

Unified resolution of the $R(D)$ and $R(D^*)$ anomalies and the lepton flavor violating decay $h \rightarrow \mu\tau$

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Abstract

Taking advantage of the fact that the flavor of the neutrino in semileptonic B decays $B \rightarrow D^{(*)}\tau\nu$ is not known, we show how a minimal set of higher-dimensional lepton flavor violating (LFV) operators can explain the $R(D^{(*)})$ anomalies, and as a spin-off, can give rise to the LFV decay of the Higgs boson, $h \rightarrow \mu\tau$. We also show how none but the minimal set of operators survive the present data.

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1 Introduction

The search for signals of lepton flavor violation (LFV) has been a long and varied quest, for it is believed to not only constitute a smoking gun for new physics (NP) beyond the Standard Model (SM), but also shed light on a variety of issues ill-understood within the SM, such as the origin of flavor on the one-hand and the generation of non-zero lepton and baryon number in the universe, on the other. While the SM can incorporate LFV, as seen, e.g., in neutrino oscillations, by the mere inclusion of right-handed neutrino fields and consequent Dirac masses, the corresponding LFV amplitudes would be too small to be manifested in processes involving charged leptons¹. Even the proposed upgrades, or new experiments, are expected to improve the limits on LFV processes by at most one order of magnitude, except for $\mu \rightarrow 3e$ and μ - e conversion [1]. Indeed, if decays such as $\mu \rightarrow e\gamma$ or $\tau \rightarrow 3\mu$ are seen in experiments currently in operation or due to start in the near future, the corresponding amplitudes would be too large to be supported by such trivial extensions of the SM.

It is in this context that the recently reported [2] hint, from the CMS experiment, of the Higgs boson decay $h \rightarrow \mu\tau$ is to be viewed. If this is not a mere background fluctuation but an actual signal, one has to entertain the possibility that such LFV decays are flavor-specific, as neither CMS nor ATLAS has seen any LFV in channels like $h \rightarrow e\tau$ or $h \rightarrow e\mu$ [3]. This, however, is not unnatural, simply because such a decay is quite likely to be generated from Yukawa couplings, and the latter are believed to be

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¹It should also be noted that total lepton number conservation is an accidental symmetry within the SM, and that the inclusion of right-handed neutrino fields would allow for unsuppressed Majorana masses as well (unless a global $U(1)_L$ is imposed), thereby further enriching the neutrino mass sector. With the Majorana/Dirac masses suffering only logarithmic corrections, ascribing appropriate (small) values to these is technically natural.

typically stronger for the higher generations, even in extensions of the SM. While the results from the ATLAS experiment on $h \rightarrow \mu\tau$ are more or less consistent with zero, these too can allow for a nontrivial branching ratio (BR) for this channel. The measurements have yielded [2, 4]

$$\text{BR}(h \rightarrow \mu\tau) = 0.84_{-0.37}^{+0.39}\% \text{ (CMS)}, \quad 0.53 \pm 0.51\% \text{ (ATLAS)}, \quad (1)$$

so that the 95% CL upper limits on the BR are 1.51% (CMS) and 1.41% (ATLAS) respectively.

While the CMS measurement *per se* does not call for new physics right away, it is interesting to juxtapose it against another long-standing anomaly, albeit in a completely different sector. The ratios of the partial widths of B mesons, $R(D)$ and $R(D^*)$, defined as

$$R(D^{(*)}) = \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu)}{\Gamma(B \rightarrow D^{(*)}\ell\nu)}, \quad (2)$$

(with $\ell = e, \mu$) are particularly clean probes of physics beyond the SM, on account of the cancellation of the leading uncertainties inherent in individual BR predictions. The values of $R(D)$ and $R(D^*)$ as measured by *BABAR* [5], when taken together, exceed SM expectations by more than 3σ , which generated interest in the first place. Furthermore, the Belle measurements for the same observables lie in between the SM expectations and the *BABAR* measurements and are consistent with both [6]. Recently, Belle has published their new result on $R(D^*)$ [7] with τ decaying semileptonically, and this agrees with the SM expectations only at the 1.6σ level, while the first measurement by LHCb [8] is also 2.1σ above the SM prediction. Taking all the results together, including the correlations, the tension between data and SM is at the level of 3.9σ . On the other hand, the recent results on the measurement of τ -polarization for the decay $B \rightarrow D^*\tau\nu$ in Belle [9] are consistent with the SM predictions, albeit with only a large uncertainty.

While the “anomalies” in either of $R(D)$ and $R(D^*)$ do not call for LFV, clearly they seem to be associated with a loss of lepton universality, and involving the very same fermions as the anomalous decay. It is therefore conceivable that the individual excesses, intriguing in their own right but not calling out for a rejection of the SM, are, together, indicative of some new physics. A combined approach to treat both these anomalies together within the scope of a particular model may be found in Ref. [10]². At this point, we may refer the reader to Refs. [12, 13], and the references therein, for a detailed analysis of the NP operators. In this paper, we investigate this more closely, coupled with the LFV Higgs decays. In particular, if anomalous Higgs interactions are indeed called for, we show that the difference between the chiral structure of the ensuing four-fermi operators and that of the SM operator could possibly explain why the experimental discrepancies are seen only in certain channels.

