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ACCELERATOR, REACTOR AND ATMOSPHERIC NEUTRINO DATA: A THREE FLAVOUR OSCILLATION ANALYSIS

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ABSTRACT

We perform a three flavour analysis of the atmospheric, accelerator and reactor neutrino data from the Kamiokande, LSND and Bugey experiments respectively. Choosing the values of Δm^2 obtained from two flavour fits of the first two experiments, the allowed ranges of the three generation mixing angles are determined. The accelerator experiments CHORUS and NOMAD are found to be most sensitive to regions of the allowed parameter space which correspond to genuine three generation solutions for the atmospheric neutrino anomaly.

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In the standard model of electroweak theory the neutrinos are assumed to be massless. But there is no compelling theoretical reason behind this assumption. If the neutrinos are massive then, as in the quark sector, the weak interaction basis of neutrinos may be different from the mass eigenstate basis – leading to mixing between different flavours. A way for probing such mixing and small neutrino masses is provided by neutrino oscillations. Two well known neutrino puzzles that can be explained by flavour oscillation of neutrinos are the solar neutrino problem and the atmospheric neutrino anomaly. The recent declaration by the Liquid Scintillator Neutrino Detector (LSND) collaboration [1] that they are observing an excess of $\overline{\nu}_e$s (over the expected background) which can be attributed to $\overline{\nu}_\mu - \overline{\nu}_e$ oscillations has added a new dimension to the issue of neutrino mass and mixing. LSND is most sensitive to $\Delta m^2 \sim 6eV^2$ [2] and the significance of this result for particle physics, astrophysics and cosmology has been investigated [3, 4]. One notes that the three phenomena mentioned above – namely, the solar neutrino problem, the atmospheric neutrino anomaly and the $\overline{\nu}_\mu - \overline{\nu}_e$ oscillations observed by the LSND group – require vastly different mass ranges. The solar neutrino problem can be explained either by Mikheyev-Smirnov-Wolfenstein oscillation [5] for $\Delta m^2 \sim 6 \times 10^{-6} eV^2$ and $\sin^2 2\theta \sim 7 \times 10^{-3}$ (non-adiabatic solution) and $\Delta m^2 \sim 9 \times 10^{-6} eV^2$ and $\sin^2 2\theta \sim 0.6$ (large mixing angle solution) [6] or by oscillation in vacuum for $\Delta m^2 \sim (0.45 - 1.2) \times 10^{-10} eV^2$ and $\sin^2 2\theta \sim (0.6 - 1.0)$ [7] in a two generation scenario. The atmospheric anomaly can be explained by either $\nu_\mu - \nu_e$ or $\nu_\mu - \nu_\tau$ oscillations in a two generation picture. The analysis of the new multi-GeV data as well as the previous sub-GeV data of the Kamiokande collaboration predicts the following best-fit parameters $(\Delta m^2, \sin^2 2\theta) = (1.8 \times 10^{-2} eV^2, 1.0)$ for $\nu_\mu - \nu_e$ oscillation and $(1.6 \times 10^{-2} eV^2, 1.0)$ for $\nu_\mu - \nu_\tau$ oscillation [8].

Although each experiment can be explained by two flavour neutrino oscillations, there are several motivations to go beyond this approximation. The LEP result that there are three light neutrinos is also supported by the requirements of nucleosynthesis in the early universe. In the quark sector, mixing between three generations is well established. A natural question then is how do experiments constrain three neutrino mixing? We stress that in a realistic three flavour framework it is important to do a combined analysis to find out the allowed range of parameters rather than using separate two flavour schemes. In particular, this might uncover regions in the parameter space sensitive to the presence of the third generation which cannot be probed in the two flavour limit.

