

Tolman IV fluid sphere in bigravity

Ksh. Newton Singh • Susmita Sarkar •
Farook Rahaman

Abstract We present Tolman IV spacetime representing compact fluid sphere in bigravity. Here we have explored the effect of scale parameter k in the local matter distribution of compact stars. We have model for three well-known compact stars and it shows that for lower values of k leads to stiffer EoS. This claim is also supported by the graphical analysis. It can be observed that the sound speed and the adiabatic index are more for lower values of k . It is also seen that all the solutions of Einstein's field equations are still satisfying the field equations in the presence of a background metric $\gamma_{\mu\nu}$. However, the density and pressure does modified by extra term from the constant curvature background, thus affecting the EoS. One can also think that the parameter $\alpha \equiv 1/k^2$ as coupling constant between the $g_{\mu\nu}$ and $\gamma_{\mu\nu}$ and consequently more the coupling stiffer is the EoS. As $k \rightarrow \infty$, the background de-Sitter spacetime reduces to Minkowski's spacetime and the coupling vanishes. The solution satisfy the causality condition, all the energy conditions and equilibrium under gravity and hydrostatic forces. The stability of the local stellar structure is enhanced by reducing the scalar curvature of the background spacetime.

Keywords Bigravity ; Tolman IV ; Compact star ; Stability

Ksh. Newton Singh

Department of Physics, National Defence Academy, Khadakwasla, Pune, Maharashtra-411023, India

email: ntnphy@gmail.com

Susmita Sarkar

Department of Mathematics, Jadavpur University, Kolkata, India.

email:susmita.mathju@gmail.com

Farook Rahaman

Department of Mathematics, Jadavpur University, Kolkata, India.

email:rahaman@associates.iucaa.in

1 Introduction

A long years back, N. Rosen (Rosen 1973, 1974, 1975, 1978) proposed a new modified theory to Einstein's general theory of relativity (GTR) involving a background metric $\gamma_{\mu\nu}$ in addition to the usual physical metric $g_{\mu\nu}$, which is known as the “*bimetric general theory of relativity* (BGTR or bigravity)”. The background metric $\gamma_{\mu\nu}$ is governed by a parameter k . In BGTR, the field equations are similar of Einstein's GTR however, with the ordinary derivatives of physical metrics are replaced by the covariant derivatives with respect to the background metric. Both $g_{\mu\nu}$ and $\gamma_{\mu\nu}$ are present in the field equations but only $g_{\mu\nu}$ interacts with matter. In the year 1978, Rosen (1978) initially written the static field equations in the form of Einstein's field equations with an additional bimetric term. In the year 1985, Harpaz & Rosen (1985) solved the field equations, which are obtained by using the procedure of Rosen (1980) corresponding to de-Sitter universe of constant curvature, to obtain a model of compact star. They also proved that for an ordinary star, the results obtained from BGTR has no difference from the result obtained from Einstein's GTR. However, for a collapsed star model the two theories yield different results. The BGTR becomes a famous among the researcher within few years of the proposal and various aspects were investigated (Khadekar & Karade 1989; Reddy & Venkateswarlu 1989; Goldman & Rosen 1977; Akrami *et al.* 2015). It is investigated that the bimetric gravity has a subtle relationship with massive gravity (Baccetti *et al.* 2012). Studies of a finite size charge and the static spherically symmetric field of an electric charge were done in the framework of BGTR in Rotbart (1979) and Rosen & Rotbart (1979), respectively. In the year 1981, Falik & Rosen (1981) have investigated a

charged point particle and then Rosen, himself discussed a classical model for elementary particles in BGTR (Rosen 1989).

In the context of the Hassan-Rosen theory, spherically symmetric systems were first studied by Comelli *et al.* (2012), where in particular, the perturbative solutions to the equations of motion was published. In presence of bimetric gravity, Volkov (2012) performed an extensive numerical study, and gave conditions for the existence of asymptotically flat black hole solutions. Star solutions and the so-called Vainshtein mechanism was studied by Babichev & Crisostomi (2013). Solutions for charged black holes and for rotating black holes can be found in Brito *et al.* (2013), Babichev & Fabbri (2014), respectively. In massive bigravity, a general review of black holes was proposed by Babichev & Brito (2022). Enander & Mörtzella proposed the phenomenology of stars and galaxies in massive bigravity by applying a parameter conditions for the existence of viable star solutions when the radius of the star is much smaller than the Compton wavelength of the graviton.