The generation of such LFV decays of the Higgs is relatively simple if the scalar sector is enlarged, as in a Type-III two-Higgs doublet model wherein the 125 GeV scalar has a tiny component of the field responsible for the LFV decays [14]. A variation is afforded by scenarios [15] wherein there are two or more nearly degenerate scalars with one of them being SM-like and the other(s) having explicitly LFV couplings. On the other hand, lepton flavor non-universality can appear in many a guise, whether it be through Higgs couplings or through gauge couplings in a theory with extended symmetry or even through the exchange of other non-standard particles such as superpartners in a supersymmetric extension of the SM, or leptoquarks. Hence, rather than adopt any particular scenario, we investigate the *structure* of the minimal alteration to the SM that can satisfactorily explain the

²There have been numerous attempts to relate the $R(D^{(*)})$ anomaly with some other anomalous observables, see, *e.g.*, Ref. [11].

anomalies while remaining consistent with the rest of low-energy phenomenology. In other words, we advocate a bottom-up approach, starting with an effective theory.

In this paper, we would like to investigate whether both these decays, namely, $h \rightarrow \mu\tau$ and $B \rightarrow D^{(*)}\tau\nu$ can be simultaneously affected by a single four-fermion operator, keeping the scalar sector to be completely SM-like at the electroweak scale. There are at least two points worth emphasizing, so let us note them down here.

- If the scalar sector is completely SM-like at all energies, *i.e.*, if the mass matrix and the Yukawa matrix are proportional, there can be no flavor-changing coupling of the Higgs boson of the form $h\bar{f}_i f_j$ with $i \neq j$, even at the one-loop level. This is in contradiction to what has been claimed in, for example, Refs. [16,17]. The reason is not difficult to understand: as soon as one generates an off-diagonal Yukawa coupling h_{ij} , an analogous term $m_{ij} = v h_{ij}$ is also generated in the mass matrix, where v is the vacuum expectation value (VEV) for the CP-even neutral component of the SM Higgs field Φ . Thus, one needs to redefine the stationary basis for the fermions again, and in that new basis, such off-diagonal effective Yukawa couplings no longer exist. However, there are possible ways out [18,19], and we will later show, with a toy model, how to achieve this. In this sense, we demonstrate how to generate the LFV decay of the Higgs boson without introducing any low-energy extension of the scalar sector.
- NP has to be there in some form or other at some high scale, but if the low-energy sector is SM-like, then any new state can exist only at a scale $\Lambda \gtrsim \mathcal{O}(1 \text{ TeV})$, the natural scale for NP. It is possible, though, that NP can appear at several (well-separated) scales, with the aforementioned Λ being the lowest of them all.

Here, we will focus on some possible dimension-6 four-fermion operators to explain both the anomalies, relating the charged current operator $b \rightarrow c\tau\nu$ with the neutral current operator, that produces $\tau\mu$ in the final state, through $SU(2)_L$. We will take advantage of two facts: first, the quark mixing in the right-chiral sector is essentially unconstrained, and second, the flavor of the neutrino that comes out in semileptonic B decays is not determined. While a similar exercise using higher dimensional effective operators has been performed [20], it was restricted only to the B -sector observables. The novelty, in our approach, lies in that we do not consider any extension of the SM scalar sector, and the Yukawa couplings remain unchanged. As we will show, the new operators that we consider produce an effective $h\mu\tau$ vertex, which we illustrate with the help of a toy model. Showing how experimental constraints already rule out most of the possible operators, we identify the minimal set of operators necessary to explain the anomalies.

The paper is arranged as follows. In Section II, we will first describe a toy model to generate flavor-changing Higgs couplings with lowest dimensional effective operators, and then elaborate our model. In Section III, we show how it affects the LFV Higgs decay $h \rightarrow \mu\tau$, and semileptonic B decays are treated in Section IV. We summarize and conclude in Section V.

2 The formalism

Assuming that the (low-energy) scalar sector is just as in the SM, the only way to explain a LFV decay of the Higgs boson h (such as the one under discussion) would be to postulate a term $\left[-y_{ij}\bar{\ell}_L^i \ell_R^j h + \text{h.c.}\right]$

(with $i \neq j$), in the Lagrangian, keeping in abeyance, for the time being, any discussion of the source of this term. Written in full, the relevant term is

$$- y_{\mu\tau} (\bar{\mu}_L \tau_R + \bar{\tau}_R \mu_L) h - y_{\tau\mu} (\bar{\tau}_L \mu_R + \bar{\mu}_R \tau_L) h, \quad (3)$$

and the corresponding branching fraction is given by

$$\text{BR}(h \rightarrow \mu\tau) = \frac{m_h}{8\pi\Gamma_h} (|y_{\tau\mu}|^2 + |y_{\mu\tau}|^2), \quad (4)$$

where $y_{\mu\tau}$ and $y_{\tau\mu}$ are effective Yukawa couplings, which need not be equal, or even of the same magnitude. If $h \rightarrow \mu\tau$ (and other possible new decay channels) have only a small BR, one can assume $\Gamma_h \approx \Gamma_h^{\text{SM}} \approx 4.07 \text{ MeV}$ for $m_h \approx 125 \text{ GeV}$.

If the scalar sector (both the field content and interactions) is restricted to being exactly as in the SM, clearly, terms as in Eq. (3) cannot occur at the tree-level. They may appear as quantum corrections though, and the required size clearly does not preclude this. However, for even this to work, either the field content of the theory has to be enlarged or non-renormalizable interactions introduced or both.

