

# Conversion of nuclear to 2-flavour quark matter in rotating compact stars: A general relativistic perspective

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The conversion of neutron star to strange star is argued to be a two step process. The first process involves the deconfinement of nuclear to two-flavour quark matter. The GR results shows that the propagating front breaks up into fragments which propagate with different velocities along different directions. The time taken for this conversion to happen is of the order of few *ms*. This calculation indicates the inadequacy of non-relativistic (NR) or even Special Relativistic (SR) treatments for these cases.

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## I. INTRODUCTION

Strange Quark Matter (SQM), consisting of approximately equal numbers of up (*u*), down (*d*) and strange (*s*) quarks, is conjectured to be the *true* ground state of strong interaction [1, 2]. SQM could naturally occur in the cores of compact stars, where central densities of about an order of magnitude higher than the nuclear matter saturation density. The transition from nuclear (hadronic) to quark matter should proceed through a conversion to an initial stage of (metastable) two flavour quark matter, which should decay to the stable SQM. Thus, neutron stars with sufficiently high central densities ought to get converted to strange, or at least hybrid, stars. These transitions could have observable signatures in the form of a jump in the breaking index and gamma ray bursts [3, 4].

There are several plausible scenarios where neutron stars could convert to quark stars, through a "seed" of external SQM [5], or triggered by the rise in the central density due to a sudden spin-down in older neutron stars [6]. Several authors have studied the conversion of nuclear matter to strange matter under different assumptions [7, 8, 9].

In our recent work [10], we have argued that the conversion process is a two step process. The first process involves the deconfinement of nuclear to two-flavour quark matter and the conversion process takes some milliseconds to occur. GR effects in such processes is studied by us [11]. In this article, we make a detailed study of this process.

## II. THEORY

We use the Nonlinear Walecka model for the nuclear matter equation of state (EOS). We consider the conversion of nuclear matter, consisting of only nucleons (*i.e.* without hyperons) to a two-flavour quark matter. The final composition of the quark matter is determined from the nuclear matter EOS by enforcing the baryon number conservation during the conversion process.

The metric describing the structure of the star, is given by [12]

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{2\alpha} (dr^2 + r^2 d\theta^2) + e^{\gamma-\rho} r^2 \sin^2\theta (d\phi - \omega dt)^2 \quad (1)$$

The four gravitational potentials  $\alpha, \gamma, \rho$  and  $\omega$  are functions of  $\theta$  and  $r$  only. The solution of the star is obtained from the 'rns' code [13].

We heuristically assume the existence of a combustive phase transition front of infinitesimal thickness, and study the outward propagation of the front. Let us assume that the conversion front is generated at the center of the star,

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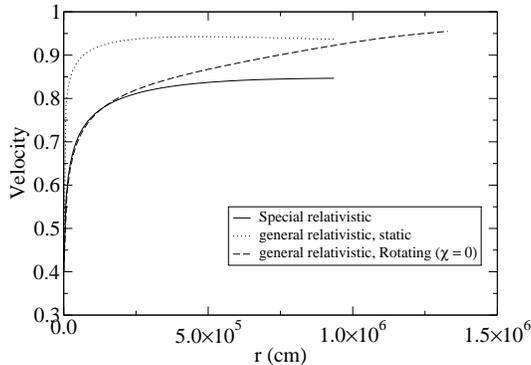


FIG. 1: Variation of velocity of the front along the radial direction of the star for three different cases, namely SR, non-rotating GR and rotating GR.

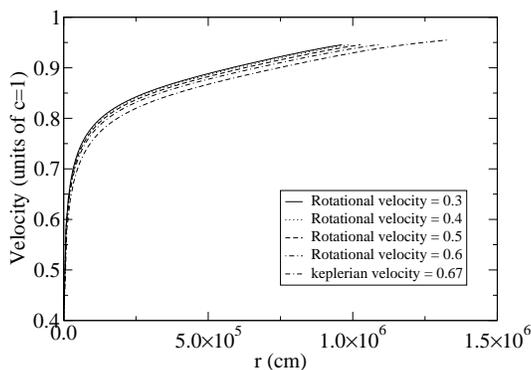


FIG. 2: Variation of velocity of the front along the radial direction for different rotational velocity of the star.

and it propagates outwards through the star with a certain velocity. Employing the conservation conditions [14] and further employing the entropy condition [15], we determine the flow velocity of matter in the two phases  $v_1$  and  $v_2$ .

It is possible to classify the various conversion mechanisms by comparing the velocities of the respective phases with the corresponding velocities of sound in these phases. For the conversion to be physically possible, velocities should satisfy an additional condition, namely,  $0 \leq v_i^2 \leq 1$ . If we plot different velocities against baryon number, we get curves from which we could calculate the initial velocity of the front at the centre of the star [10]. The results [10] also show that the range of values of baryon density, for which the flow velocities are physical, increases with temperature. Starting with this initial velocity, we investigate the evolution of the front with time. Treating both nuclear and strange matters as ideal fluids, the system is governed, together with the metric and the EOS, by the Einstein's equation  $R_i^k - \frac{1}{2}\delta_i^k R = \kappa T_i^k$  and the equation of motion  $T^k_{i;k} = 0$  [16].

