

# Bulk viscosity coefficient of hadronic matter

Sabyasachi Ghosh<sup>1,2</sup>, Sandeep Chatterjee<sup>2</sup>, and Bedangadas Mohanty<sup>2</sup>

<sup>1</sup> Department of Physics, University of Calcutta, 92, A. P. C. Road, Kolkata - 700009, India

<sup>2</sup> School of Physical Sciences, National Institute of Science Education and Research Bhubaneswar, HBNI, Jatni, 752050, India

**Abstract.** The bulk viscosity coefficient of hadronic matter has been estimated in this present work, where the thermodynamical equilibrium quantity like speed of sound in the medium has been obtained by using standard hadron resonance gas model. Whereas, the non-equilibrium quantity like thermal widths of medium constituents have been calculated in the framework field theory at finite temperature. Our values of bulk viscosity coefficient are in agreement with some earlier estimations.

**Keywords:** Transport coefficient, Thermal field theory, HRG model

## 1 Introduction

Recent research of heavy ion physics has concluded that the medium, formed in relativistic heavy ion collisions, must have very small shear viscosity, which is in contrast to the weak coupling picture, described by the standard finite temperature calculation of quantum chromo dynamics (QCD). Owing to this motivation, several microscopic calculations of shear viscosity have been done in recent times. Estimation of other transport coefficient like bulk viscosity of this strongly interacting matter is also becoming a contemporary research interest. In this context, this present article has addressed the bulk viscosity calculation for hadronic matter, where equilibrium thermodynamics for all hadrons in medium are described by Hadron Resonance Gas (HRG) model. The non-equilibrium part involving the thermal widths of medium constituents have been calculated by using effective Lagrangian densities for the hadronic medium. Here we have assumed pions and nucleons as the most abundant constituents of the hadronic medium and we have calculated their thermal widths, which basically reflect their in-medium scattering with different mesonic and baryonic resonances.

## 2 Formalism

For the equilibrium part of the medium, we have used the ideal HRG model, where the hadrons and resonances with masses up-to 2 GeV have been taken from the Particle Data Book. Constructing total partition function of hadronic medium, one can easily derived all thermodynamical quantities like pressure ( $P$ ),

energy density ( $\epsilon$ ), entropy density ( $s$ ). Their temperature dependence at zero baryon chemical potential ( $\mu_B = 0$ ) are quite close to the corresponding results in the hadronic temperature domain, obtained by the lattice quantum chromo dynamics (LQCD) calculations [1]. In terms of  $P$  and  $\epsilon$ , the square of the speed of sound for constant baryon density ( $n_B$ ) is defined as  $v_n^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_{n_B}$ , plays an important role in bulk viscosity calculation. Either from the Relaxation Time Approximation (RTA) in kinetic theory approach or from the one-loop diagram representation in Kubo framework, one can get standard expressions of bulk viscosity coefficient for pion and nucleon components [2]:

$$\zeta_\pi = \left(\frac{g_\pi}{T}\right) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{n_\pi [1 + n_\pi]}{\omega_\pi^2 \Gamma_\pi} \left\{ \left(\frac{1}{3} - v_n^2\right) \mathbf{k}^2 - v_n^2 m_\pi^2 \right\}^2 \quad (1)$$

and

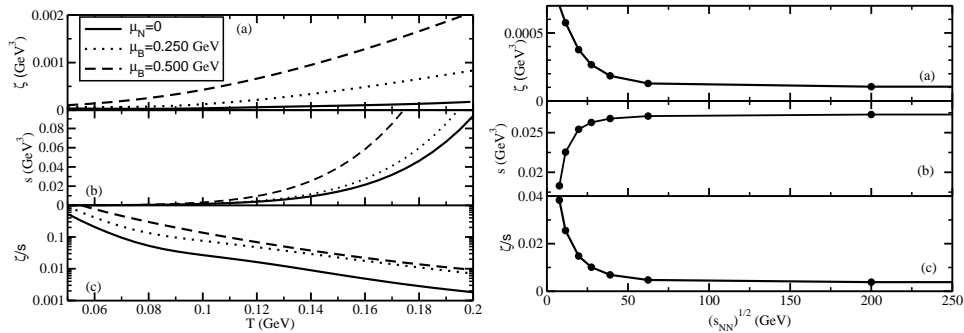
$$\begin{aligned} \zeta_N = & \left(\frac{g_N}{T}\right) \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega_N^2 \Gamma_N} \left[ \left\{ \left(\frac{1}{3} - v_n^2\right) \mathbf{k}^2 - v_n^2 m_N^2 - \omega_N \left(\frac{\partial P}{\partial n_B}\right)_\epsilon \right\}^2 n_N^+ (1 - n_N^+) \right. \\ & \left. + \left\{ \left(\frac{1}{3} - v_n^2\right) \mathbf{k}^2 - v_n^2 m_N^2 + \omega_N \left(\frac{\partial P}{\partial n_B}\right)_\epsilon \right\}^2 n_N^- (1 - n_N^-) \right], \quad (2) \end{aligned}$$