2.1 Flavor-changing Higgs couplings: A toy model

As was discussed earlier, one cannot simply postulate such an off-diagonal coupling for the Yukawa and the mass matrices often turn out to be proportional to each other (not only at the tree level, but to any given order in perturbation theory). To circumvent this argument, let us consider a toy model. Suppose the Lagrangian contains dimension-5 terms like

$$\frac{1}{\Lambda} \left[a_t \bar{t}_R Q_L \tilde{\Phi} X + a_l \bar{\tau}_R L_L \Phi X^* \right] + \text{H.c.} \quad (5)$$

where Φ is the SM doublet ($\tilde{\Phi} = i\sigma_2 \Phi^*$), and X is a complex $\text{SU}(2)_L$ triplet with hypercharge $Y = 2$. We will assume that the mass-squared term for X is positive and $\mathcal{O}(\text{TeV}^2)$. Consequently, the components of X receive no vacuum expectation value, thereby trivially satisfying the constraints from the ρ -parameter. A further consequence is that they are almost degenerate in mass, which allows the scenario to evade the remaining constraints from electroweak precision observables. Λ above is a cutoff scale, with $\Lambda \gg m_X$ so as to validate the effective Lagrangian approach.

Written in full, with $X = (x^{++}, x^+, x_0)$, the relevant terms look like

$$\mathcal{L} \supset \frac{1}{\sqrt{3}\Lambda} \left[a_t \left(\bar{t}_R t_L \phi^{0*} x_0 - \frac{1}{\sqrt{2}} \bar{t}_R b_L \phi^{0*} x^+ \right) + a_l \left(\bar{\tau}_R \mu_L \phi^0 x_0^* - \frac{1}{\sqrt{2}} \bar{\tau}_R \nu_{\mu L} \phi^0 x^- \right) \right] + \text{H.c.} \quad (6)$$

Integrating out the X fields yields a dimension-8 term in the Lagrangian of the form

$$\frac{-a_t a_l}{3\Lambda^2 m_X^2} |\phi^0|^2 \bar{t}_L t_R \bar{\mu}_L \tau_R + \text{h.c.}, \quad (7)$$

valid at scales well below m_X . Here, analogous terms involving the putative Goldstones have been suppressed. On the breaking of the electroweak symmetry, one may write $\phi^0 = (h + v)/\sqrt{2}$, with h being the physical Higgs field. This yields not only a four-Fermi term of the form

$$\mathcal{L}_{4\text{fer}} = \frac{a_t a_l}{6\Lambda^2} \frac{v^2}{m_X^2} (\bar{t}_L t_R) (\bar{\tau}_R \mu_L) + \text{H.c.}, \quad (8)$$

but also couplings of the same set of fields with both a single higgs and a pair of higgses, or, in other words, a five-field and a six-field vertex each. Of immediate concern are the first two of these terms. Clearly the $(2vh)\bar{t}_L t_R \bar{\mu}_L \tau_R$ term, on contracting the top-fields, would lead to an effective LFV coupling $h\bar{\mu}_L \tau_R$. Similarly, the term in Eq. (8) would contribute to an off-diagonal mass term connecting the muon and the tau. Importantly, these one loop contributions to the Yukawa and the mass matrices bear a relation different from the tree-level terms, viz. $\delta y_{\mu\tau} = 2\delta m_{\mu\tau}/v$. The extra factor of 2 destroys the overall proportionality of the Yukawa and the mass matrices, thereby allowing for a LFV Higgs coupling when the fermions are rotated into the stationary basis.

The evaluation of the loop contributions is quite straightforward. While they are, formally, quadratically divergent, it needs to be realized that the effective theory under consideration has a natural cutoff at m_X . The leading term, apart from the overall coupling, is thus $-4N_c m_X^2 m_t / 16\pi^2$, where the minus sign comes from the fermion loop and m_t from the chirality flip. Thus, the effective LFV Yukawa coupling is given by

$$\frac{1}{2} \times \frac{a_t a_l v}{3\Lambda^2} \frac{N_c}{4\pi^2} m_t \bar{\mu}_L \tau_R h. \quad (9)$$

The factor of half needs explaining. As mentioned above, the term proportional to v^2 generates an off-diagonal term in the mass matrix and, consequently, an extra rotation is needed to get back to the new mass basis. This absorbs half of the effect (which is why a coupling proportional to $(h+v)$ cannot lead to flavor-changing Yukawa couplings), leaving us with the remaining half.

It should be noted that much the same low-energy phenomenology could have been obtained, had we started with an $Y=0$ triplet instead, with the Lagrangian now being

$$\frac{1}{\Lambda} \left[a_t \bar{t}_R Q_L \Phi X + a_l \bar{\tau}_R L_L \tilde{\Phi} X^* \right] + \text{H.c.} .$$

Similarly, had we started with a scalar leptoquark field, coupling to both a t - τ and a t - μ current, the ensuing effective Lagrangian, on Fierz-rearrangement, would yield terms analogous to those above, but with (axial-)vector couplings instead.

