

Can radiative magnification of mixing angles occur for two-zero neutrino mass matrix textures?

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Abstract

Neutrino Majorana masses and mixings can be generated from a dimension-5 operator within the standard model particle content. After a review of the mechanism of radiative enhancement of the mixing angle in a two-neutrino case, we consider three-flavour mass matrices of two-zero texture generated from such an operator and investigate the possibility of implementing the mechanism here. We observe that radiative magnification of only the solar angle is consistent with oscillation data on masses and mixings, and that too for nearly degenerate neutrinos, with two of them having opposite CP parities, while for hierarchical masses the mechanism does not work. In supersymmetry or in an extra-dimensional scenario the above features are qualitatively unchanged.

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I Introduction

The mixing angles of quarks are experimentally known to be small. It is a natural expectation, mainly boosted by the idea of quark-lepton unification, that lepton mixings will be small too. But, quite contrary to this, recent neutrino experiments indicate [1] that two of the three mixing angles, namely the solar and the atmospheric ones, are large [2, 3], while the third mixing angle (CHOOZ) is small [4]. So, it is of interest to look for mechanisms which can naturally explain the large mixing angles in the neutrino sector without conflicting with the spirit of grand unification. Renormalization Group (RG) evolution of neutrino masses and mixing angles offers one such mechanism ([5]–[12]).

To implement this mechanism, the first step is to make an assumption about the elements of the neutrino mass matrix at some scale. In general, an arbitrary three flavour neutrino mass matrix involves nine model-independent complex parameters. As emphasized in [13], from the results of the ongoing and foreseen experiments it is not possible to fully determine this mass matrix. So, as the authors of [13] argued, scenarios with structural simplicity need to be conjectured in which some elements of the mass matrix in the flavour basis are identically zero. The number of such zeros depends on the symmetry of these so called ‘texture zero’ mass matrices. They observed that a mass matrix with more than two zeros is incompatible with data, and seven out of the fifteen possible two-zero texture mass matrices are consistent with experiments. The authors of [14] have carried this investigation one step further by showing that only three of the above seven mass matrices – the ones which predict hierarchical neutrino masses (normal or inverted) – survive if one takes the atmospheric mixing angle to be *exactly* maximal ($\pi/4$), while those structures which yield quasi-degenerate masses are excluded. Calculation of Majorana-type CP violating phases associated with two-zero textures has been presented in [15]. It should also be mentioned that texture zero up and down quark mass matrices have been successfully used in the past to relate the ratios of quark masses to their mixing angles [16].

In view of this recent interest in texture two-zero structures, and more so for their predictive properties, in this paper we investigate whether radiative magnification of neutrino mixing angles can occur in such schemes. We will consider both quasi-degenerate and hierarchical mass matrices at a high scale and examine whether a significant running of one or more angles is consistent with data on masses and mixings. This way we complement the analyses in [13] and [14] to seek whether such textures can be embedded in a unification framework.

Let us briefly summarize what is already known. It has been demonstrated in [5, 6] that starting from a tiny mixing angle between *two* active neutrinos at some high energy scale, one can achieve large mixing at low energy through RG evolution. The analyses in [7, 8] are of very general nature and they contain explicit expressions for RG evolution of masses and mixing angles. A detailed discussion of how to promote the analysis from two to three generations with a special reference to the existence of fixed points of mixing angles at low energy is contained there. The existence of these fixed points has been shown by the authors of [9] to lead to a stable atmospheric mixing angle close to the maximum value. In [10], it has been shown that starting from small mixing angles at a high scale, radiative correction can generate large atmospheric mixing at low scale while keeping the other two angles small, thus leading to a small angle MSW solution. The authors of [11] have shown that, in a see-saw model, starting from a bimaximal mixing at the GUT scale, the solar angle is driven by RG evolution to a smaller value at a low scale while the other two angles do not move appreciably. In [12], it has been shown that starting from a very small solar mixing at the GUT scale, the LMA solution

can be reached by RG running. It should be noted that many of the above analyses discuss SM and supersymmetry together in the same breath as, except for some numerical alteration, the general parametrization remains the same for both scenarios.

The purpose of our analysis is two-fold. First, much new data have accumulated, particularly from SNO. Bimaximal mixing is now disfavoured, as is the SMA solution for solar neutrinos. Though the best-fit atmospheric mixing has remained at the maximum value over the years, solar mixing is now expected to be large but not maximal. Thus the conditions that have to be met through radiative enhancement have changed. Second, we choose a restricted framework, namely, the two-zero mass matrix textures. With the aim of implementing the radiative enhancement mechanism starting from small mixing at a high scale, we perform a case-by-case consistency check with experimental data for a variety of two-zero conjectures. We find that only those textures which result in a quasi-degenerate neutrino mass spectrum support a large RG enhancement of the solar mixing angle. We also briefly remark about scenarios beyond the SM, namely, supersymmetry and extra dimensions.

To generate neutrino masses and mixings, we set M_X as the scale at which lepton number is broken. The Majorana masses of the left-handed neutrinos result from the following dimension-5 operator which involves only the SM fields:

$$L^{SM} = \frac{\kappa_{ij}}{M_X} \bar{\ell}_i^c \ell_j H H + \text{h.c.} \quad (1)$$

where ℓ and H are the left-handed lepton and Higgs doublets of the SM, respectively. Here i, j are flavour indices. $SU(2)$ indices have been suppressed in (1). It results in a neutrino mass matrix $M_{ij} \sim \kappa_{ij}(v^2/M_X)$, where v is the vacuum expectation value of the SM Higgs boson.