where  $n_\pi = 1/\{e^{\omega_\pi/T} - 1\}$  with  $\omega_\pi = \{\mathbf{k}^2 + m_\pi^2\}^{1/2}$  for pion,  $n_N^\pm = 1/\{e^{(\omega_N \mp \mu_B)/T} + 1\}$  with  $\omega_N = \{\mathbf{k}^2 + m_N^2\}^{1/2}$  for nucleon and their respective degeneracy factors are  $g_\pi = 3$  and  $g_N = 2 \times 2$ . The  $m_\pi, m_N$  stand for masses of pion, nucleon and their momenta are denoted by  $\mathbf{k}$ . The  $\Gamma_\pi$  and  $\Gamma_N$  are thermal widths of pion and nucleon, which have been calculated from the imaginary part of their self-energies (on-shell) at finite temperature. We have considered different mesonic and baryonic loops for pion self-energy, whereas, pion-baryon intermediate states are taken in nucleon self-energy [3,4].

### 3 Results and Conclusion

Using the explicit momentum  $\mathbf{k}$ ,  $T$  and  $\mu_B$  dependent thermal widths of pion and nucleon in the integrands of Eqs. (1) and (2), we can calculate bulk viscosity of pion and nucleon components respectively. Adding these two components, we get total bulk viscosity, which is plotted in the left panel of Fig. 1(a). The entropy density  $s$ , obtained from HRG and the ratio  $\zeta/s$  are also drawn in the left panel of Fig. 1(b) and (c), which reveal their temperature dependence at  $\mu_B = 0$  (solid line), 0.250 (dotted line) and 0.500 GeV (dashed line). We notice that  $\zeta$  and  $s$  both increase with  $T$  and  $\mu_B$ . Whereas,  $\zeta/s$  is a decreasing function of  $T$  (but still increase in  $\mu_B$ ) because the increment of  $s(T)$  is larger than the increment of  $\zeta(T)$ .

Next, right panel of Fig. 1(a), (b) and (c) show respectively the variation of  $\zeta$ ,  $s$  and  $\zeta/s$  with the variation of center of mass energy  $\sqrt{s_{NN}}$ . We have used the parameterization from Ref. [5], where beam energy dependence of  $T$  and  $\mu_B$  used in computation are those obtained from fits to hadron yields. We notice in



**Fig. 1.** Left :  $T$  dependence of bulk viscosity (a), entropy density (b) and their ratio  $\zeta/s$  (c) at  $\mu_B = 0$  (solid line), 0.250 GeV (dotted line) and 0.500 GeV (dash line). Right : Variation of same quantities  $\zeta$  (a),  $s$  (b) and  $\zeta/s$  (c) with the variation of center of mass energy ( $\sqrt{s_{NN}}$ ).

the right panel of Fig. 1 that  $\zeta$  (a) as well as  $\zeta/s$  (c) are decreasing with  $\sqrt{s_{NN}}$ , which is qualitatively agreeing with the results of earlier Ref. [6]. The decreasing trend of  $\zeta$  and  $\zeta/s$  with  $\sqrt{s}$  can be understood from the fact that  $\mu_B$  decreases with  $\sqrt{s}$  while  $T$  remains fairly constant in the range of  $\sqrt{s}$  analyzed here and the  $\zeta$  and  $\zeta/s$  decrease with decreasing of  $\mu_B$  as we have noticed in the left panel of Fig. (1). At  $\mu_B = 0$ , the decreasing nature of our  $\zeta/s(T)$  agrees with most of the earlier works as addressed elaborately in Ref. [4]. However, some investigations have reported it to increase with  $T$  and some time also have a peak structure near transition temperature [7].

**Acknowledgments:** Authors thank UGC, DAE, DST Govt. of India for financial support during the period, when this work was carried out.

## References

1. Bazavov, A. et al. (HotQCD Collaboration) : Equation of state in (2+1)-flavor QCD. Phys. Rev. **D 90**, 094503 (2014).
2. Gavin, S. : Transport coefficients in ultra-relativistic heavy-ion collisions. Nucl.Phys. **A 435** (1985) 826.
3. Ghosh, S. : The nucleon thermal width due to pion-baryon loops and its contributions in Shear viscosity. Phys. Rev. **C 90**, 025202 (2014).
4. Ghosh, S., Chatterjee, S., Mohanty, B. : Bulk viscosity for pion and nucleon thermal fluctuation in the hadron resonance gas model. Phys. Rev. **C 94**, 045208 (2016).
5. Karsch, F. and Redlich, K. : Probing freeze-out conditions in heavy ion collisions with moments of charge fluctuations. Phys. Lett. **B 695**, 136 (2011).
6. Kadam, G. P., Mishra, H. : Dissipative properties of hot and dense hadronic matter in an excluded-volume hadron resonance gas model. Phys. Rev. **C 92** (2015) 035203.
7. Saha, K., Upadhaya, S., Ghosh, S. : A comparative study on two different approaches of bulk viscosity in the Polyakov Nambu Jona-Lasinio model. Mod. Phys. Lett. **A 32**, 1750018 (2017).