2.2 The minimal operator basis

Having argued that it is indeed possible to generate flavor-changing Higgs couplings (for a theory with a single scalar doublet) within the stationary basis, and that this may be achieved quite naturally within the paradigm of an effective theory, we now turn to the other anomalies at hand, namely $R(D^{(*)})$. To this end, we augment the SM by postulating at most a couple of effective dimension-6 operators obeying the full symmetry of the SM. These operators will be shown to generate an effective $h\mu\tau$ vertex, by a mechanism similar to that outlined above, which is of the right magnitude. While a similar approach was adopted in Ref. [21] to explain $h \rightarrow \mu\tau$ alone, we go much beyond and relate the operators to the anomalies in $R(D)$ and $R(D^*)$.

Following Refs. [22, 23], let us consider an effective charged-current Hamiltonian of the form

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} [O_{\text{SM}} + C_{S_1} O_{S_1} + C_{S_2} O_{S_2} + C_T O_T], \quad (10)$$

where

$$\begin{aligned}
O_{\text{SM}} &= (\bar{c}_L \gamma^\lambda b_L)(\bar{\tau}_L \gamma_\lambda \nu_{\tau L}), \\
O_{S_1} &= (\bar{c}_L b_R)(\bar{\tau}_R \nu_{\mu L}), \\
O_{S_2} &= (\bar{c}_R b_L)(\bar{\tau}_R \nu_{\mu L}), \\
O_T &= (\bar{c}_R \sigma^{\mu\lambda} b_L)(\bar{\tau}_R \sigma_{\mu\lambda} \nu_{\mu L}),
\end{aligned} \tag{11}$$

and the fermion fields are weak-eigenstates, as befits operators in an effective theory defined above the electroweak scale. While O_{S_1} and O_{S_2} might result from the mechanism discussed in the previous subsection (albeit with different fermionic fields), the generation of O_T is more non-trivial, and the ultraviolet completion of the same would, typically, require the introduction of exotic fields³, such as a doublet scalar leptoquark with a hypercharge of $\frac{7}{6}$. Note that this set is not exactly identical to that given in Ref. [23]. For one, the new operators contain ν_μ instead of ν_τ . With the neutrinos in a decay being unidentified, this does not affect the analysis of $R(D)$ and $R(D^*)$ except for the fact that, now, no interference between the SM operator O_{SM} and the new operators would exist. Furthermore, we have dropped some operators, involving (axial-)vector currents, as they (to be demonstrated shortly) not only do not lead to $h \rightarrow \mu\tau$, but, in addition, cause disagreements with other observables. Later on, we will show that C_{S_1} should be of the order of unity to produce a good fit with the data, and it is almost trivial to show that this leads to an unacceptably large contribution to the decay $B_s \rightarrow \mu\tau$, which is yet to be observed. Thus, even the operator O_{S_1} falls out of favor, but we will keep this in our analysis for the time being.

The origin of the specific set of operators is, of course, uncertain. Given that the family number is conserved, it is quite conceivable, for example, that these arise on account of flavor dynamics. We do not, however, attempt to answer such questions, but only offer the argument that this leads us to the minimal set of new operators required to explain the data. To further reduce the number of free parameters, we shall consider an additional simplification and consider two reduced sets, namely

- Model 1: $C_{S_1}, C_{S_2} \neq 0, C_T = 0$;
- Model 2: $C_{S_2}, C_T \neq 0, C_{S_1} = 0$.

In other words, only two new Wilson coefficients are introduced in each case.

The new operators also imply the existence of their SU(2) conjugates, with identical Wilson coefficients, namely

$$\begin{aligned}
O'_{S_1} &= (\bar{s}_L b_R)(\bar{\tau}_R \mu_L), \\
O'_{S_2} &= (\bar{c}_R t_L)(\bar{\tau}_R \mu_L), \\
O'_T &= (\bar{c}_R \sigma^{\mu\lambda} t_L)(\bar{\tau}_R \sigma_{\mu\lambda} \mu_L).
\end{aligned} \tag{12}$$

This, immediately, puts into perspective our earlier assertion about O_{S_1} being highly constrained, for O'_{S_1} would readily generate semileptonic LFV decays like $B \rightarrow K^{(*)} \tau \mu$ and the purely leptonic decay $B_s \rightarrow \tau \mu$. In fact, if the corresponding Wilson coefficient C_{S_1} is of order unity, the BR of $B_s \rightarrow \tau \mu$ becomes so large ($\sim \mathcal{O}(0.1)$) that it should certainly have been observed. Thus, unless C_{S_1} is of the order of at least 10^{-3} , it is hard, but not entirely impossible, to entertain O_{S_1} (and hence Model I as mentioned before) as a possible candidate for the minimal set of operators.

³It should be noted, though, that such a rendition would require the simultaneous introduction of other operators as well.

Having been written in terms of the weak-interaction eigenstates, the operators need to be re-expressed in terms of the stationary states (i.e., the mass eigenstates). With the fermion mass-matrices being diagonalized through a bi-unitary transformation, we have, in principle, as many as four 3×3 unitary matrices ($U_{L,R}$, $D_{L,R}$) in play, one each for the (left-) right-handed (up-) down-quarks. Thanks to the right-handed fields being $SU(2)_L$ singlets and universality of the gauge-structure across generations, within the SM, two of these matrices (U_R and D_R) play no dynamic role, and only the combination $U_L^\dagger D_L$ is manifested physically (as the Cabibbo-Kobayashi-Maskawa matrix). In the presence of these new operators, this would no longer be the case. In particular, both of U_R and D_R would now play a nontrivial role. Once again, rather than consider the most general case, we simplify the analysis by retaining only the most important term, namely

