attain 10^8 K. At this temperature the burning of helium gas in the stellar core takes place [25]. In the temperature range $10^8 - 10^9$ K the gas pressure is dominating over the radiation pressure; though the effect of radiation pressure cannot be neglected in this region. In addition, the region will be non-degenerate if density $< 2 \times 10^6$ gm/cc. The electron neutrino bremsstrahlung process may have some effect in this region though the energy loss rate is not so high as it is in the extreme relativistic case. In the low density the energy loss rate increases rapidly with rising core temperature. Eventually the process contributes in both degenerate and non-degenerate cases, whereas degenerate electrons participate only when the density of the medium is very high. Hence, the electron-neutrino bremsstrahlung process is an important energy-generation mechanism during the evolution of stars, particularly in the later stages.

5 Acknowledgement:

It is my pleasure to thank Prof. **Probhas Raychaudhri**, University of Calcutta, India for his valuable suggestions and guidance in the preparation of this manuscript. I would also like to thank **CSIR**, India for funding this research work and **ICTP**, Trieste, Italy for providing me some excellent facilities and information, which have facilitated to carry out this work. My special thank goes to **Biplob Bhattacherjee**, Department of Pure Physics, University of Calcutta for helping me in the computational work.

References

[1] G.Gamow and M.Schoenberg, *Phys.Rev.* **59**, 539 (1941)

[2] B.M. Pontecorvo, *JETP* **36**, 1615 (1959)[*Soviet Phys. JETP* **9**, 1148 (1959)]

[3] G.M. Gandel'man and V.S. Pinaev, *JETP* **37**, 1072 (1959) [*Soviet Phys. JETP* **10**, 764 (1960)]

[4] H.Y.Chiu and P.Morrison, *Phys.Rev. Lett.* **5**, 573 (1960)

[5] H.Y.Chiu and R.C.Stabler, *Phys.Rev.* **122**, 1317 (1961)

[6] H.Y.Chiu, *Phys.Rev.* **123**, 1040 (1961)

[7] D.A.Dicus, *Phys.Rev.D* **6**, 941 (1972)

[8] R.Feynman and M.Gell-Mann, *Phys.Rev.* **109**, 193 (1958)

[9] P.Raychaudhuri, *Astrophys. Sp.Sc.* **8**, 432 and 448 (1970)

[10] P.Raychaudhuri, *Progress. Theo. Phys.* **42**, 116 (1969)

[11] D.A.Dicus and W.W.Repko, *Phys.Rev.D* **48**, 5106 (1993)

[12] A.Abbasabadi,A.Devoto,D.A.Dicus and W.W.Repko *Phys.Rev.D* **59**, 013012 (1998)

[13] A.Abbasabadi, A.Devoto and W.W.Repko, *Phys.Rev.D* **63**, 093001 (2000)

[14] L.Rosenberg, *Phys.Rev.* **129**, 2786 (1963)

[15] I.Bhattacharyya, *Astropart.Phys.* **22**, 369 (2005)

[16] N.Itoh, H.Hayashi, A.Nishikawa and Y. Kohyama, *Astrophys. Journ. suppl.* **102**, 411 (1996)

[17] A.De Santo, *Int.J.Mod.Phys.A* **16**, 4085 (2001)

[18] D.A.Dicus, W.W.Repko and R.Vega, *Phys.Rev.D* **62**, 093027 (2000)

[19] S.Dodelson and G.Feinberg, *Phys.Rev.D* **43**,913 1991

[20] P.Cazzola and A.Saggion, *IL Nuvo Cim.* **63A** 354 and 367 (1969)

[21] V.Vortuba, *Phys.Rev.* **73**, 1468 (1948)

[22] J.Wheeler and W.E.Lamb, *Phys.Rev.* **55**, 858 and 1956 (1939) and *Phys.Rev.* **101**, 1836 (1939)

[23] J.Joseph and F.Rohrlich, *Rev.Mod.Phys.* **30**, 354 (1958)

[24] S.Chandrasekhar, *An Introduction to the Study of Stellar Structure* (The Univeristy of Chicago Press, Chicago,1939, and Dover Publications, New York 1958)

[25] C.Hayashi, R.Hoshi and D.Sugimoto, *Supp. Progress. Theo. Phys.* **22**, 1 (1962)

Table-1 :

CM Energy (MeV)	$\sigma_{\nu_e}(cm^2)$	
	Software generated result	Our result
10	3.45×10^{-48}	3.64×10^{-48}
20	1.19×10^{-47}	1.88×10^{-47}
30	4.47×10^{-47}	4.84×10^{-47}
40	1.17×10^{-46}	0.92×10^{-46}
50	1.23×10^{-46}	1.56×10^{-46}
60	1.95×10^{-46}	2.32×10^{-46}
70	4.06×10^{-46}	3.28×10^{-46}
80	4.45×10^{-46}	4.44×10^{-46}

Table 1: Comparison of the scattering cross-section for electron type of neutrino obtained by our method relative to that generated by CalcHep software.

Figure Caption :

Figure-1: Feynman diagrams for the direct process

Figure-2: Exchange diagrams

Figure-3: Feynman diagrams for the direct process having $e - W^- - \nu_e$ effect

Figure-4: Exchange diagrams having $e - W^- - \nu_e$ effect