In this paper we perform a three flavour analysis of the atmospheric and LSND data assuming that the presently reported values will not change significantly as more results accumulate. The constraints obtained from the reactor experiments are also incorporated. We take the Δm^2s as: $\Delta_{12} \simeq \Delta_{13} = 6 eV^2$ in the LSND range and $\Delta_{23} = 10^{-2} eV^2$ as preferred by the atmospheric neutrino data. The $\simeq$ sign means we neglect 10^{-2} as compared to 6. It will become clear as we proceed that most of our analysis does not depend on this specific choice as long as the order of magnitude remains the same. The three mixing angles are allowed to cover the whole range from 0 to $\pi/2$. For atmospheric neutrinos, we find, in addition to the two flavour results, genuine three generation solutions where both $\nu_\mu - \nu_e$ and $\nu_\mu - \nu_\tau$ oscillation channels simultaneously contribute. The implications of the allowed areas thus obtained for the accelerator experiments CHORUS and NOMAD searching for $\nu_\mu - \nu_\tau$ osillations are also discussed. Such an analysis for constraining the mixing angles has been performed in [9] under the approximation of an effective two flavour interpretation of the atmospheric neutrino problem either in the $\nu_\mu - \nu_e$ or $\nu_\mu - \nu_\tau$ oscillation mode, instead of a full three flavour investigation. A detailed analysis combining the accelerator, reactor, solar and atmospheric neutrino data had been carried out earlier (pre-LSND) [10] taking a different spectrum for Δm^2 and assuming the mixing angles to be less than $\pi/4$.

The measurement of atmospheric neutrino fluxes is being carried out by the following groups – Kamiokande, IMB, Fréjus, Nusex and Soudan2. So far, data of most statistical significance have been collected by the Kamiokande and the IMB collaborations. To reduce the uncertainty in the absolute flux values the usual practice is to present the ratio of ratios R which is defined as,

$$R = \frac{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)_{\text{obsvd}}}{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)_{\text{MC}}} \tag{1}$$

where MC denotes the Monte-Carlo simulated ratio. For neutrinos of energy less than ~ 1 GeV, IMB finds R $= 0.54 \pm 0.05 \pm 0.12$ [11] in agreement with the Kamiokande data R $= 0.60^{+0.06}_{-0.05} \pm 0.05$ in this energy range [8, 12]. Recently the Kamiokande collaboration has published the results of the mesurement of the flux ratio in the multi-GeV energy range [8]. They found R $= 0.57^{+0.08}_{-0.07} \pm 0.07$ in good agreement with the sub-GeV value. All these data show that R is smaller than the expected value of unity, a result that might be explained by neutrino flavour oscillation [13]. Another aspect of this measurement that can independently point towards neutrino oscillation is the dependence of R on the zenith-angle. The multi-GeV Kamiokande data reveals a dependence on the zenith-angle unlike the sub-GeV data, though the statistical significance of this result has been questioned [14]. For the purpose of this paper we use the sub-GeV Kamiokande results.

The LSND group searches for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations using $\overline{\nu}_e$ appearance. The $\overline{\nu}_e$s produce neutrons *via* the reaction $\overline{\nu}_e p \to e^+ n$ which in turn are captured by protons producing a 2.2 MeV γ. An excess of beam-on events with a γ of the above energy in time and space coincidence with an electron in the energy range 36 MeV $< E_e <$ 60 MeV is considered as a signal for $\overline{\nu}_e$. The mean source-detector distance is 30 metres. The initial LSND data reports an excess of $16.4^{+9.7}_{-8.9} \pm$ 3.3 events over the estimated background which, if interpreted in terms of neutrino oscillations, corresponds to a probability $P_{\overline{\nu}_\mu \overline{\nu}_e}$ of $(0.34^{+0.20}_{-0.18} \pm 0.07)\%$.