In section II, we write down the RG equation of κ_{ij} and review the two flavour formalism. In section III, we consider the radiative corrections to three flavour two-zero mass matrix textures classified in three different subsections based on the hierarchy of mass patterns. We identify those where the radiative enhancement mechanism can be implemented. In section IV, we carry the discussion to supersymmetric and extra-dimensional scenarios. In section V, we draw our conclusions.

II Two flavour case

Let us consider the following mixing matrix

$$U = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

such that $M = U M_d U^T$, where $M_d = \text{diag}(m_1, m_2)$. The neutrino mixing angle is then given by

$$\tan 2\theta = \frac{2\kappa_{12}}{\kappa_{22} - \kappa_{11}} = -\frac{2(\kappa_{12}/\kappa_{22})}{d\kappa}, \quad (2)$$

where we have defined

$$d\kappa \equiv \frac{\kappa_{11} - \kappa_{22}}{\kappa_{22}}.$$

The evolution of the κ matrix is governed by the equation [5, 6]¹

$$16\pi^2 \frac{d\kappa}{d\ln\mu} = \{-3g_2^2 + 2\lambda + 2S\}\kappa - \frac{3}{2}\{\kappa(Y_l^\dagger Y_l) + (Y_l^\dagger Y_l)^T \kappa\}, \quad (3)$$

where μ is the energy scale, g_2 is the SU(2) coupling constant, Y_l is the Yukawa coupling matrix of charged leptons², and

$$S = \text{Tr} [3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_l^\dagger Y_l].$$

Above, $Y_{u,d}$ are the Yukawa coupling matrices of the up- and down-type quarks, respectively. The RG equations for the SM couplings are well known [18] and are not presented. Here, the gauge and Yukawa couplings at the electroweak scale are chosen as input parameters. It is necessary to exercise some caution in the choice of the Higgs mass to fix the quartic scalar coupling λ . Although the mixing angle evolution is quite insensitive to this choice, unless $m_H \gtrsim 150$ GeV, the RG evolution drives λ to negative values making the scalar potential unbounded from below.

The first term in braces on the r.h.s. of (3), which we denote by $D \equiv -3g_2^2 + 2\lambda + 2S$, treats all elements of the κ matrix identically – a universal contribution – while the term in the second braces involving the leptonic Yukawa matrices distinguishes between different elements. Further, the equation is linear in κ so that the net effect of the RG evolution is a multiplicative change for each element. More specifically,

$$\begin{aligned} 16\pi^2 \frac{d\kappa_{11}}{d\ln\mu} &= D\kappa_{11} - \frac{3}{2}(2Y_1^2)\kappa_{11}, \\ 16\pi^2 \frac{d\kappa_{12}}{d\ln\mu} &= D\kappa_{12} - \frac{3}{2}(Y_1^2 + Y_2^2)\kappa_{12}, \\ 16\pi^2 \frac{d\kappa_{22}}{d\ln\mu} &= D\kappa_{22} - \frac{3}{2}(2Y_2^2)\kappa_{22}. \end{aligned} \quad (4)$$

For our subsequent discussions, we take $Y_2 = Y_\tau$ and $Y_1 = Y_e$ or Y_μ . Now, to a good approximation, the effects of RG running can be summarized by introducing two parameters r and a as:

$$\begin{aligned} \kappa_{11}(M_X) &\rightarrow \kappa_{11}(M_Z) = a\kappa_{11}(M_X), \\ \kappa_{12}(M_X) &\rightarrow \kappa_{12}(M_Z) = a(1 + r/2)\kappa_{12}(M_X), \\ \kappa_{22}(M_X) &\rightarrow \kappa_{22}(M_Z) = a(1 + r)\kappa_{22}(M_X), \end{aligned} \quad (5)$$

where M_X is some high scale and M_Z characterizes the electroweak scale. The universal contribution, $a \sim 0.7$ for $M_X = 10^{18} - 10^{19}$ GeV, and is dominant. The other piece, namely, $r \simeq (3Y_\tau^2/16\pi^2) \ln(M_X/M_Z) \sim Y_\tau^2 \sim 10^{-4}$ is crucial in determining the running of the mixing angle. By considering a simultaneous one loop running of all the necessary couplings, we have numerically checked that the parametrization of (5), which we use only for illustrative purposes, works extremely well to order $r^2 \sim 10^{-8}$. We should note that the value of r is really controlled by the τ -lepton Yukawa coupling, and its order-of-magnitude does not vary appreciably if M_X is altered even by a few orders.

The following points, based on which our subsequent arguments will proceed, are worth noticing:

a) It follows from (5) that the quantity $d\kappa \tan 2\theta = -2\kappa_{12}/\kappa_{22}$ is renormalization scale invariant to a

¹It was pointed out in [17] that the coefficient of the second term on the r.h.s. of Eq. (3) is $-\frac{3}{2}$ in place of $-\frac{1}{2}$, commonly used in the earlier literature.