$$c_R = \cos \alpha c'_R + \sin \alpha t'_R, \quad t_R = -\sin \alpha c'_R + \cos \alpha t'_R, \quad (13)$$

where the primed fields are in the mass basis. This immediately leads to

$$\begin{aligned} O_{S_2} &= \cos \alpha (\bar{c}'_R b_L)(\bar{\tau}_R \nu_{\mu L}) + \dots, \\ O_T &= \cos \alpha (\bar{c}'_R \sigma^{\mu\lambda} b_L)(\bar{\tau}_R \sigma_{\mu\lambda} \nu_{\mu L}) + \dots, \\ O'_{S_2} &= \sin \alpha (\bar{t}'_R t_L)(\bar{\tau}_R \mu_L) + \dots, \\ O'_T &= \sin \alpha (\bar{t}'_R \sigma^{\mu\lambda} t_L)(\bar{\tau}_R \sigma_{\mu\lambda} \mu_L) + \dots. \end{aligned} \quad (14)$$

The left-chiral quark fields are also rotated to the mass basis as per the Cabibbo-Kobayashi-Maskawa paradigm. These rotations have important physical consequences. For example, even if the mixing is confined to the down quark sector alone, O'_{S_1} , after field rotation, can lead to $\Upsilon \rightarrow \mu\tau$, which, within the SM, is highly suppressed compared to the electromagnetic decay $\Upsilon \rightarrow \ell^+ \ell^-$. This particular mode, though, is not very restrictive once the aforementioned constraints from $B_s \rightarrow \tau\mu$ are satisfied. Similarly, if the mixing is for the up-type quarks, O'_{S_2} and O'_T can lead to LFV charmonium decays, which are also yet to be observed. While eq. (14) lists all the operators relevant for our study, it is instructive, at this stage, to examine the ramifications thereof. Clearly, engendering the flavor-changing Yukawa coupling $h\bar{\mu}\tau$ by Wick-contracting the top-fields is possible only for O'_{S_2} . Thus, only this operator (and its sibling, O_{S_2}) are relevant for this aspect. On the other hand, $O_{S_1}^{(\prime)}$ and $O_T^{(\prime)}$ appear at the same order in the effective theory and, like $O_{S_2}^{(\prime)}$, can contribute to both $R(D)$ and $R(D^*)$. Thus, the inclusion of at least two operators is necessary to maintain agreement for these decays.

Before we end this section, we would like to point out that, in obtaining the operators in Eq. (12) from those in Eq. (11) through basis transformations, we would also generate many other operators, designated by the ellipses in Eq. (12). These would have their own consequences, such as the FCNC top decay $t \rightarrow c\mu\tau$. We have checked that, for the sizes of the Wilson coefficients (C_{S_2} accompanied by one of C_{S_1} and C_T) that we would need, such effects are negligible.

3 LFV decays of the Higgs

The presence of an operator such as $(\bar{f}\Gamma_a f)(\bar{\tau}\Gamma_a \mu)$, where f is a SM fermion and Γ_a a Dirac matrix, denotes the violation of both N_τ and N_μ while preserving their difference. Clearly, this can result in $h \rightarrow \mu\tau$, at least at the loop-level. Fig. 1 shows two typical diagrams, in the context of the toy model discussed before, that contributes to such a process.

It is easy to see that O_T cannot contribute to this amplitude, for, to obtain a $h\mu\tau$ vertex, we would need to contract the leptonic current with two external momenta which, of course, is not possible. For (axial-)vector operators (not listed in Eq. (11)), on the other hand, only one such contraction is needed and, consequently, the amplitude is proportional to the lepton mass. Furthermore, the very structure of the operator ensures that the loop integral is logarithmically divergent and scales only as $m_X^{-2} \ln(m_h^2/m_X^2)$ at the most. While this suppression is not necessarily an overwhelming one (provided m_X is not too large), it should be realized that corresponding diagrams exist where the Higgs field is replaced by the Z . The latter would lead to an unsuppressed contribution [24] to the decay $Z \rightarrow \tau\mu$, well beyond the experimental limits, unless the Wilson coefficient for the four-fermion interaction is suppressed enough. This, though, would imply that the operator has a negligibly small effect in Higgs decays.

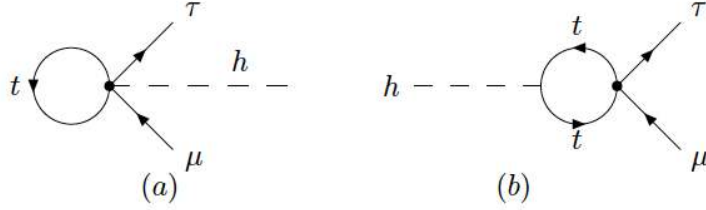


Figure 1: Typical contributions to the decay $h \rightarrow \mu^+\tau^-$ initiated by the new operators. Diagrams for the conjugate process would be analogous.