Other appearance experiments searching for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations are KARMEN at the ISIS spallation neutron facility [15] and the BNL-E776 [16]. These experiments are consistent with no neutrino oscillation. KARMEN has so far quoted an upper limit on the oscillation probability as $P_{\overline{\nu}_\mu \overline{\nu}_e} \leq 3.1 \times 10^{-3}$ (90% C.L.) wheareas from the two flavour exclusion areas presented by BNL one gets $P_{\overline{\nu}_\mu \overline{\nu}_e} \leq 1.5 \times 10^{-3}$ (90% C.L.). In ref. [1] the LSND group has shown that some of the areas allowed by them in a two flavour analysis are disfavoured by KARMEN and BNL-E776. In this paper we confine ourselves to the LSND data for constraining the paremeters.

Reactor experiments searching for neutrino oscillation are GÖSGEN, KRASNOYARSK and Bugey. These experiments provide exclusion regions in the Δm^2 - $\sin^2 2\theta$ parameter space by non-observance of neutrino oscillation. The maximum exclusion is by Bugey which measures the spectrum of $\overline{\nu}_e$, coming from the Pressurized Water Reactors running at the Bugey nuclear power plant, at 15, 40, and 95 metres using neutron detection techniques. The 90% C.L. exclusion contour implies $1 - P_{\overline{\nu}_e \overline{\nu}_e} \leq 0.05$ [17].

The general expression for the probability that an initial ν_α of energy E gets converted to a ν_β after travelling a distance L in vacuum is

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4\ \Sigma_{j>i}\ U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2(\frac{\pi L}{\lambda_{ij}}) \tag{2}$$

where $\lambda_{ij} = 2.47m(E_\nu/MeV)(eV^2/\Delta_{ij})$, $\Delta_{ij} = {m_j}^2 - {m_i}^2$. The actual forms of the various survival and transition probabilities depend on the spectrum of Δm^2 assumed and the choice of the mixing matrix U relating the flavour eigenstates to the mass eigenstates. The most suitable parametrisation of U for the mass spectrum chosen by us is $U = R_{13} R_{12} R_{23}$ where R_{ij} denotes the rotation matrix in the ij-plane. This yields:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}c_{23} - s_{13}s_{23} & c_{13}s_{12}s_{23} + s_{13}c_{23} \\ -s_{12} & c_{12}c_{23} & c_{12}s_{23} \\ -s_{13}c_{12} & -s_{13}s_{12}c_{23} - c_{13}s_{23} & -s_{12}s_{13}s_{23} + c_{13}c_{23} \end{pmatrix} \tag{3}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. We have assumed CP-invariance so that U is real. The above choice of U has the advantage that θ_{23} does not appear in the expressions for the probability for LSND and Bugey. We now turn to the implications of the above mixing matrix and the chosen mass ranges on the various probabilities.

(i) LSND

In order to see the impact of three neutrino generations, we first note that for the energy and length scales relevant for LSND $\lambda_{23} >> L$ and the term involving $\sin^2(\pi L/\lambda_{23}) \rightarrow 0$. Further, $\lambda_{13} \simeq \lambda_{12}$ and (2) simplifies to

$$P_{\overline{\nu}_\mu \overline{\nu}_e} = 4c_{12}^2 s_{12}^2 c_{13}^2 \sin^2(\pi L/\lambda_{12}) \tag{4}$$

(ii) Bugey

For Bugey, the neutrino energy ranges from 2.8 - 7.8 MeV whereas L is typically $\sim$ 40 metres. Then $\lambda_{23} >> L$, so that $\sin^2(\pi L/\lambda_{23}) \rightarrow 0$. On the other hand $\lambda_{12} = \lambda_{13} << L$ so that $\sin^2(\pi L/\lambda_{12})$ and $\sin^2(\pi L/\lambda_{13})$ average out to 1/2. Then the relevant probability is

$$P_{\overline{\nu}_e \overline{\nu}_e} = 1 - 2c_{13}^2 c_{12}^2 + 2c_{13}^4 c_{12}^4 \tag{5}$$