²We work in a basis in which Y_l is diagonal.

good approximation.

b) It also follows from (5) that

$$d\kappa(M_Z) \simeq d\kappa(M_X) - r.$$

c) Combining a) and b) we obtain

$$\tan 2\theta(M_Z) = \tan 2\theta(M_X)/\Delta, \quad \text{where } \Delta \equiv 1 - r/d\kappa(M_X). \quad (6)$$

Now let us pay attention to the case when a small mixing angle at M_X becomes large near M_Z by a resonant enhancement. Since both the solar and atmospheric neutrino data suggest large neutrino mixing, it is of more interest to consider this class of running than other possibilities. Eq. (6) indicates that a resonant enhancement will be possible if $d\kappa(M_X)$ is chosen to be as close to r as possible. The quantity Δ is a fine-tuning parameter which determines this closeness. Smaller the value of Δ , more fine-tuned is the initial texture; but paying this price we attain an enhancement of the mixing by a factor $1/\Delta$, which is the ratio of the two tangents as shown in the first equality of (6). It also turns out that

$$d\kappa(M_Z) = \Delta \cdot d\kappa(M_X) \simeq \Delta \cdot r \sim 10^{-4} \cdot \Delta, \quad (7)$$

to ensure a significant running ending up in large mixing near the electroweak scale.

Let us now estimate the impact of (7) on the neutrino masses and mass splittings. For the sake of illustration, assuming $m_1 \sim m_2 \gg |m_1 - m_2|$, we can express

$$d\kappa(M_Z) \simeq 0.5 \frac{\Delta m^2}{m^2} \cos 2\theta(M_Z), \quad (8)$$

where $m = 0.5(m_1 + m_2)$ is an average mass, and $\Delta m^2 \equiv |m_1^2 - m_2^2|$, both defined at the low scale. Combining Eqs. (7) and (8), we can write,

$$\Delta m^2 (\text{eV}^2) \lesssim 10^{-3} \left[\frac{\Delta}{\cos 2\theta(M_Z)} \right] \left[\frac{m}{2.2 \text{ eV}} \right]^2, \quad (9)$$

where the inequality arises from the upper limit on the absolute neutrino masses (see discussions later).

The relation (9) is fairly general. It only assumes that ν_τ has to participate in the oscillation, and that there is a significant running of the mixing angle. Quantitatively, the latter signifies that one can substitute $d\kappa(M_X)$ by r in (7). Now we have to make a judgement of what is the maximum value of m that we are allowed to take and how much fine-tuning, parametrized by the quantity Δ , we can tolerate.

The mass matrix that we have generated is of Majorana nature. The neutrinoless double beta decay constraint [19] applies on the (ee) -element of the mass matrix, and considering large mixing at low scale, this means $0.05 < m < 0.84$ eV at 95% C.L. But this result needs further confirmation. Hence, a more conservative constraint is to use $m < 2.2$ eV from the Tritium beta decay experiment [20].

The size of Δ is indeed a tricky issue. It follows from (6) that $\Delta/\cos 2\theta(M_Z) \simeq \tan 2\theta(M_X)$ as $\theta(M_Z)$ gets closer to $\pi/4$. To get an intuitive feeling of the size of this fine-tuning, let us consider a toy scenario in which a small angle $\alpha \equiv \theta(M_X)$ becomes $(\pi/4 - \alpha) = \theta(M_Z)$ by a resonant enhancement. Then $\tan 2\theta(M_X) = \cot 2\theta(M_Z) \simeq \cos 2\theta(M_Z) \leq 0.22$ (post-SNO fit) [21]. In this example, $\Delta \sim$

$\cos^2 2\theta(M_Z) \leq 0.04$. All in all, it is perhaps not unfair to take $\tan 2\theta(M_X) \sim 0.1 - 0.5$, which corresponds to $\theta(M_X) \sim (3^\circ - 13^\circ)$ to cover a rather wide range of what we can call a ‘small’ initial mixing. Plugging this input in (9), we observe that $\Delta m^2 \leq 5 \times 10^{-4} \text{ eV}^2$ is a very safe prediction (assuming $m \leq 2.2 \text{ eV}$).

Now, if we attempt to explain the solar neutrino problem with the above formalism, it is possible to arrange for both the LMA ($\Delta m^2 \sim 10^{-5} \text{ eV}^2$) and LOW ($\Delta m^2 \sim 10^{-7} \text{ eV}^2$) solutions. However, if the neutrinoless double beta decay observation is confirmed, the situation will become rather tight. For atmospheric neutrinos, the preferred value of Δm^2 is $\sim 3 \times 10^{-3} \text{ eV}^2$. It is obvious from (9) that this cannot be explained if we have to respect the Tritium beta decay limit on the absolute neutrino mass.

We conclude this section by noting that instead of involving ν_τ if we consider $\nu_\mu - \nu_e$ oscillation, then, for resonance, $d\kappa$ would have to be $\sim Y_\mu^2$ for which the corresponding Δm^2 is too small to fit the experimental data.

III Three flavour case with two-zero mass matrix textures

Now we turn to the question whether it is possible to accommodate both the solar and atmospheric neutrino solutions in a picture of radiative enhancement of mixing within a three flavour scenario, keeping in mind the constraint from the CHOOZ experiment. We consider a solution acceptable if both the mass splittings as well as the *approximately* bimaximal nature of the mixing matrix are reproduced at low energy, *with the radiative effects playing a significant role*. The analysis in the case of the most general (3×3) structure is not very tractable. Some discussions are available in the literature ([7]-[11]). We restrict ourselves to the two-zero texture mass matrices which have recently attracted attention. These matrices are defined in terms of six real parameters and are written in a basis in which the charged lepton mass matrix is diagonal. We show that for these textures the conclusions can be drawn in a rather simple and instructive fashion without taking recourse to numerical calculations.

