The conversion of Neutron star to Strange star : A two step process

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Abstract

The conversion of neutron matter to strange matter in a neutron star have been studied as a two step process. In the first step, the nuclear matter gets converted to two flavour quark matter. The conversion of two flavour to three flavour strange matter takes place in the second step. The first process is analysed with the help of equations of state and hydrodynamical equations, whereas, in the second process, non-leptonic weak interaction plays the main role. Velocities and the time of travel through the star of these two conversion fronts have been analysed and compared.

I. Introduction

It has been conjectured that strange quark matter, consisting of almost equal numbers of u, d and s quarks, may be the true ground state of strongly interacting matter [1, 2] at high density and/or temperature. This conjecture is supported by bag model calculations [3] for certain range of values for the strange quark mass and the strong coupling constant. By considering realistic values for the strange quark mass (150 - 200 MeV [4]), it may be shown that the strangeness fraction in a chemically equilibrated quark matter is close to unity for large baryon densities. Such bulk quark matter would be referred to as "strange quark matter (SQM)" in what follows.

The above hypothesis may lead to important consequences both for laboratory experiments as well as for astrophysical observations. Normal nuclear matter at high enough density and/or temperature, would be unstable against conversion to two flavour quark matter. The two flavour quark matter would be metastable and would eventually decay to SQM, releasing a finite amount of energy in the process. Such conversion may take place in the interior of a neutron star where the densities can be as high as (8-10)ρ_0 with ρ_0 being the nuclear matter density at saturation [5, 6]. If Witten's conjecture [1] is correct, the whole neutron star may convert to a strange star with a significant fraction of strange quarks in it. (Neutron star may also become a hybrid star with a core of SQM in case the entire star is not converted to a strange star. Such a hybrid star would have a mixed phase region consisting of both the quark matter and the hadronic matter [7].) Hadron to quark phase transition inside a compact star may also yield observable signatures in the form of Quasi-Periodic Oscillations (QPO) and the Gamma ray bursts [8, 9].

There are several ways in which the conversion may be triggered at the centre of the star. A few possible mechanisms for the production of SQM in a neutron star have been discussed by Alcock *et al* [10]. The conversion from hadron matter to quark matter is expected to start as the star comes in contact with a seed of external strange quark nugget. Such a seed would then grow by 'eating up' baryons in the hadronic matter during its travel to the centre of the star, thus converting the neutron star either to a strange star or a hybrid star. Another mechanism for the initiation of the conversion process was given by Glendenning [7] . It was suggested there that a sudden spin down of the star may increase the density at its core, thereby triggering the conversion process spontaneously.

Conversion of neutron matter to strange matter has been studied by several authors. Olinto [11] viewed the conversion process to proceed via weak interactions as a propagating slow-combustion (i.e. a deflagration) front and derived the velocity of such a front. Olsen and Madsen [12] and Heiselberg *et al* [13] estimated the speed of such conversion front to range between 10 m/s to 100 km/s. The combustive conversion front was assumed to have a microscopic width of a few tens to a few hundreds of fm in these calculations.

Collins and Perry [14], on the other hand, assumed that the hadronic matter gets converted first to a two flavour quark matter that eventually decays to a three flavour strange matter through weak interactions. Lugones *et al* [15] argued that the hadron to SQM conversion process may rather proceed as a detonation than as a deflagration even in the case of strangeness production occuring through seeding mechanisms [10].

Horvarth and Benvenuto [16] examined the hydrodynamic stability of the combustive conversion in a non-relativistic framework. These authors inferred that a convective instability may increase the velocity of the deflagration front, so that a transition from slow combustion to detonation may occur. They argued that such a detonation may as well be responsible for the type II supernova explosions[17]. In a relativistic framework, Cho *et al* [18] examined the conservation of the energy-momentum and the baryonic density flux across the conversion front. Using Bethe-Johnson [19] and Fermi-Dirac neutron gas [20] equations of state (EOS) for the nuclear matter (NM) and the Bag model for SQM, they found that the conversion process was never a detonation but a slow combustion only for some special cases. Recently, Tokareva *et al* [21] modelled the hadron to SQM conversion process as a single step process. They argued that the mode of conversion would vary with temperature of SQM and with the value of bag constant in the Bag model EOS. Berezhiani *et al* [22], Bombaci *et al* [23] and Drago *et al* [24], on the other hand, suggest that the formation of SQM may be delayed if the deconfinement process takes place through a first order transition [25] so that the purely hadronic star can spend some time as a metastable object.

In this paper, we model the conversion of nuclear matter to SQM in a neutron star as occuring through a two step process. Deconfinement of nuclear matter to a two (up and down) flavour quark matter takes place in the first step in strong interaction time scale. The second step concerns with the generation of strange quarks from the excess of down quarks via a weak process. We may add here that this is the first instance where a realistic nuclear matter EOS is used to study the nuclear matter to SQM conversion as two step process. Drago *et al* [24], on the other hand, studied the burning of nuclear matter directly to SQM in detail by using the conservation conditions and the compact star models.

To study the conversion of nuclear matter to a two-flavour quark matter, we here consider relativistic EOSs describing the forms of the matter in respective phases. Along with such EOSs, we would also consider hydrodynamical equations depicting various conservation conditions to examine such conversion process in a compact neutron star. Development of the conversion front, as it propagates radially through the model star, would be examined. We would next study the conversion of two-flavour quark matter to a three-flavour SQM through a non-leptonic weak interaction process by assuming β equilibrium for the SQM. The paper is organised as follows. In section II, we discuss the EOSs used for the present work. In section III, we discuss the conversion to two-flavour quark matter. Conversion to three flavour SQM is discussed in section IV. In section V, we summarise the results. Conclusions that may be drawn from these results regarding the actual conversion process that may take place in a neutron star are also presented in this final section.