This leaves us with the (pseudo-)scalar operators O_{S_1} and O_{S_2} . Let us concentrate on the latter, and take our toy model as a concrete example. This gives

$$\frac{4G_F}{\sqrt{2}} V_{cb} C_{S_2} \sin \alpha = \frac{a_t a_l}{6\Lambda^2} \frac{v^2}{m_X^2}, \quad (15)$$

and hence, the first diagram of Fig.1 yields

$$y_{\mu\tau} = \frac{G_F}{\sqrt{2}\pi^2} m_X^2 V_{cb} N_c \frac{m_t}{v} C_{S_2} \sin \alpha \approx 0.076 \left(\frac{m_X}{1 \text{ TeV}} \right)^2 C_{S_2} \sin \alpha, \quad (16)$$

where $N_c = 3(1 + \alpha_s/\pi) \approx 3.11$ is the effective number of colors, and $h_t \approx 1$ is the top quark Yukawa coupling. We have also used $m_t = 175 \text{ GeV}$, $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$, and $|V_{cb}| = (41.1 \pm 1.3) \times 10^{-3}$. The contribution of the second diagram of Fig. 1 is further suppressed by a factor of $\sim v/m_X$. This gives

$$\text{BR}(h \rightarrow \mu\tau) \approx 7.1 \left(\frac{m_X}{1 \text{ TeV}} \right)^4 [C_{S_2} \sin \alpha]^2 < 0.014 \Rightarrow C_{S_2} \sin \alpha < 4.4 \times 10^{-2} \left(\frac{1 \text{ TeV}}{m_X} \right)^2. \quad (17)$$

Thus, if $|C_{S_2}|$ is of order unity, one needs a small mixing in the t_R - c_R sector, namely, $\tan \alpha \sim 10^{-2}$, to explain the LFV Higgs decay. Note that while the estimation has been done for a particular toy model, the essence is model-independent.

4 The B -decay anomalies

In terms of the differential distributions $d\Gamma/dq^2$ for the decay $B \rightarrow X\ell\nu$, where $q_\mu \equiv (p_B - p_X)_\mu$ is the momentum transfer, the ratios $R(D)$ and $R(D^*)$ are defined as

$$R(D^{(*)}) = \left[\int_{m_\tau^2}^{q_{max}^2} \frac{d\Gamma(\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}_\tau)}{dq^2} dq^2 \right] \left[\int_{m_\ell^2}^{q_{max}^2} \frac{d\Gamma(\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}_\ell)}{dq^2} dq^2 \right]^{-1} \quad (18)$$

with $q_{max}^2 = (m_B - m_{D^{(*)}})^2$, and $\ell = e$ or μ . In each case, both isospin channels are taken into account. Using the effective Hamiltonian in Eq. (10), the expressions for these distributions are given as

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D\tau\bar{\nu}_\tau)}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} q^2 \sqrt{\lambda_D(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times \left\{ \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) H_{V,0}^{s,2} + \frac{3}{2} \frac{m_\tau^2}{q^2} H_{V,t}^{s,2} \right] \right. \\ &\quad \left. + \frac{3}{2} |C_{S_1} + C_{S_2}|^2 H_S^{s,2} + 8 |C_T|^2 \left(1 + \frac{2m_\tau^2}{q^2}\right) H_T^{s,2} \right\}, \end{aligned} \quad (19)$$

and

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau)}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} q^2 \sqrt{\lambda_{D^*}(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \\ &\quad \times \left\{ \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) (H_{V,+}^2 + H_{V,-}^2 + H_{V,0}^2) + \frac{3}{2} \frac{m_\tau^2}{q^2} H_{V,t}^2 \right] \right. \\ &\quad \left. + \frac{3}{2} |C_{S_1} - C_{S_2}|^2 H_S^2 + 8 |C_T|^2 \left(1 + \frac{2m_\tau^2}{q^2}\right) (H_{T,+}^2 + H_{T,-}^2 + H_{T,0}^2) \right\}, \end{aligned} \quad (20)$$

with $\lambda_X(q^2) \equiv m_B^4 + m_X^4 + q^4 - 2m_B^2 m_X^2 - 2m_B^2 q^2 - 2m_X^2 q^2$. Here, H_i s are the respective form factors as defined within the Heavy Quark Effective Theory [25], and we use the values determined by the Heavy Flavor Averaging Group (HFAG) [26]. For more details, we refer the reader to Ref. [22]. While the results for the lighter leptons are obtained by substituting $m_\tau \rightarrow m_\ell \approx 0$, putting all the C_i s equal to zero would yield the SM results.

4.1 $R(D)$ and $R(D^*)$

Let us first focus on $R(D)$ and $R(D^*)$. Several experiments have measured these ratios, and the current status is summarized in Fig. 2 as well as in Table 1. However, while Table 1 includes the latest Belle result [9] on $R(D^*)$, Fig. 2 takes into account only the Belle update till August 2016. Though the change is quite small and can easily be neglected, we have used the updated result [9] in our analysis.