Two-zero (3×3) mass matrix textures of three different types (A, B , and C) have been shown to be compatible with the present experimental data [13]. These textures correspond to hierarchical, quasi-degenerate, and inverted hierarchical neutrino masses, respectively [14]. Since zeros of κ_{ij} are unaffected by RG evolution, the pattern of these neutrino mass matrices – i.e., the zeros in the texture – is unaltered at the high scale. The question we investigate here is whether RG evolution can significantly affect the mixing angles for these mass matrix textures. We consider the three cases in turn.

III.1 Quasi-degenerate masses

If at a high energy the neutrino mass matrix is of the form

$$M_\nu^h = \begin{bmatrix} x_1 & x_2 & 0 \\ x_2 & 0 & x_3 \\ 0 & x_3 & x_4 \end{bmatrix}, \quad (10)$$

then, in view of our discussions in the previous section, see (5), at the low scale it becomes

$$M_\nu^l = a \begin{bmatrix} x_1 & x_2 & 0 \\ x_2 & 0 & x_3(1 + \frac{r}{2}) \\ 0 & x_3(1 + \frac{r}{2}) & x_4(1 + r) \end{bmatrix}.$$

Notice that the overall scale factor $a \sim \mathcal{O}(1)$ does not affect the mixing; only the mass eigenvalues are scaled by it. The above mass matrix belongs to the type B_1 texture in the notation of [13]. It corresponds to neutrinos with nearly degenerate masses [13, 14].

Our goal will be to extract the mass splittings and mixing angles which follow from this low energy neutrino mass matrix. The 3×3 matrix M_ν can be diagonalized according to

$$V^T M_\nu V = \text{diag}(m_1, m_2, m_3),$$

where V is the Maki-Nakagawa-Sakata [22] unitary matrix which relates flavour (α) and mass (i) eigenstates of neutrinos through $\nu_\alpha = V_{\alpha i} \nu_i$ and m_1, m_2, m_3 are the eigenvalues. It can be expressed as $V = U_{23} U_{13} U_{12}$, where U_{ij} are the standard rotation matrices.

The rotation angle θ_{23} is given by

$$\tan 2\theta_{23}^l = \frac{2x_3(1 + \frac{r}{2})}{x_4(1 + r)}.$$

It is obvious that there is no scope of getting large $\tan 2\theta_{23}$ at the low scale starting from a small one at the high scale through a resonance induced mechanism. So, for this texture, we have to keep $\tan 2\theta_{23}$ large for the whole scale of running and the condition for this is³

$$\frac{|x_3|}{|x_4|} \gg 1. \quad (11)$$

Applying the rotation through U_{23} , we get

$$M_{\nu 23}^l = a \begin{bmatrix} x_1 & x_2 c_{23}^l & x_2 s_{23}^l \\ x_2 c_{23}^l & \lambda_1 & 0 \\ x_2 s_{23}^l & 0 & \lambda_2 \end{bmatrix}. \quad (12)$$

where s_{ij}^l and c_{ij}^l are the sines and cosines of θ_{ij}^l . Also,

$$\lambda_1 = x_4 s_{23}^{l2} (1 + r) - 2x_3 s_{23}^l c_{23}^l (1 + \frac{r}{2}) \simeq -x_3 (1 + \frac{r}{2}), \quad (13)$$

and

$$\lambda_2 = x_4 c_{23}^{l2} (1 + r) + 2x_3 s_{23}^l c_{23}^l (1 + \frac{r}{2}) \simeq x_3 (1 + \frac{r}{2}). \quad (14)$$

where in the final step we have set $\theta_{23} = \pi/4$.

For the next rotation U_{13} , we set

$$\tan 2\theta_{13}^l = \frac{2x_2 s_{23}^l}{\lambda_2 - x_1} \simeq \frac{\sqrt{2}x_2}{x_3(1 + \frac{r}{2}) - x_1}.$$

³Here, and in the following, $\tan 2\theta_{ij}$, ($i, j = 1, 2, 3$), are allowed to be positive as well as negative (the so-called ‘dark side’).

The bound on $(V^l)_{e3}$ from CHOOZ requires the angle $\theta_{13}^l \rightarrow 0$. A significant RG evolution of this angle will demand a large $\tan 2\theta_{13}^h$, which is possible if

$$x_3 \simeq x_1. \quad (15)$$

This, together with $\tan 2\theta_{13}^l \rightarrow 0$, and the smallness of r ($\sim 10^{-4}$), fixes the following relation between mass matrix elements

$$|x_2|, |x_4| \ll |x_3| \sim |x_1|. \quad (16)$$

Now, after the second rotation⁴

$$M_{\nu 23,13}^l \simeq a \begin{bmatrix} x_1 & x_2/\sqrt{2} & 0 \\ x_2/\sqrt{2} & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}.$$

The next step is to diagonalize the (12)-block through the θ_{12} rotation:

$$\tan 2\theta_{12}^l = \frac{\sqrt{2}x_2}{\lambda_1 - x_1}.$$

It is obvious from (13), (15), and (16) that $\tan 2\theta_{12}^l$ is small; a direct contradiction of the empirical requirement.