II. The equation of state

The nuclear matter EOS has been evaluated using the nonlinear Walecka model [26]. The Lagrangian density in this model is given by:

$$\mathcal{L}(x) = \sum_i \bar{\psi}_i(i\gamma^\mu\partial_\mu - m_i + g_{\sigma i}\sigma + g_{\omega i}\omega_\mu\gamma^\mu - g_{\rho i}\rho^a_\mu\gamma^\mu T_a)\psi_i - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu}$$

$$+\frac{1}{2}m^2_\omega\omega_\mu\omega^\mu + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m^2_\sigma\sigma^2) - \frac{1}{4}\rho^a_{\mu\nu}\rho^{\mu\nu}_a + \frac{1}{2}m^2_\rho\rho^a_\mu\rho^\mu_a$$

$$-\frac{1}{3}bm_n(g_{\sigma N}\sigma)^3 - \frac{1}{4}C(g_{\sigma N}\sigma)^4 + \bar{\psi}_e(i\gamma^\mu\partial_\mu - m_e)\psi_e \tag{1}$$

The Lagrangian in equation (1) includes nucleons (neutrons and protons), electrons, isoscalar scalar, isoscalar vector and isovector vector mesons denoted by ψ_i, ψ_e, σ, ω^μ and $\rho^{a,\mu}$, respectively. The Lagrangian also includes cubic and quartic self interaction terms of the σ field. The parameters of the nonlinear Walecka model are meson-baryon coupling constants, meson masses and the coefficient of the cubic and quartic self interaction of the σ mesons (b and c, respectively). The meson field interacts with the baryons through linear coupling. The ω and ρ meson masses have been chosen to be their physical masses. The rest of the parameters, namely, nucleon-meson coupling constant ($\frac{g_\sigma}{m_\sigma}$, $\frac{g_\omega}{m_\omega}$ and $\frac{g_\rho}{m_\rho}$) and the coefficient of cubic and quartic terms of the σ meson self interaction (b and c, respectively) are determined by fitting the nuclear matter saturation properties, namely, the binding energy/nucleon (-16 MeV), baryon density (ρ_0=0.17 fm^{-3}), symmetry energy coefficient (32.5 MeV), Landau mass (0.83 m_n) and nuclear matter incompressibility (300 MeV).

In the present paper, we first consider the conversion of nuclear matter, consisting of only nucleons (*i.e.* without hyperons) to a two flavour quark matter. The final composition of the quark matter is determined from the nuclear matter EOS by enforcing the baryon number conservation during the conversion process. That is, for every neutron two down and one up quarks and for every proton two up and one down quarks are produced, electron number being same in the two phases. While describing the state of matter for the quark phase we consider a range of values for the bag constant. Nuclear matter EOS is calculated at zero temperature, whereas, the two flavour quark matter EOS is obtained both at zero temperature as well as at finite temperatures.

III. Conversion to two flavour matter

In this section we discuss the conversion of neutron proton (n-p) matter to two flavour quark matter, consisting of u and d quarks along with electrons for the sake of ensuring charge neutrality. We heuristically assume the existence of a combustion front. Using the macroscopic conservation conditions, we examine the range of densities for which such a combustion front exists. We next study the outward propagation of this front through the model star by using the hydrodynamic (*i.e.* Euler) equation of motion and the equation of continuity for the energy density flux [27]. In this study, we consider a non-rotating, spherically symmetric neutron star. The geometry of the problem effectively reduces to a one dimensional geometry for which radial distance from the centre of the model star is the only independent spatial variable of interest.

Let us now consider the physical situation where a combustion front has been generated in the core of the Neutron star. This front propagates outwards through the neutron star with a certain hydrodynamic velocity, leaving behind a u-d-e matter. In the following, we denote all the physical quantities in the hadronic sector by subscript 1 and those in the quark sector by subscript 2.

Condition for the existence of a combustion front is given by [28]

$$\epsilon_2(p, X) < \epsilon_1(p, X), \tag{2}$$

where p is the pressure and $X = (\epsilon + p)/{n_B}^2$, n_B being the baryon density. Quantities on opposite sides of the front are related through the energy density, the momentum density and the baryon number density flux conservation. In the rest frame of the combustion front, these conservation conditions can be written as [21, 27, 29]:

$$\omega_1 v_1^2 \gamma_1^2 + p_1 = \omega_2 v_2^2 \gamma_2^2 + p_2, \tag{3}$$

$$\omega_1 v_1 \gamma_1^2 = \omega_2 v_2 \gamma_2^2, \tag{4}$$

and

$$n_1 v_1 \gamma_1 = n_2 v_2 \gamma_2. \tag{5}$$

In the above three conditions v_i (i=1,2) is the velocity, p_i is the pressure, $\gamma_i = \frac{1}{\sqrt{1-v_i^2}}$ is the Lorentz factor, $\omega_i = \epsilon_i + p_i$ is the specific enthalpy and ϵ_i is the energy density of the respective phases.

Besides the conservation conditions given in (2-5), the condition of entropy increase across the front puts an additional constraint on the possibility of the existence of the combustion front. This entropy condition is given by [30],

$$s_1 v_1 \gamma_1 \leq s_2 v_2 \gamma_2 \tag{6}$$

with s_i being the entropy density.