While the two scenarios ($C_{S_1} = 0$ vs. $C_T = 0$) are identical as far as $h \rightarrow \mu\tau$ is concerned, their effects are quite markedly different on $R(D^{(*)})$. We perform a χ^2 goodness-of-fit analysis to fit the new physics Wilson coefficients through their effects as summarized in Eqs. 19 and 20. In our analysis, we use the q^2 -integrated data on $R(D)$ and $R(D^*)$, given in Tables 1 and 2 for different isospin channels (*i.e.*, both B^+ and B^0 decays) with appropriate correlations wherever the data is available. However, we have not used the isospin-constrained data measured by *BABAR* (given in Table 1) as an input in our analysis as those are not independent data-points. Our analysis involves 11 data-points: 4 from Ref. [5], 2 from Ref. [6], 2 from Ref. [30], and 1 each from Refs. [7], [8], and [9]. Ref. [30] supplies the

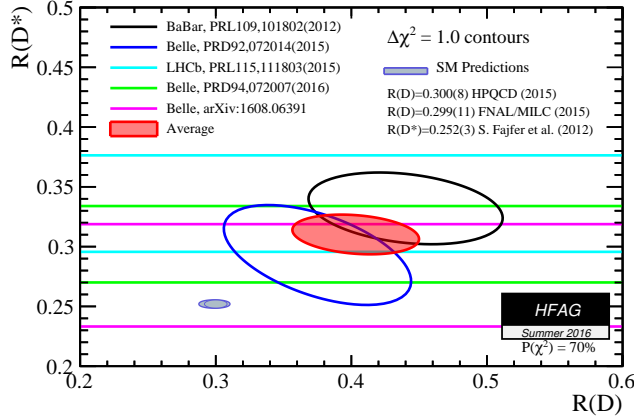


Figure 2: Current experimental status in the measurements of $R(D)$ and $R(D^*)$ [27].

	$R(D)$	$R(D^*)$
SM prediction	0.300 ± 0.008 [28]	0.252 ± 0.003 [29]
<i>BABAR</i> (Isospin constrained)	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018$ [5]
Belle (2015)	$0.375 \pm 0.064 \pm 0.026$	$0.293 \pm 0.038 \pm 0.015$ [6]
Belle (2016)	-	$0.302 \pm 0.030 \pm 0.011$ [7]
Belle (2016, Full Dataset)	-	$0.270 \pm 0.035^{+0.028}_{-0.025}$ [9]
LHCb	-	$0.336 \pm 0.027 \pm 0.030$ [8]

Table 1: The SM predictions for and the data on $R(D)$ and $R(D^*)$. While *BABAR* considers both charged and neutral B decay channels, LHCb and Belle results, as quoted here, are based only on the analysis of neutral B modes.

Experiment	Channel	$R(D^{(*)})$
<i>BABAR</i> [5]	$B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$	$0.429 \pm 0.082 \pm 0.052$
	$\bar{B}^0 \rightarrow D^+ \tau^- \bar{\nu}_\tau$	$0.469 \pm 0.084 \pm 0.053$
	$B^- \rightarrow D^{*0} \tau^- \bar{\nu}_\tau$	$0.322 \pm 0.032 \pm 0.022$
	$\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau$	$0.355 \pm 0.039 \pm 0.021$
Belle [30]	$B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$	0.339 ± 0.112
	$B^- \rightarrow D^{*0} \tau^- \bar{\nu}_\tau$	0.372 ± 0.071

Table 2: The measured values of $R(D^*)$ in different isospin channels. Only Belle 2010 and not the later Belle papers gives the isospin break-up.

data in the form of branching fractions. We have converted them to $R(D^{(*)})$ by normalizing them with $\text{BR}(B \rightarrow D^{(*)} \ell \nu)$ [31] while propagating the errors.

An important point to note is that the expressions depend only on $|C_{S_1}|$ and $|C_{S_2}|$ (or $|C_T|$ and $|C_{S_2}|$) and hence there is a fourfold ambiguity on the position of the minimum. This is best understood

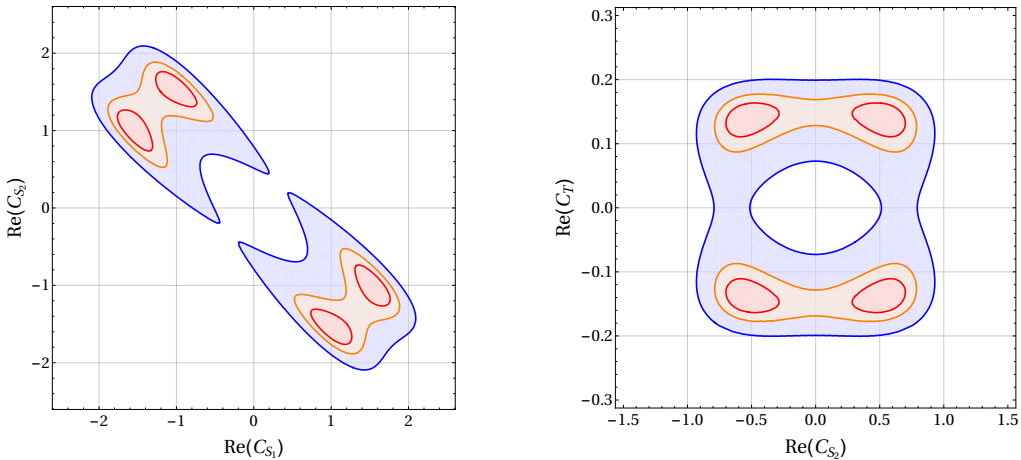


Figure 3: The χ^2 contours for Model 1 (left) and Model 2 (right). The 1σ (68.27%), 2σ (95.45%), and 4σ (99.99%) confidence levels are shown by red, orange, and blue lines respectively.