Now we can check the only remaining possibility, i.e., keeping $\tan 2\theta_{13}$ *small* while, as before, $\tan 2\theta_{23}$ remaining large throughout the energy range, whether there can be a prominent RG evolution of θ_{12} . In this case, we get the relation

$$|x_3 - x_1| \gg \sqrt{2}|x_2|.$$

It is seen that

$$\tan 2\theta_{12}^h \simeq \frac{\sqrt{2}x_2}{-x_3 - x_1}, \quad \tan 2\theta_{12}^l \simeq \frac{\sqrt{2}x_2}{-x_3(1 + \frac{r}{2}) - x_1}. \quad (17)$$

A resonant enhancement of $\tan 2\theta_{12}$ requires $rx_3 \simeq -2(x_3 + x_1)$. Thus,

$$x_3 \sim -x_1 \quad (18)$$

and, further, for significant running of θ_{12} , from (17)

$$rx_3 \gg 2\sqrt{2}|x_2|. \quad (19)$$

It now remains to verify whether it is possible to satisfy (11), (18) and (19) and at the same time reproduce the correct mass splittings at low energies for atmospheric and solar neutrino oscillations. For this texture, the mass eigenstates are quasi-degenerate⁵ and it can be shown that:

$$\Delta m_{atm}^2 \sim 2a^2 x_1 x_4, \quad \Delta m_{sol}^2 \sim 2\sqrt{2}a^2 x_1 x_2. \quad (20)$$

Experiment demands that $\Delta m_{atm}^2/\Delta m_{sol}^2 \sim 100$ for the LMA solution to the solar neutrino problem while it is $\sim 10^4$ for the LOW solution. From (20) it is clear that there is no obstruction in achieving

⁴The smallness of θ_{13} is a consequence of the (13)- and (31)-elements of (12) being negligible compared to the (11)- and (33)-elements. The second rotation therefore amounts to simply dropping the former elements. This approximation is also used in the following subsections.

⁵ $m_1 \simeq m_2 \simeq -m_3$; the negative mass eigenvalue representing opposite CP-phase.

Scale	$\tan \theta_{23}$	$\sin 2\theta_{13}$	$\sin 2\theta_{12}$	Δm_{atm}^2 (eV ²)	Δm_{sol}^2 (eV ²)
High	0.9995	0.0	0.132	6.5×10^{-3}	3.5×10^{-4}
Low	0.9995	0.0	0.907	3.2×10^{-3}	2.5×10^{-5}

Table 1: The mixing angles and mass splittings from the chosen values of x_i (see text).

the desired ratio by suitably choosing x_2, x_3 , and x_4 . Since the relationships amongst the x_i are linear, they can all be scaled to achieve the right absolute magnitudes of the mass splittings.

We have numerically checked that this is true. For example, with the inputs $x_1 = -1.799196, x_2 = 9.0 \times 10^{-6}, x_3 = 1.8$ and $x_4 = 1.8 \times 10^{-3}$ (all in eV), we observe a running of the solar mixing angle and the mass splittings as given in Table 1⁶.

The fine-tuning between x_1 and x_3 above is an essential ingredient of the radiative enhancement of the θ_{12} mixing angle. This is reminiscent of the fine-tuning in the two-flavour case discussed in the previous section. Note that $\tan \theta_{23}$ is large but not maximal, a consequence of the non-zero value of x_4 – a feature noted in [14]. The quasi-degenerate neutrino spectrum with one neutrino with an opposite CP-phase from the other two is necessary for an RG evolution of θ_{12} . This is consistent with the observations in [24].

There are three other two-zero texture mass matrices ($B_i, i = 2,3,4$, in the notation of [13]) for which RG running of mixing angles can be significant as in the example discussed above. The B_3 texture differs from Eq. (10) in that the (13)- and (31)-elements are non-zero while the (12)- and (21)-elements vanish. Obviously, this does not affect the θ_{23} prediction, which is again maximal. After the θ_{23} rotation through $(\pi/4)$ the second and third rows (and columns) become the same in the B_1 and the B_3 textures and the remaining discussions are identical. Finally, the B_2 (B_4) texture can be obtained from the B_3 (B_1) texture by placing the zero diagonal entry in the (33) position rather than in the (22) position. This only affects the first step of the argument in that $\tan 2\theta_{23}$ now changes sign. Obviously this does not affect the conclusions in any way. *Thus the four two-zero textures $B_i, (i = 1, \dots, 4)$, which correspond to a quasi-degenerate neutrino mass spectrum, can all have prominent RG running of the solar mixing angle.*

III.2 Hierarchical masses

Here we consider

$$M_\nu^h = \begin{bmatrix} 0 & 0 & x_1 \\ 0 & x_2 & x_3 \\ x_1 & x_3 & x_4 \end{bmatrix}, \quad (21)$$

then at the low scale it becomes

$$M_\nu^l = a \begin{bmatrix} 0 & 0 & x_1(1 + \frac{r}{2}) \\ 0 & x_2 & x_3(1 + \frac{r}{2}) \\ x_1(1 + \frac{r}{2}) & x_3(1 + \frac{r}{2}) & x_4(1 + r) \end{bmatrix}.$$

⁶Variations of x_i around the quoted values are tightly constrained by the experimental data. Interestingly, the effective Majorana mass parameter relevant for neutrinoless double beta decay is predicted in the 1 eV range. We have also checked that both the Tritium beta decay bound (mentioned earlier) and the cosmological bound from the recent 2dF Galaxy Redshift Survey, namely $\sum_i m_i \lesssim 2.2$ eV [23], are satisfied.

This is the type A_1 texture of [13].