The velocities of the matter in the two phases, given by equations (3-5), are written as [27]:

$$v_1^2 = \frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)}, \tag{7}$$

and

$$v_2^2 = \frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}. \tag{8}$$

It is possible to classify the various conversion mechanisms by comparing the velocities of the respective phases with the corresponding velocities of sound, denoted by c_{si}, in these phases. Thes conditions are [31],

$$\text{strong detonation}: \; v_1 > c_{s1}, \qquad v_2 < c_{s2},$$

$$\text{Jouget detonation}: \; v_1 > c_{s1}, \qquad v_2 = c_{s2},$$

$$\text{supersonic or weak detonation}: \; v_1 > c_{s1}, \qquad v_2 > c_{s2},$$

$$\text{strong deflagration}: \; v_1 < c_{s1}, \qquad v_2 > c_{s2},$$

$$\text{Jouget deflagration}: \; v_1 < c_{s1}, \qquad v_2 = c_{s2},$$

$$\text{weak deflagration}: \; v_1 < c_{s1}, \qquad v_2 < c_{s2}.$$

For the conversion to be physically possible, velocities should satisfy an additional condition, namely, $0 \leq v_i^2 \leq 1$. We here find that the velocity condition, along with the eq.(2), puts severe constraint on the allowed equations of state.

To examine the nature of the hydrodynamical front, arising from the neutron to two flavour quark matter conversion, we plot, in fig.1, the quantities v_1, v_2, c_{s1} and c_{s2} as functions of the baryon number density (n_B). As mentioned earlier, the u and d quark content in the quark phase is kept same as the one corresponding to the quark content of the nucleons in the hadronic phase. With these fixed densities of u and d quarks and electrons, the EOS of the two flavour matter has been evaluated using the bag model prescription. We find that the energy condition (eqn.(2)) and velocity condition (${v_i}^2 > 0$) both are satisfied only for a small window of $\approx \pm 5.0 MeV$ around the bag pressure $B^{1/4} = 160 MeV$. The constraint imposed by the above conditions results in the possibility of deflagration, detonation or supersonic front as shown in the figs.(1-3).

In fig.1, we considered both the phases to be at zero temperature. A possibility, however, exist that a part of the internal energy is converted to heat energy, thereby increasing the temperature of the two flavour quark matter during the exothermic combustive conversion process. Instead of following the prescription for the estimation of temperature as given in references [17, 24], we study the changes in the properties of combustion with the temperature of the newly formed two flavour quark phase in the present paper. In figures 2 and 3, we plot the variation of velocities with density at two different temperatures, namely, $T = 50 MeV$ and $T = 100 MeV$, respectively. These figures show that the range of values of baryon density, for which the flow velocities are physical, increases with temperature. Figure 4 shows the variation of velocities with temperature for values of baryon number densities given by $n_B \approx 3\rho_0$ and $7\rho_0$, respectively. In this figure, the difference between velocities v_1 and v_2 increases with temperature of the two flavour quark matter. In the present paper we have considered only the zero temperature nuclear matter EOS. On the other hand, equation of state of quark matter has a finite temperature dependence and hence the difference between v_1 and v_2, varies with temperature.

The preceding discussion is mainly a feasibility study for the possible generation of the combustive phase transition front and its mode of propagation. Having explored such possibilities, we now study the evolution of the hydrodynamical combustion front with position as well as time. This might give us some insight regarding the actual conversion of a hadronic star to a quark star and the time scale involved in such a process. To examine such an evolution, we move to a reference frame in which the nuclear matter is at rest. The speed of the combustion front in such a frame is given by $v_f = -v_1$ with v_1 being the velocity of the nuclear matter in the rest frame of the front.

In the present work, we use the special relativistic formalism to study the evolution of combustion front as it moves outward in the radial direction inside the model neutron star. The relevant equations are the equation of continuity and the Euler's equation, that are given by [21]:

$$\frac{1}{\omega}(\frac{\partial \epsilon}{\partial \tau} + v\frac{\partial \epsilon}{\partial r}) + \frac{1}{W^2}(\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial \tau}) + 2\frac{v}{r} = 0 \tag{9}$$

and

$$\frac{1}{\omega}(\frac{\partial p}{\partial r} + v\frac{\partial p}{\partial \tau}) + \frac{1}{W^2}(\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial r}) = 0, \tag{10}$$

where, $v = \frac{\partial r}{\partial \tau}$ is the front velocity in the nuclear matter rest frame and $k = \frac{\partial p}{\partial \epsilon}$ is taken as the square of the effective sound speed in the medium.

Substituting these expressions for v and k in equations (9) and (10) we get

$$\frac{2v}{\omega}\frac{\partial \epsilon}{\partial r} + \frac{1}{W^2}\frac{\partial v}{\partial r}(1+v^2) + \frac{2v}{r} = 0 \tag{11}$$

and

$$\frac{n}{\omega}\frac{\partial \epsilon}{\partial r}(1+v^2) + \frac{2v}{W^2}\frac{\partial v}{\partial r} = 0 \tag{12}$$

Equations (11) and (12) ultimately yield single differential equation that is written as:

$$\frac{dv}{dr} = \frac{2vkW^2(1+v^2)}{r[4v^2 - k(1+v^2)^2]}. \tag{13}$$

The equation (13) is integrated, with respect to r(t), starting from the centre towards the surface of the star. The nuclear and quark matter EOS have been used to construct the static configuration of compact star, for different central densities, by using the standard Tolman-Oppenheimer-Volkoff (TOV) equations [32]. The velocity at the centre of the star should be zero from symmetry considerations. On the other hand, the 1/r dependence of the dv/dr, in eq.(13) suggests a steep rise in velocity near the centre of the star.