The above two equations are the starting point for deriving the appropriate continuity and Euler's equations [11]. After a bit of algebra, we get a single differential equation for  $v$ :

$$\frac{\partial v}{\partial r} = \frac{W^2 v [K + K1]}{2[v^2(1 + G)^2 - n(1 + v^2 G)^2]}. \quad (2)$$

$\omega = 0$  and  $\sin\theta = 1$  in this equation yield the equation for the static star and if we put all potentials equal to zero, we recover our equation for the SR case [10].

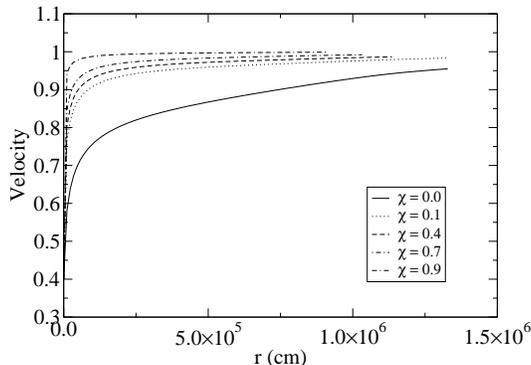


FIG. 3: Variation of velocity of the front along the radial direction for different  $\chi$ .

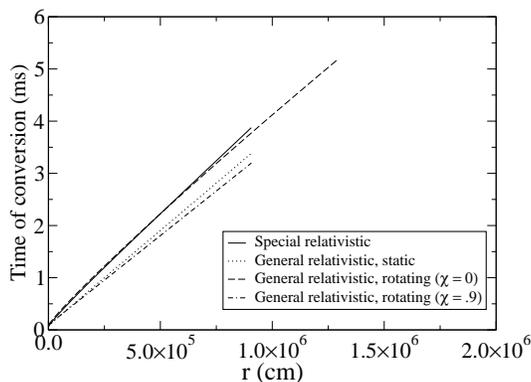


FIG. 4: Variation of time of arrival of the front at certain radial distance for different cases.

### III. RESULTS AND DISCUSSION

Having constructed the density profile of the star for a fixed central density, the respective flow velocities  $v_1$  and  $v_2$  of the nuclear and quark matter in the rest frame of the front, at a radius infinitesimally close to the center of the star. This would give us the initial velocity of the front ( $-v_1$ ), at that radius, in the nuclear matter rest frame. With this, we integrate eqn. (2) outwards along the radius of the star. The solution gives the variation of the velocity with position as a function of the time of arrival of the front. Using this velocity profile, we can calculate the time required to convert the whole star using the relation  $\frac{dr}{dt} = vG$ .

For a rotating star, due to the asymmetry, we introduce a new parameter  $\chi = \cos\theta$ , along the vertical axis of the star. We start our calculation by choosing the central density of the star to be 7 times the nuclear matter density, for which the Keplerian velocity of the star is  $0.67 \times 10^{-4} \text{sec}^{-1}$  (the rotational velocities given in fig. (2) are all in units of  $10^{-4} \text{sec}^{-1}$ ). For this central density, the initial velocity of the front comes out to be 0.45. In fig. (1) the propagation velocity of the front along the radial direction of the star for three cases. The unbroken curve is for the SR case, the broken curve for non-rotating GR case and the dotted curve for the rotating GR case with  $\chi = 0$ , *i.e.* at the equator. Due to the asymptotic behaviour the velocity shoots up at the centre and saturates at larger radii. It can also be clearly seen that the GR effect increases the velocity of the front considerably (maximum by 30%) and the effect is most pronounced for the static case. The rotational effect of the star seems to suppress the GR effect and therefore the velocity of the front decreases. The result becomes clearer if we look at fig. (2) where we have plotted the front velocity with equatorial radius for different rotational velocities; as the rotational velocity increases, the velocity of the front decreases.

From fig. (3), we find that the front velocity is maximum along the polar direction and minimum along the equator.

Therefore, at any particular instant of time, we may have a situation where the polar part of the star has been converted while along the equatorial direction, the front is still propagating.

From fig. (4) we can see that the time taken by the conversion front to convert the neutron star to two-flavour quark star is of the order of few  $ms$ . The static star takes the minimum time ( $3.3ms$ ) whereas the rotating star takes the maximum time ( $5.1ms$ ) due to the enlarged equatorial radius. The polar part of the star needs much lesser time for conversion ( $3.1ms$ ), even less than static star, as its radius gets compressed.

To summarize, we have shown in this article that the conversion of nuclear matter to quark matter in compact stars, especially rotating stars which are more realistic than static stars, is strongly affected by GR effects. The emergence of different conversion fronts, propagating with different velocities along different radial directions was not anticipated by Newtonian or SR calculations. It remains to be explored whether the incorporation of dissipative effects materially changes the results. Though the calculation is much involved, such an investigation is on our immediate agenda.

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