After the rotation through U_{23} ,

$$M_{\nu 23}^l = a \begin{bmatrix} 0 & -x_1 s_{23}^l (1 + \frac{r}{2}) & x_1 c_{23}^l (1 + \frac{r}{2}) \\ -x_1 s_{23}^l (1 + \frac{r}{2}) & \lambda_1 & 0 \\ x_1 c_{23}^l (1 + \frac{r}{2}) & 0 & \lambda_2 \end{bmatrix},$$

where

$$\tan 2\theta_{23}^l = \frac{2x_3(1 + \frac{r}{2})}{x_4(1 + r) - x_2}. \quad (22)$$

To get maximal mixing in the atmospheric sector through RG running, one must therefore have

$$x_4(1 + r) = x_2. \quad (23)$$

Further,

$$\lambda_1 = x_2 c_{23}^{l2} + x_4 s_{23}^{l2} (1 + r) - 2x_3 s_{23}^l c_{23}^l (1 + \frac{r}{2}) \sim x_2 - x_3 (1 + \frac{r}{2}), \quad (24)$$

$$\lambda_2 = x_2 s_{23}^{l2} + x_4 c_{23}^{l2} (1 + r) + 2x_3 s_{23}^l c_{23}^l (1 + \frac{r}{2}) \sim x_2 + x_3 (1 + \frac{r}{2}), \quad (25)$$

where (23) has been used in the second step.

The next rotation is through U_{13} and is given by

$$\tan 2\theta_{13}^l = \frac{2x_1 c_{23}^l (1 + \frac{r}{2})}{\lambda_2}.$$

Experiments indicate $\tan 2\theta_{13}^l \rightarrow 0$. Therefore, we must have $|l_2| \gg |\sqrt{2}x_1(1 + r/2)|$ which, in view of (25), implies

$$|x_2 + x_3(1 + \frac{r}{2})| \gg \sqrt{2}|x_1(1 + \frac{r}{2})|. \quad (26)$$

After this second rotation, the mass matrix $M_{\nu 23,13}^l$ looks like

$$M_{\nu 23,13}^l = a \begin{bmatrix} 0 & -x_1 s_{23}^l (1 + \frac{r}{2}) & 0 \\ -x_1 s_{23}^l (1 + \frac{r}{2}) & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}.$$

For the remaining (12)-rotation, using (24), we have

$$\tan 2\theta_{12}^l = \frac{-2x_1 s_{23}^l (1 + \frac{r}{2})}{x_2 - x_3(1 + \frac{r}{2})},$$

and the condition to get large $\tan 2\theta_{12}^l$ is (using $\sin \theta_{23}^l \sim 1/\sqrt{2}$)

$$|x_2 - x_3(1 + \frac{r}{2})| \ll \sqrt{2}|x_1(1 + \frac{r}{2})|. \quad (27)$$

Thus all the angles in the low energy scale will be compatible with the data so long as the requirements in Eqs. (23), (26), (27) are met. Our aim now is to check whether these angles can be generated from significantly different ones at the high scale. Several alternatives are possible.

First, assume that θ_{23} is small at the high scale. From (22), this implies

$$2|x_3| \ll |x_4 - x_2| = r|x_4|,$$

where we have used (23) in the last step. Since the magnitude of $r \sim 10^{-4}$, Eq. (23) implies that $x_2 \simeq x_4$ and consequently $|x_2| \gg |x_3|$. Thus, $|x_2 \pm x_3| \simeq |x_2|$ and hence conditions (26) and (27) cannot be satisfied simultaneously. This establishes that a radiative enhancement of θ_{23} is not possible with large θ_{12}^l and small θ_{13}^l for this mass matrix texture.

For the remainder of this subsection we restrict ourselves to the situation where the angle θ_{23} does not evolve much, i.e., $\theta_{23}^h \sim \theta_{23}^l = \pi/4$. This requires that

$$|x_3| \gg |x_4 - x_2| = r|x_4|. \quad (28)$$

An evolution of $\tan 2\theta_{13}$ from a large value at the high scale will occur if, in addition to condition (26), one also has

$$|x_2 + x_3| \ll \sqrt{2}|x_1|. \quad (29)$$

Together, they require $|rx_3| \gg |x_1|$, which in view of the smallness of r implies $|x_3| \gg |x_1|$. From (29), then $x_2 \simeq -x_3$ and hence (27) cannot be satisfied. Thus a significant running of θ_{13} is also excluded.

Finally, there is one remaining avenue for important RG evolution. This is the case where $\tan 2\theta_{23}$ remains large over the entire range while $\tan 2\theta_{13}$ is small, but $\tan 2\theta_{12}$ starts off small at the high scale and evolves to a near maximal value at low scale. We now show that even this is inadmissible. For the running of $\tan 2\theta_{12}$ one must satisfy (27) as well as

$$|x_2 - x_3| \gg \sqrt{2}|x_1|. \quad (30)$$

These conditions can be simultaneously met if $|rx_3| \gg |x_1|$. Therefore, $|x_3| \gg |x_1|$, which together with (27), (28) and (30) requires

$$|x_4| \simeq |x_2| \simeq |x_3| \gg |x_1|/r.$$

Here it is useful to note that this texture corresponds to a hierarchical mass spectrum with eigenvalues $\sim m, -m, M$ with $M \gg m$ [14]. Therefore, $M \sim \sqrt{\Delta m_{atm}^2} \sim 5 \times 10^{-2}$ eV. From the trace and the determinant of the mass matrix (21) one finds

$$M = a(x_2 + x_4) \simeq 2ax_2 \quad \text{and} \quad Mm^2 = a^3x_2x_1^2.$$

Thus, $m \sim x_1 \ll rx_2 \sim 10^{-6}$ eV, which is way too small to accommodate a possible solar mass square splitting at the level of 10^{-5} eV² (LMA) or 10^{-7} eV² (LOW).