Our calculation proceeds as follows. We first construct the density profile of the star for a fixed central density. Equations (7) and (8) then specify the respective flow velocities v_1 and v_2 of the nuclear and quark matter in the rest frame of the front, at a radius infinitesimally close to the centre of the star. This would give us the initial velocity of the front ($-v_1$), at that radius, in the nuclear matter rest frame. We next start with equation (13) from a point infinitesimally close to the centre of the star and integrate it outwards along the radius of the star. The solution gives us the variation of the velocity with the position as a function of time of arrival of the front, along the radius of the star. Using this velocity profile, we can calculate the time required to convert the whole star using the relation $v = dr/d\tau$.

In figure 5, we show the variation of the velocity for values of the central baryon densities $3\rho_0$, $4.5\rho_0$ and $7\rho_0$, respectively. The respective initial velocities corresponding to such central densities are taken to be 0.43, 0.67 and 0.68. The figure shows that the velocity of the front, for all the central densities, shoots up near the centre and then saturates at a certain velocity for higher radius. Such a behaviour of velocity near the central point is apparent from the equation (13) above. The numerically obtained saturation velocity varies from 0.92 for central baryon density $3\rho_0$ to 0.98 for $7\rho_0$. The existence of a saturation velocity, at large r, is apparent from the asymptotic behaviour of equation (13). A comparison with fig.1 shows that for the densities $3\rho_0$ and $4.5\rho_0$, the conversion starts as weak detonation and stays in the same mode throughout the star. On the other hand, for $7\rho_0$, initial detonation front changes over to weak detonation and the velocity of front becomes almost 1 as it reaches the outer crust. The corresponding time taken by the combustion front to propagate inside the star is plotted against the radius in figure 6. The time taken by the front to travel the full length of the star is of the order of few milliseconds. According to the present model, the initial neutron star thus becomes a two flavour quark star in about 10^{-3} sec. The results discussed above correspond to the case in which both nuclear as well as quark matter are at zero temperature. For finite temperature quark matter results vary only by a few percent of the front velocities for the quark matter at zero temperature.

We would like to mention here that in the above discussions, the equations governing the conversion of nuclear to quark matter are purely hydrodynamic. There is no dissipative process, so that the combustion front continues to move with a finite velocity depending on the density profile. Furthermore, there is no reaction rate involved here as the deconfinement process occurs in the strong interaction time scale and hence can be taken to be instantaneous. This is certainly very different from the second step process, to be discussed in the next section, where the two flavour matter converts to a three flavour matter. Here, the governing rate equations are weak interaction rates which play a decisive role in the conversion. Comparing the total time ($\equiv 10^{-3}$ sec) taken by the combustion front to travel through the star with the weak interaction time scale

($10^{-7} - 10^{-8}$ sec), it is evident that the second step may start before the end of the first step process. In that case, perhaps, one should ideally considers two fronts, separated by a finite distance, moving inside the star [27]. In the present paper, we have taken a much simplified picture and considered the conversion of a chemically equilibrated two flavour to three flavour quark matter as the second step process. Our results may provide us with more information regarding the necessity of considering two fronts.

IV. The conversion to three flavour SQM

In this section we discuss the conversion of two flavour quark matter to three flavour SQM in a compact star. Similar to the discussion above, we assume the existence of a conversion front at the core of the star that propagates radially outward leaving behind the SQM as the combustion product. This conversion is governed by weak interactions that take place inside the star.

For a three flavour quark matter, the charge neutrality and the baryon number conservation conditions yield

$$3n_B = n_u + n_d + n_s \tag{14}$$

$$2n_u = n_d + n_s + 3n_e \tag{15}$$

where n_i is the number density of particle i (i= u, d, s and e).

The weak reactions which govern the conversion of excess down quark to strange quark can be written as,

$$d \rightarrow u + e^- + \overline{\nu}_{e-}; \quad s \rightarrow u + e^- + \overline{\nu}_{e-}; \quad d + u \rightarrow s + u \tag{16}$$

We assume that the neutrinos escape freely from the site of reaction and the temperature of the star remains constant. The non-leptonic weak interaction in such a case becomes the governing rate equation. The semi-leptonic weak decays, then, are solely responsible for the chemical equilibration which can be incorporated through the relations given below.

$$\mu_{e^-} = \mu_d - \mu_u; \quad \mu_d = \mu_s, \tag{17}$$

where μ_i is the chemical potential of the i'th particle. The number densities (n_i) of the quarks and electrons are related to their respective chemical potentials by,

$$n_i = g_i \int_0^\infty d^3p/(2\pi)^3 [f^+ - f^-], \tag{18}$$

where $f^+$and f^- are given by

$$f^+ = \frac{1}{exp[(E_p - \mu)/T] + 1} \qquad f^- = \frac{1}{exp[(E_p + \mu)/T] + 1}. \tag{19}$$

In equations (18) and (19), g_i is the degeneracy factor and T the temperature. Eqns.(14-19) can be solved self consistently to calculate the number densities of quarks and electrons.

The conversion to SQM starts at the centre ($r = 0$) of the two flavour star. Assuming that the reaction region is much smaller than the size of the star, we have considered the front to be one dimensional. Moreover, as we are considering spherical static stars only, there is no angular dependence. The combustion front, therefore, moves radially towards the surface of the star. As the front moves outwards, excess d quarks get converted to s quark through the non leptonic weak process. The procedure employed in the present work is somehow similar to that of ref.[11], although the physical boundary conditions are different.