(iii) Atmospheric neutrinos

In a three flavour mixing scheme (1) is given in terms of the neutrino transition and survival probabilities as

$$R = \frac{P_{\nu_\mu \nu_\mu} + r_{MC} P_{\nu_\mu \nu_e}}{P_{\nu_e \nu_e} + \frac{1}{r_{MC}} P_{\nu_\mu \nu_e}} \tag{6}$$

where $r_{MC} = (\nu_e + \overline{\nu}_e)/(\nu_\mu + \overline{\nu}_\mu)$ as obtained from a Monte-Carlo simulation. Notice that for neutrinos in the energy range $\sim (0.1 - 1)$ GeV travelling through a distance ranging from $\sim (10 - 10^4)$ km, $\lambda_{12} = \lambda_{13} << L$ and $\sin^2(\pi L/\lambda_{12})$ and $\sin^2(\pi L/\lambda_{13})$ can be replaced by their average value 1/2. Taking this into account, the probabilities appearing in (6) can be expressed as

$$P_{\nu_e \nu_e} = 1 - 2c_{13}^2 c_{12}^2 + 2c_{13}^4 c_{12}^4 - 4(c_{13} s_{12} c_{23} - s_{13} s_{23})^2 (c_{13} s_{12} s_{23} + s_{13} c_{23})^2 \sin^2(\pi L/\lambda_{23}) \tag{7}$$

$$P_{\nu_\mu \nu_e} = 2c_{13}^2 c_{12}^2 s_{12}^2 - 4c_{12}^2 c_{23} s_{23} (c_{13} s_{12} c_{23} - s_{13} s_{23})(c_{13} s_{12} s_{23} + s_{13} c_{23}) \sin^2(\pi L/\lambda_{23}) \tag{8}$$

$$P_{\nu_\mu \nu_\mu} = 1 - 2c_{12}^2 s_{12}^2 - 4c_{12}^4 c_{23}^2 s_{23}^2 \sin^2(\pi L/\lambda_{23}) \tag{9}$$

The results of the combined analysis of the above three experiments are presented in figs. 1 and 2 in the large s_{13}^2 and small s_{13}^2 limits respectively. It is sufficient to consider these limits as the allowed values of s_{13}^2 are confined in these ranges. As seen in (4), the parametrisation chosen for the mixing matrix U ensures that the probability relevant for LSND is independent of the mixing angle s_{23}. From the value of $P_{\overline{\nu}_\mu \overline{\nu}_e}$ quoted by the LSND group [1] one can find the allowed area in the s_{12}^2 - s_{13}^2 parameter space for fixed values of the ratio $\Delta m^2 L/E$. The following constraints are found: for s_{12} very small (~ 0) or very large (~ 1), s_{13} ranges from $0 \leq s_{13} < 1$ while for intermediate values of s_{12}, only very large s_{13} values are allowed. This is between the solid lines in fig. 1 (2) for large (small) values of s_{13}, in the limit of $\sin^2(\pi L/\lambda_{12}) \sim 1$. From eq. (5) the probability for Bugey is also a function of the same mixing angles s_{12} and s_{13} only, so that, using their result one can further rule out a portion of the parameter space – namely, intermediate s_{13} values at small s_{12} – which were allowed by LSND. This is shown by the dashed lines in figs. 1 and 2 implying the following limits for small $s_{12}^2 (< 0.0018)$: either $s_{13}^2 >\sim 0.97$ (fig. 1) or $s_{13}^2 < 0.026$ (fig. 2). In the other regions of the parameter space the LSND data puts more severe constraints than Bugey.