from the χ^2 contours shown in Fig. 3. For example, the best fit points are

Model 1 :

$$\begin{aligned}
 & C_{S_1} \cos \alpha = \pm(1.55 \pm 0.11), \quad C_{S_2} \cos \alpha = -\text{sgn}(C_{S_1} \cos \alpha)(1.01 \pm 0.12), \\
 \text{or } & C_{S_1} \cos \alpha = \pm(1.01 \pm 0.12), \quad C_{S_2} \cos \alpha = -\text{sgn}(C_{S_1} \cos \alpha)(1.55 \pm 0.11), \\
 & \text{Correlation coefficient} = -0.71
 \end{aligned} \tag{21}$$

Model 2 :

$$\begin{aligned}
 & |C_{S_2} \cos \alpha| = 0.53 \pm 0.09, \quad |C_T \cos \alpha| = 0.14 \pm 0.01, \\
 & \text{Correlation coefficient} = -0.29
 \end{aligned} \tag{22}$$

with almost identical $\chi^2/\text{d.o.f} \approx 4.50/9$, whereas the SM has $\chi^2 = 33.05$. From the smallness of α , it is clear that Model 1, with the operator O_{S_1} , is almost ruled out from the non-observation of $B_s \rightarrow \mu\tau$.

For the best fit points, the values of $R(D)$ and $R(D^*)$ are given in Table 3. We also show, in Fig. 4, how the 1σ contours in the C_{S_2} - C_T plane translate to the $R(D)$ - $R(D^*)$ plane. The plot is for Model 2, but it would have been the same for Model 1 if it were not disfavored, as the goodness-of-fit is the same in both cases. While the operator O_{S_2} can lead to the chirally unsuppressed decay through weak annihilation $B_c \rightarrow \tau\nu$, whose partial width is bounded from the lifetime of the B_c meson [32], it is easy to check that the Wilson coefficient C_{S_2} is not so large as to put that bound in jeopardy.

5 Conclusions

In this paper, we have tried to explain, with the introduction of a minimal set of operators, two apparently uncorrelated anomalies. The first one is that of the normalized $B \rightarrow D^{(*)}\tau\nu$ decay widths, denoted as $R(D)$ and $R(D^*)$, for which almost all the experiments find a nontrivial pull from the SM expectations. The second one is the hint of the LFV decay $h \rightarrow \mu\tau$ as seen by the CMS collaboration. While none of them immediately calls for a beyond-SM explanation right now, it is nevertheless interesting to see whether one can relate these two sets of data following the principle of Occam's razor, *i.e.*

Decay	Model	$R(D)$	$R(D^*)$
From B^+	1	0.419 ± 0.072	0.317 ± 0.008
	2	0.419 ± 0.040	0.317 ± 0.011
From B^0	1	0.377 ± 0.064	0.316 ± 0.008
	2	0.377 ± 0.036	0.316 ± 0.011

Table 3: New physics model predictions of $R(D^{(*)})$ with the fitted Wilson coefficients as given in Eq. (22).

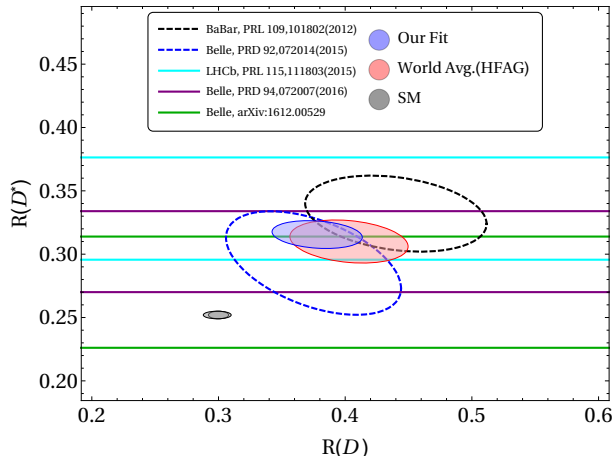


Figure 4: The 1σ contour in the $R(D)$ - $R(D^*)$ plane with the best fit points for Model 2. The current experimental results and the world average are also shown for comparison.

by the introduction of a minimal set of higher-dimensional operators.

We find that this is indeed possible. However, not all operators invoked in the literature to explain the $R(D^{(*)})$ can do the job. The situation apparently becomes even more complicated from the fact that no LFV Higgs coupling can survive if the scalar sector is SM-like. However, this can be circumvented by postulating the existence of new degrees of freedom at a higher scale while the low-energy scalar sector remains completely SM-like. This also leads to new four-fermion operators which can possibly contribute to $b \rightarrow c\tau\nu$ decays. Arguing that the undetermined nature of the neutrino flavor allows for the anomaly to be explained in terms of the muon-neutrino, we relate it, through the $SU(2)_L$ symmetry to the $\tau\mu$ final state. While many Lorentz structures, *per se.*, could explain the anomaly(ies), only some survive the stringent limits imposed by the Z and B_s decays.

We find that it is indeed possible to find a parameter space where both the anomalies can be successfully explained, with the fit showing a very marked improvement over the SM. This region is also physically meaningful in the sense that all the Wilson coefficients for the new operators are of the order of unity.

This scenario can be tested in a number of ways. First, the τ polarization, P_τ , can be measured with much improved precision in future B factories. The SM τ s are all left-chiral, while our model predicts a large number of right-chiral τ s as well. The second way is to investigate the LFV couplings of the

Higgs boson in future electron-positron colliders. As has been shown in Ref. [33], the International Linear Collider can have a reach one order of magnitude better than the LHC. As for which models can produce such effective operators, we leave that to the model builders.

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