We now define a quantity,

$$a(r) = [n_d(r) - n_s(r)]/2n_B \tag{20}$$

such that, $a(r = 0) = a_0$ at the core of the star. The quantity a_0 is the number density of the strange quarks, at the centre, for which the SQM is stable and its value lies between 0 and 1. For equal numbers of d and s quarks, $a(r) = 0$. Ideally at the centre of the star a_0 should be zero for strange quark mass $m_s = 0$. Since s quark has a mass $m_s \sim 150$MeV, at the centre of the star a_0 would be a small finite number, depending on the EOS. The s quark density fraction, however, decreases along with the decrease of the baryon density towards the surface of the star, so that, $a(r \to R) \to 1$ with R being the radius of the star. At any point along the radius, say $r = r_1$, initial $a(r_1)$, before the arrival of the front, is decided by the initial two flavour quark matter EOS. The final $a(r_1)$, after the conversion is obtained from the equilibrium SQM EOS at the density corresponding to r_1.

The conversion to SQM occurs via decay of down quark to strange quark ($u+d \to s+u$) and the diffusion of the strange quark from across the front [11]. The corresponding rate of change of $a(r)$ with time is governed by following two equations:

$$\frac{da}{dt} = -R(a), \tag{21}$$

and

$$\frac{da}{dt} = D\frac{d^2a}{dr^2}, \tag{22}$$

In the equation (21) $R(a)$ is the rate of conversion of d to s quarks. Equation (22) yields the rate of change of $a(r)$ due to diffusion of s quarks, with D being the diffusion constant. Following Olinto [11], assuming the one dimensional steady state solution and using equations (21) and 22 we get:

$$Da^{''} - va^{'} - R(a) = 0, \tag{23}$$

where v is the velocity of the fluid. In equation (23) $a^{'} = \frac{da}{dr}$.

Conservation of baryon number flux at any position yields $n_q v_q = n_s v_s$. The subscripts q and s denote the two flavour quark matter phase and SQM phase, respectively. The baryon flux conservation condition yields the initial boundary condition at any point r along the radius of the star:

$$a'(r) = -\frac{v}{D}(a_i(r) - a_f(r)), \tag{24}$$

where $a_i(r)$ and $a_f(r)$ are the values for the $a(r)$ before and after the combustion, respectively.

The reaction rate for the non-leptonic weak interaction $u + d \rightarrow u + s$ is in general a five dimensional integral for non zero temperature and m_s [33, 34]. Here, instead, we have taken the zero temperature, small a limit [11].

$$R(a) \approx \frac{16}{15\pi} {G_F}^2 cos^2\theta_c sin^2\theta_c {\mu_u}^5 \frac{a}{3}^3 , \tag{25}$$

where, G_F is the weak coupling constant and θ_c is the Cabibbo angle. The above equation can be written in the following form:

$$R(a) \approx \frac{a^3}{\tau}, \tag{26}$$

where $\tau = \frac{16}{(3^3)15\pi} {G_F}^2 cos^2\theta_c sin^2\theta_c {\mu_u}^5$, here, depends on the position of the front.

Following the line of arguments given in ref.[11], we write down the analytic expressions for D and v as:

$$D = \frac{\lambda\overline{v}}{3} \simeq 10^{-3} (\frac{\mu}{T})^2 cm^2/s, \tag{27}$$

and

$$v = \sqrt{\frac{D}{\tau} \frac{{a_f(r)}^4}{2(a_i(r) - a_f(r))}}. \tag{28}$$

Our calculation proceeds as follows. First we get the star characteristics for a fixed central baryon density ρ_c. For a given ρ_c, number densities of u, d and s quarks, in both the two and three flavour sectors, are known at any point inside the star. That means $a_i(r)$ and $a_f(r)$ is fixed. Eqns.(25-28) are then used to get the diffusion constant and hence the radial velocity of the front.

The central baryon densities considered here are same as those of section III. Assuming that the neutrinos leave the star, the temperature is kept constant at some small temperature so that we can use the equation (25), evaluated in the zero temperature limit. The variation of $a(r)$ with the radius of the star is given in figure 7. The plot shows that $a(r)$ increases radially outward, which corresponds to the fact that as density decreases radially, the number of excess down quark which is being converted to strange quark by weak interaction also decreases. Hence, it takes less time to reach a stable configuration and hence the front moves faster, as shown in the figs. 8 and 9.

In Fig.8, we have plotted the variation of velocity along the radius of the star. The velocity shows an increase as it reaches sufficiently low density and then drops to zero near the surface as $d \rightarrow s$ conversion rate becomes zero. Fig.9 shows the variation of time taken to reach a stable configuration at different radial position of the star. The total time needed for the conversion of the star, for different central densities, is of the order of 100 seconds, as can be seen from the figure.

V. Summary and discussion

We have studied the conversion of a neutron star to strange star. This conversion takes place in two stages. In the first stage a detonation wave is developed in the hadronic matter (containing neutrons, protons and electrons). We have described this hadronic matter with a relativistic model. For such an equation of state the density profile of the star is obtained by solving Tolman-Openheimer-Volkoff equations. The corresponding quark matter equation of state is obtained by using the bag model. However, this quark matter equation of state is not equilibrated and contains two flavours. Matter velocities in the two media, as measured in the rest frame of the front, have been obtained using conservation conditions. These velocities have been compared with the sound velocity in both phases.