A variant of this mass matrix texture – type A_2 – has the zero off-diagonal entry in the (13) position. It is readily seen that this does not affect the discussion concerning θ_{23} above. Since this angle is maximal, after this rotation the A_2 texture and A_1 texture, considered above, give rise to identical structures and the rest of the discussion goes through without change. *Thus, we draw the conclusion that the hierarchical mass structures do not admit a radiative enhancement of the mixing angles.*

III.3 Inverted mass hierarchy

Finally, we consider the neutrino mass matrix texture

$$M_\nu^h = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & 0 & x_4 \\ x_3 & x_4 & 0 \end{bmatrix},$$

and therefore

$$M_\nu^l = a \begin{bmatrix} x_1 & x_2 & x_3(1 + \frac{r}{2}) \\ x_2 & 0 & x_4(1 + \frac{r}{2}) \\ x_3(1 + \frac{r}{2}) & x_4(1 + \frac{r}{2}) & 0 \end{bmatrix}.$$

This texture (type C) results in a neutrino mass spectrum with an inverted hierarchy [13, 14].

We see that this structure with $(M_\nu)_{22} = (M_\nu)_{33} = 0$ ensures maximal mixing in the (23)-sector at all energies. Therefore,

$$M_{\nu 23}^l = a \begin{bmatrix} x_1 & (x_2 - x_3(1 + \frac{r}{2}))/\sqrt{2} & (x_2 + x_3(1 + \frac{r}{2}))/\sqrt{2} \\ (x_2 - x_3(1 + \frac{r}{2}))/\sqrt{2} & \lambda_1 & 0 \\ (x_2 + x_3(1 + \frac{r}{2}))/\sqrt{2} & 0 & \lambda_2 \end{bmatrix},$$

where

$$\lambda_1 = -x_4(1 + \frac{r}{2}) \quad \text{and} \quad \lambda_2 = x_4(1 + \frac{r}{2}).$$

Now we have,

$$\tan 2\theta_{13}^l = \frac{\sqrt{2}[x_2 + x_3(1 + \frac{r}{2})]}{x_4(1 + \frac{r}{2}) - x_1}. \quad (31)$$

In order to have a small θ_{13}^l as required by the CHOOZ constraint, one must ensure

$$|\sqrt{2}[x_3(1 + \frac{r}{2}) + x_2]| \ll |x_4(1 + \frac{r}{2}) - x_1|. \quad (32)$$

After the (13)-rotation, we then have

$$M_{\nu 23,13}^l = a \begin{bmatrix} x_1 & (x_2 - x_3(1 + \frac{r}{2}))/\sqrt{2} & 0 \\ (x_2 - x_3(1 + \frac{r}{2}))/\sqrt{2} & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}. \quad (33)$$

Then diagonalizing the (12)-sector, we obtain

$$\tan 2\theta_{12}^l \simeq \frac{\sqrt{2}[x_2 - x_3(1 + \frac{r}{2})]}{\lambda_1 - x_1}. \quad (34)$$

Now, we require a near maximal mixing in the solar sector, which implies

$$|\sqrt{2}[x_3(1 + \frac{r}{2}) - x_2]| \gg |x_4(1 + \frac{r}{2}) + x_1|. \quad (35)$$

We should note that a simultaneous fulfillment of (32) and (35), as dictated by experimental data, requires that either one or both of (a) $x_1 \simeq -x_4$ and (b) $x_2 \simeq -x_3$ have to be necessarily satisfied.

Using (32) and (35), we now calculate the solar and atmospheric mass splittings from (33) as

$$\Delta m_{sol}^2 \simeq \sqrt{2}a^2 \left| [x_2 - x_3(1 + \frac{r}{2})][x_1 - x_4(1 + \frac{r}{2})] \right|, \quad \Delta m_{atm}^2 \simeq 0.5a^2 \left[x_2 - x_3(1 + \frac{r}{2}) \right]^2. \quad (36)$$

Now let us first study whether it is at all possible to accommodate the running of θ_{13} . To evolve this mixing angle from a large value at a high scale to a small value at the low scale, one must ensure the following high scale condition

$$\sqrt{2}|x_2 + x_3| \gg |x_4 - x_1|. \quad (37)$$

But if we now ensure that (a) $x_1 \simeq -x_4$ and (b) $x_2 \simeq -x_3$, the two near-equalities emerging from (32) and (35), are simultaneously satisfied, it automatically follows that an enforcement of (37) runs into contradiction with experimental data. To demonstrate this, first notice that the solar and atmospheric mass splittings, in this case, take the simple form

$$\Delta m_{sol}^2 \simeq 4\sqrt{2}a^2 x_3 x_4, \quad \Delta m_{atm}^2 \simeq 2a^2 x_3^2. \quad (38)$$