Our approach next is to determine how much of the combined allowed area from LSND and Bugey is consistent with the atmospheric data for fixed values of s_{23}. The sub-GeV Kamiokande data implies

$$0.48 \leq R \leq 0.73 \; (90\%\text{C.L.}) \tag{10}$$

Imposing this constraint, one finds that the allowed parameter space (shown shaded in figs. 1 and 2) depends on the chosen s_{23}^2. In general there are three regions:
(i) The large $s_{13}^2 (>\sim 0.97)$ - small $s_{12}^2 (<\sim 0.1)$ region shown in fig. 1. In this limit it is the $\nu_\mu - \nu_e$ oscillation that dominates. Considering the limiting case of $s_{12} \to 0$ and $s_{13} \to 1$, the relevant probabilities assume the forms:
$P_{\nu_e \nu_e} \simeq 1 - 2c_{23}^2 s_{23}^2$, $P_{\nu_\mu \nu_e} \simeq 2c_{23}^2 s_{23}^2$, $P_{\nu_\mu \nu_\mu} \simeq 1 - 2c_{23}^2 s_{23}^2$
From these expressions it is clear that in this limit $P_{\nu_\mu \nu_\tau} \simeq 0$.
(ii) The large s_{13}^2 and intermediate s_{12}^2 zone also shown in fig. 1. To understand the

transitions in this range we examine the various probabilities in the limit $s_{13}^2 \to 1$. In this limit eqs. (7) - (9) become

$P_{\nu_e\nu_e} \simeq 1 - 2c_{23}^2 s_{23}^2$, $P_{\nu_\mu\nu_e} \simeq 2c_{12}^2 c_{23}^2 s_{23}^2$, $P_{\nu_\mu\nu_\mu} \simeq 1 - 2c_{12}^2 s_{12}^2 - 2c_{12}^4 c_{23}^2 s_{23}^2$

This is the region where the depletion can be due to both the channels simultaneously excepting in the special case of $s_{23} \to 0$ when this reduces to solely $\nu_\mu - \nu_\tau$ oscillation. From fig. 1 one also notices that irrespective of the choice of s_{23}, large values of s_{12}^2 around $\sim$ (0.85-1) are disfavoured by the atmospheric data. In this region $\nu_e - \nu_\tau$ conversion is effective.

(iii) The small s_{12}^2 - s_{13}^2 zone – $0 < s_{12}^2 < 1.8 \times 10^{-3}$, $0 \leq s_{13}^2 \leq 0.01$ – a look at the various survival and transition probabilities reveals that this is a region where the depletion is mainly due to $\nu_\mu - \nu_\tau$ oscillation. This can be easily seen by considering the limiting cases s_{12}, $s_{13} \to 0$, when eqs. (7) - (9) give $P_{\nu_e\nu_e} \simeq 1$, $P_{\nu_\mu\nu_e} \simeq 0$, $P_{\nu_\mu\nu_\mu} \simeq 1 - 2c_{23}^2 s_{23}^2$. Substituting these in (10) one finds $0.162 < s_{23}^2 < 0.838$. There is a sharp cut-off as s_{23}^2 crosses 0.162 and for practically all intermediate values upto 0.838, the whole of the parameter space allowed by LSND and Bugey is consistent with the atmospheric neutrino data. Thus in this regime we show the allowed region for only one representative s_{23}^2. We have numerically checked that the allowed region is the same as the one presented in fig. 2 for all other s_{23}^2 in the above range.

In our analysis we have fixed $\Delta_{12} \simeq \Delta_{13}$ at $6eV^2$, where LSND is most sensitive and $\sin^2(\pi L/\lambda_{12}) \to 1$, maximising the oscillatory term. As discussed in [4] it remains to be seen what best-fit value, consistent with KARMEN and BNL-E776, emerges when more data is accumulated. Our results remain unchanged as long as it is permissible to use the above limit.

For the atmospheric neutrino case we approximate the $\sin^2(\pi L/\lambda_{23})$ factor by its averaged value 0.5 as is often done in the context of the sub-GeV data [18, 19, 20]. This approximation can be improved by an averaging over the incident neutrino energy spectrum, the zenith-angle of the beam as well as the final lepton energy [18, 10]. r_{MC} is taken to be 0.45 from a detailed Monte-Carlo simulation including the effects of muon polarisation [21].