For a particular density inside the star, flow velocities of the matter on the two sides of the front is now fixed. Starting from a point, infinitesimally close to the centre, hydrodynamic equations are solved radially outward. The solution of the hydrodynamic equations gives the velocity profiles for different central densities. The velocity of the front shoots up very near to the core and then saturates at a value close to 1. The mode of combustion is found to be weak detonation for lower central densities. For higher central densities, the initial detonation becomes weak detonation as the the front moves radially outward inside the star. This result is different from that of ref.[24], where the conversion process always correspond to a strong deflagration. The time required for the conversion of the neutron star to a two flavour quark star is found to be of the order of few milliseconds. After this front passes through, leaving behind a two flavour matter a second front is generated. This second front converts the two flavour matter via weak interaction processes. The velocity of the front varies along the radius of the star. As the front moves out from the core to the crust, its velocity increases, implying faster conversion. The time for the second conversion to take place comes out to be ~ 100 seconds. This is comparable to the time scale obtained in ref.[11].

The comparison of the time of conversion from neutron star to two flavour quark star and the weak interaction time scale suggests that at some time during the passage of the first combustion front, the burning of two flavour matter to strange matter should start. This means that at some point of time, there should be two fronts moving inside the star. On the other hand, our results show that, inside the model star, the burning of the nuclear matter to two flavour quark matter takes much smaller time compared to the conversion from two flavour quark matter to SQM. However, the consideration of two fronts might provide us with some more information regarding the conversion of neutron star to a final stable strange star. In the present case we have considered a two step process, there being only one type of front, inside the star, at any instant of time.

Here we would also like to mention that ideally the second step should start with a non-equilibrated two flavour matter [35, 33]. Since this is a numerically involved calculation, in the present case we have taken the simplified picture of equilibrated quark matter.

Finally, the burning of the nuclear matter to two flavour quark matter is studies using special relativistic hydrodynamic equations. The actual calculation should involve general relativity, taking into account the curvature of the front for the spherical star. We propose to explore all these detailed features in our subsequent papers.

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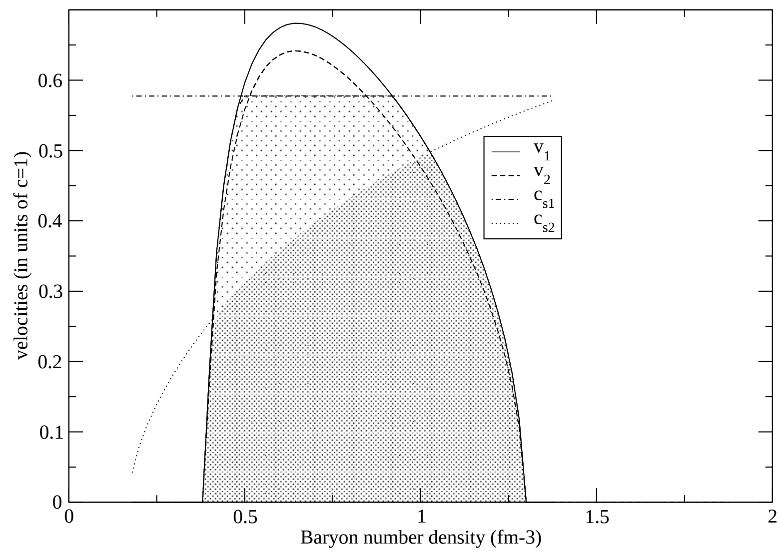

Figure 1: Variation of different velocities with baryon number density for T = 0 MeV. The dark-shaded region correspond to deflagration, light-shaded region correspond to detonation and the unshaded region correspond to supersonic conversion processes.

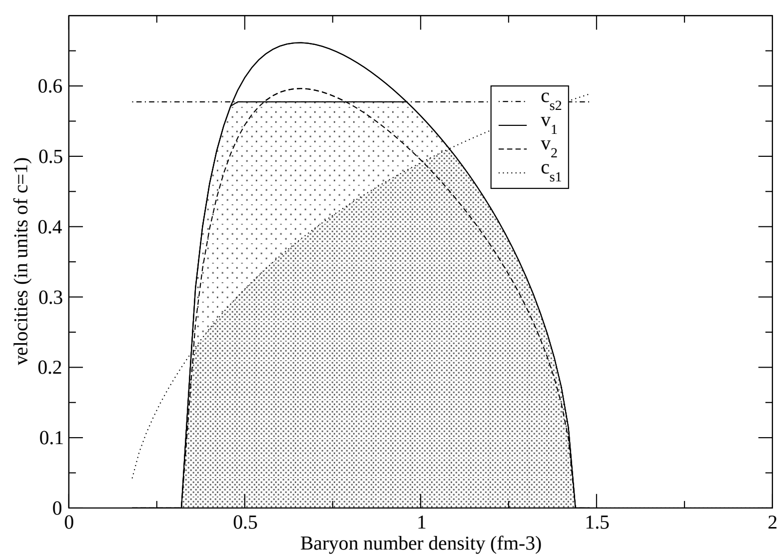

Figure 2: Variation of velocities with baryon number density for T = 50 MeV. Different regions correspond to different modes of conversion, where the notations are the same as in fig 1.