From this we can infer that $|x_3| \sim 10^{-2}$ and $|x_4| \sim 10^{-3}$, as we have already found the parameter a to be order unity, to reproduce the experimental Δm_{sol}^2 and Δm_{atm}^2 . Incidentally, the values of x_i so obtained correspond to an inverted mass hierarchy. Now let us relate x_2 and x_3 as $x_2 = -x_3(1 + \delta)$, where δ is some small parameter. Substituting this relation in (32) and (37), we observe that $\delta \simeq r/2 \sim 10^{-4}$ for a successful running of θ_{13} . From (37) it then follows $|x_4| \ll 10^{-6}$, which is very different from the value $|x_4| \sim 10^{-3}$ obtained above directly from the mass splittings. This leads to the conclusion that a significant RG evolution of θ_{13} is not possible. Through a somewhat more lengthy chain of arguments, which we do not advance here, we can as well demonstrate that even if any one of (a) $x_1 \simeq -x_4$ and (b) $x_2 \simeq -x_3$ is satisfied, θ_{13} running is not possible. Indeed, satisfying both (a) and (b), i.e., the case we presented above, helps to simplify the illustration.

Next we consider the possibility of significant RG evolution of the solar angle θ_{12} , keeping θ_{13} small throughout the scale. The condition for a small θ_{12} at the high scale is

$$\sqrt{2}|x_3 - x_2| \ll |x_4 + x_1|. \quad (39)$$

Now the question is whether (39) can be ensured without contradicting experimental data. First, one should observe that this case is very similar to the case of θ_{13} running, and the two cases can be handled in similar fashion. Proceeding in the same way as we did for θ_{13} , we can demonstrate that θ_{12} running is not possible as well. In other words, the necessary compliance of any one or both of (a) $x_1 \simeq -x_4$ and (b) $x_2 \simeq -x_3$, in conjunction with the experimental values of Δm_{sol}^2 and Δm_{atm}^2 , would lead to results which are inconsistent with (39). *Thus the inverted hierarchical neutrino mass matrix texture will also not support a significant RG running of the mixing angles.*

IV Non-standard models

In the previous sections we have based our discussion on the RG evolution of the neutrino masses within the framework of the standard model. Essentially all these results can be extended to the case of supersymmetry and models of extra dimensions. In both these scenarios, a and r , which capture the essence of the RG effect on neutrino masses, satisfy $a \sim \mathcal{O}(1)$ and $|r| \ll 1$. The conclusions which we have drawn in the previous sections rest on these features alone. Though the precise numerical values

of the mass matrix parameters will undergo appropriate modifications due to the changes in r and a , the basic conclusion that RG running of mixing angles can be prominent in the quasi-degenerate case but not in the hierarchical or inverted hierarchical alternatives will still continue to hold. For the sake of completeness, we summarize the main new ingredients of these two scenarios below.

IV.1 Supersymmetry

In the Minimal Supersymmetric Standard Model, the RG equation for κ is slightly different from Eq. (3) [5, 6]. There are two Higgs doublets, the quartic scalar coupling λ is determined by the gauge coupling, and supersymmetric particles can appear in the internal lines of the one-loop Feynman diagrams contributing to the different beta-functions. We do not present the modified equation here. It is of similar form but the coefficients are different. Suffice it to say that

$$r \simeq -(Y_\tau^2/8\pi^2) \ln(M_X/M_Z).$$

Note that r is of opposite sign from the SM. It needs to be borne in mind that in supersymmetry Y_τ depends on $\tan\beta$ and can be larger than in the standard model. In fact, $|r|$ can become as large as $\sim 10^{-2}$ in this case.

IV.2 Extra Dimensions

The RG evolution of neutrino masses in models with compact extra dimensions has been examined in [25]. The main differences from the SM are that (a) the coupling constants evolve with energy scale as a power law rather than logarithmically, and (b) the higher scale, where the coupling constants unify, is not very large $\sim \mathcal{O}(10 \text{ TeV})$. If there are δ extra dimensions and if the compactification radius is given by μ_0^{-1} then due to the power law running above $\mu_0 \sim 1 \text{ TeV}$:

$$r \simeq \left(\frac{3Y_\tau^2}{16\pi^2} \right) \frac{X_\delta}{\delta} \left[\left(\frac{M_X}{\mu_0} \right)^\delta - 1 \right] \sim 10^{-4}, \quad \text{where} \quad X_\delta = \frac{2\pi^{\delta/2}}{\delta\Gamma(\delta/2)}.$$

r is of the same order as in the SM in spite of power law evolution. This is a consequence of the curtailed evolution range – from $\mu_0 \sim 1 \text{ TeV}$ to $M_X \sim 30 \text{ TeV}$.

V Conclusions

Two of the three neutrino mixing angles are large – a situation not encountered in the quark sector. Radiative enhancement of neutrino mixings could be a possible mechanism behind this. First, we have considered the generation of neutrino Majorana masses and mixings via a dimension-5 nonrenormalizable interaction. Then we have reviewed the radiative mechanism for the two flavour neutrino mass matrix before moving to three flavour cases. For the latter, we concentrate on two-zero mass matrix textures. Radiative corrections of different categories of such structures, namely quasi-degenerate and hierarchical (normal and inverted), have been considered. We observe that in order to maintain consistency with experimental data on masses and mixings, the atmospheric and the CHOOZ angle cannot have appreciable running. Only the solar angle can have a possibility to evolve from a low value at a

high scale to a large value at a low scale, and that too only for the quasi-degenerate mass structures with one of the neutrinos having opposite CP parity from the other two. The overall conclusions do not change for supersymmetric and extra-dimensional scenarios.

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