Finally let us discuss the implications of the parameter space allowed by the Kamiokande atmospheric neutrino, LSND and Bugey data for the $\nu_\mu - \nu_\tau$ oscillation search at CHORUS [22] and NOMAD [23]. These experiments use the ν_μ beam from the CERN SPS with the mean energy ~ 30 GeV and the approximate source-detector distance is 0.8 km so that $\lambda_{23} >> L$ and

$$P_{\nu_\mu \nu_\tau} = 4c_{12}^2 s_{12}^2 s_{13}^2 \sin^2(\pi L/\lambda_{13}) \tag{11}$$

With the CERN SPS designed to deliver 2.4 $\times 10^{19}$ protons, CHORUS and NOMAD are sensitive to a minimum oscillation probability of 10^{-4}. For $\Delta_{12} = \Delta_{13}$ in the LSND range of $\sim 6eV^2$, $\sin^2(\pi L/\lambda_{13}) \sim 0.04$, whence (11) is $P_{\nu_\mu\nu_\tau} \simeq 0.16 c_{12}^2 s_{12}^2 s_{13}^2$. Then for the three allowed regions in the $s_{12}^2 - s_{13}^2$ plane one gets:

(i) In the large s_{13}^2, small s_{12}^2 zone $P_{\nu_\mu\nu_\tau}$ can be slightly greater than 10^{-4} being marginally within the reach of these experiments. This is the $\nu_\mu - \nu_e$ oscillation zone for atmospheric neutrinos.

(ii) For large s_{13}^2 and intermediate values of s_{12}^2, $P_{\nu_\mu\nu_\tau}$ is $\sim 10^{-2}$, which is well within the reach of CHORUS and NOMAD. Recall that this is the genuine three generation oscillation regime for atmospheric neutrinos where both $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_e$ modes are operative, excepting for the special case of $s_{23} \simeq 0$ for which it is just $\nu_\mu - \nu_\tau$.

(iii) In the limit of both s_{12}^2, s_{13}^2 small, $P_{\nu_\mu\nu_\tau}$ is very small and this regime, where the atmospheric anomaly is due to $\nu_\mu - \nu_\tau$ oscillation, cannot be probed by CHORUS and NOMAD.

For the chosen values of the mass-differences a simultaneous solution to the solar neutrino problem is unobtainable unless one invokes a sterile neutrino. Work is in progress in this direction [24].

In conclusion, we have obtained the mixing angles compatible with atmospheric, LSND and reactor experiments (in particular Bugey) in the context of three flavour mixing. Keeping Δm^2 fixed at the best fit values obtained from two generation analyses of the LSND and atmospheric data, we find three regions of parameter space that can account for all three experiments simultaneously. Our results differ from an analysis presented in [9] in that we find the mixing angles to be less restricted. Our method, which takes into account the possibility that the depletion of the atmospheric neutrinos can be simultaneously due to $\nu_\mu - \nu_e$ and $\nu_\mu - \nu_\tau$ oscillations, is more general and includes the constraints obtained in [9] as a special case. A direct comparison of the values obtained for the mixing angles is, however, not proper because the definitions of the mixing matrices are different. The sensitivity of the accelerator based neutrino oscillation experiments at CERN, CHORUS and NOMAD, is different in the three allowed zones and thus they can distinguish between these sectors of the parameter space. We find that CHORUS and NOMAD are most sensitive to that part of the combined allowed area where the atmospheric neutrino anomaly is due to $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_e$ oscillation modes simultaneously.

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FIGURE CAPTIONS

Figure 1: The 90 % C.L. allowed region in the $s_{12}^2 - s_{13}^2$ plane from LSND is between the solid lines, that from Bugey is above the dashed line while the combined allowed area including the Kamiokande sub-GeV data is shown shaded.

Figure 2: Same as in fig. 1 excepting the region allowed from Bugey is below the dashed line.

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