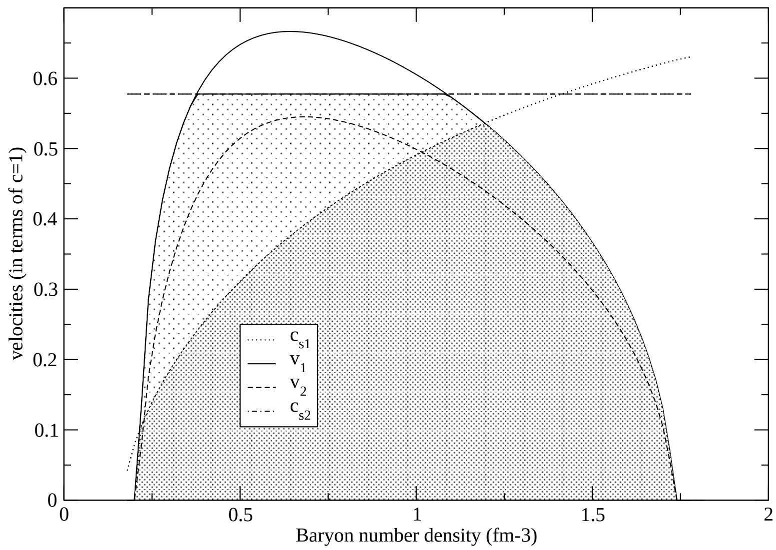

Figure 3: Variation of velocities with baryon number density for T = 100 MeV. Different regions correspond to different modes of conversion, where the notations are the same as in fig 1.

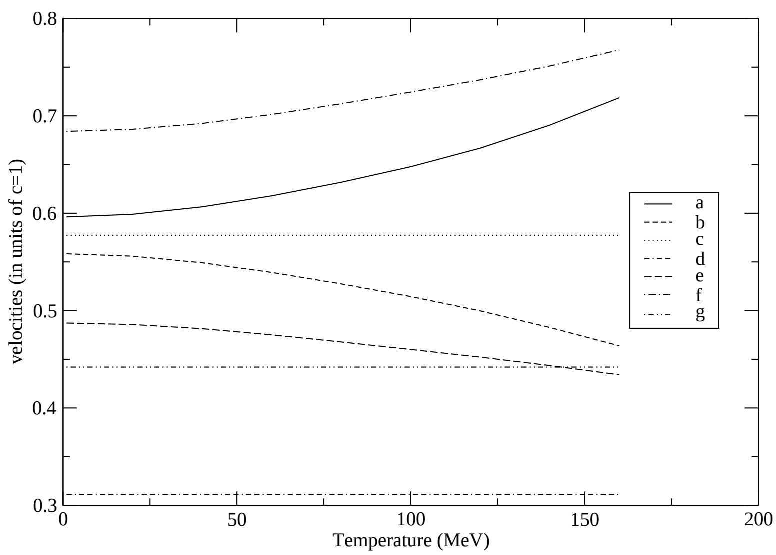

Figure 4: Variation of velocities with temperature: (a)v_1 for density $3\rho_0$, (b) v_2 for density $3\rho_0$, (c) c_{s1}, (d) c_{s2} for $3\rho_0$, (e) v_2 for $7\rho_0$, (f) v_1 for $7\rho_0$ and (g) c_{s2} for $7\rho_0$.

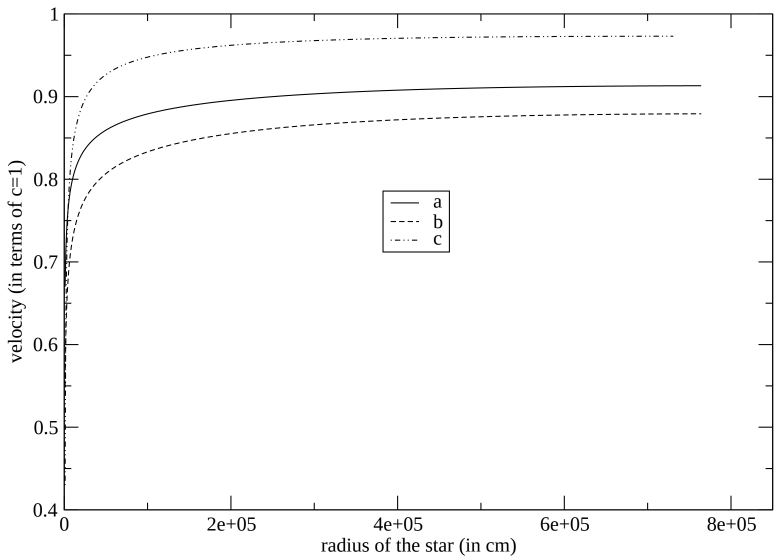

Figure 5: Variation of velocity of the conversion front with radius of the star, for three different values of the central densities, namely, (a) $3\rho_0$, (b) $4.5\rho_0$ and (c) $7\rho_0$, respectively. Here ρ_0 is the nuclear density. The initial velocity for the three cases are 0.66, 0.65 and 0.47, respectively.

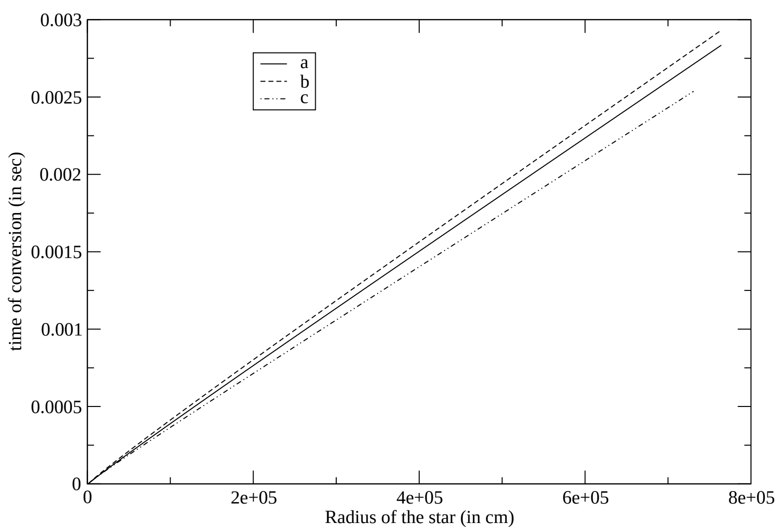

Figure 6: Variation of the time of arrival of the conversion front at a certain radial distance inside the star as a function of that radial distance from the centre of the star for three different central densities. Values of the central density corresponding to the curves (a), (b) and (c) are the same as shown in fig 5.

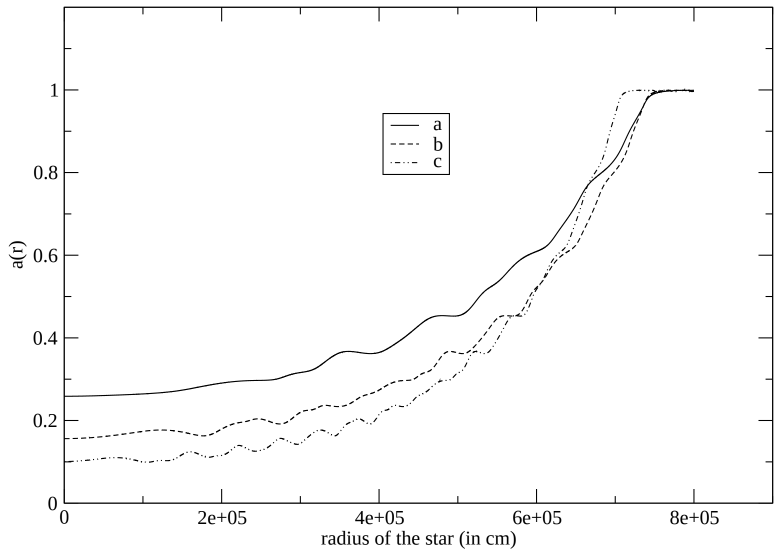

Figure 7: Variation of a(r) (as in the text) with radius of star, for different central densities, where, (a) corresponds to case for which the central density is $3\rho_0$, (b) for $4.5\rho_0$ and (c) for $7\rho_0$.

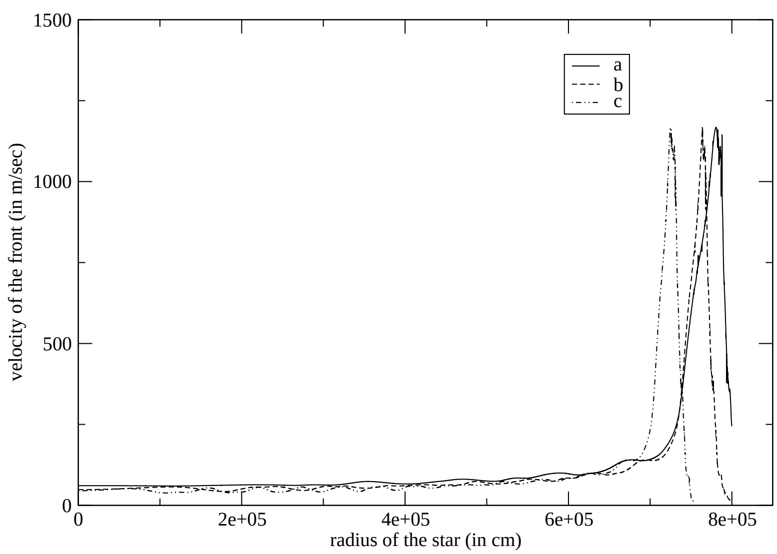

Figure 8: Variation of velocity of the two to three flavour quark conversion front with radius of the star for different central densities, where, (a) corresponds to case for which the central density is $3\rho_0$, (b) for $4.5\rho_0$ and (c) for $7\rho_0$.

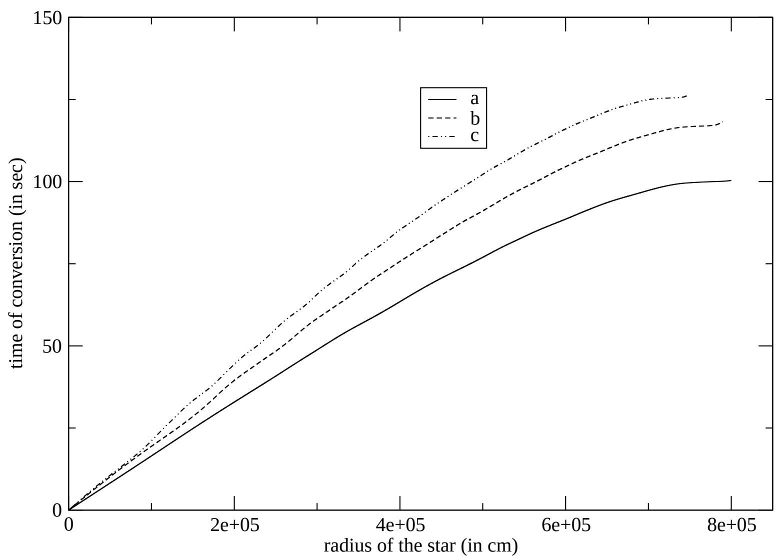

Figure 9: Variation of time taken for the two to three flavour quark conversion front with radius of the star, for different central densities, where, (a) corresponds to case for which the central density is $3\rho_0$, (b) for $4.5\rho_0$ and (c) for $7\rho_0$.