Avakian *et al.* (1991) have studied on superdense celestial objects in generalized bimetric gravity with a variable gravitational constant. In the framework of bimetric gravity, the solution for spherically symmetric self-gravitating anisotropic matter distribution was investigated by Khadekar & Kandalkar (2004) and that solution agrees with the Einstein's GRT for a physical system compared to the solar system size of universe. A charged fluid was also studied in BGTR (Kandalkar & Gawande 2010). Recently, Grigorian *et al.* (2018) have shown that the masses for superdense compact objects in bimetric theory can be essentially larger compared to the objects in GTR depending on the value of the bimetric parameter.

It is well known that, in the presence of an extra spin-2 field, bimetric theory describes gravitational interactions. Recently, Hassan *et al.* (Hassan & Rosen 2012,a) have proposed a particular bimetric theory (or bigravity) to avoid the ghost instability. This theory describes a nonlinear interactions of the gravitational metric with an additional spin-2 field. By assuming the Planck mass M_f of the second metric, the ghost instability can be avoid by pushing back to early unobservably times. This limit the uses an effective cosmological constant in general relativity (Akrami *et al.* 2015).

It is quite familiar that non-linear bimetric theories of gravity suffer from the same Boulware-Deser ghost instability (Boulware & Deser 1972). To describe the interaction of gravity with a massive spin-2 meson, such theories were introduced. Recently, there has

been renewed interest in bigravity due to their accelerating cosmological solutions (Damour *et al.* 2002). Hassan & Rosen (2012) showed that the introduction of a kinetic term for the background metric in the ghost-free massive gravity leads to a bigravity theory which is also ghost free. A details study of cosmological solutions in bigravity can be found in refs. Volkov (2012); Strauss *et al.* (2012); Comelli *et al.* (2012).

In present paper, we have developed a model of compact star in bimetric gravity where the metric potentials are chosen as Tolman IV metric potential. In one of our previous paper, we have investigated a new model of anisotropic compact star in (3+1)-dimensional space-time. The model was obtained in the background of Tolman IV g_{rr} metric potential as input and the field equations were solved by assuming a suitable expression for radial pressure p_r (Bhar *et al.* 2016). The present paper is organized as follows: In sect. 2, field equations in bigravity is developed. In sect. 3 and 4, the model parameters are obtained by solving the field equations and physical properties are discussed, respectively. In sect. 5, we match our interior spacetime to the exterior Schwarzschild line element. Final two sections are devoted on a brief discussion.

2 Field equations in bimetric gravity

The bigravity modifies the Einstein's theory by accounting a background space-time. Therefore, one can defined the two symmetric tensor $g_{\mu\nu}$ and $\gamma_{\mu\nu}$ associated with the two spacetime and are defined as

$$ds_-^2 = g_{\mu\nu} dx^\mu dx^\nu \quad , \quad ds_+^2 = \gamma_{\mu\nu} dx^\mu dx^\nu. \quad (1)$$

These two metric tensor are having non-vanishing determinants. Adopting the notation used by Rosen (1940), $\left\{ \begin{smallmatrix} \lambda \\ \mu \ \nu \end{smallmatrix} \right\}$ is the Christoffel symbol in g -differentiation and $\Gamma_{\mu\nu}^\lambda$ in γ -differentiation. The relationship between these two symbols are given as

$$\left\{ \begin{smallmatrix} \lambda \\ \mu \ \nu \end{smallmatrix} \right\} = \Gamma_{\mu\nu}^\lambda + \Delta_{\mu\nu}^\lambda, \quad (2)$$

where, $\Delta_{\mu\nu}^\lambda$ is found to be related to $g_{\mu\nu}$ as

$$\Delta_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\sigma} (g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}). \quad (3)$$

Now the Riemann tensor related to the two metric functions can be written as

$$\begin{aligned} \mathcal{R}_{\mu\nu\sigma}^\lambda &= \mathcal{P}_{\mu\nu\sigma}^\lambda - \Delta_{\mu\nu,\sigma}^\lambda + \Delta_{\mu\sigma,\nu}^\lambda + \Delta_{\alpha\nu}^\lambda \Delta_{\mu\sigma}^\alpha - \Delta_{\alpha\sigma}^\lambda \Delta_{\mu\nu}^\alpha \\ &= -\Delta_{\mu\nu,\sigma}^\lambda + \Delta_{\mu\sigma,\nu}^\lambda + \Delta_{\alpha\nu}^\lambda \Delta_{\mu\sigma}^\alpha - \Delta_{\alpha\sigma}^\lambda \Delta_{\mu\nu}^\alpha. \end{aligned} \quad (4)$$

One can easily see that the tensor $\Delta_{\mu\nu}^\lambda$ is related to the normal Riemann curvature tensor. The curvature

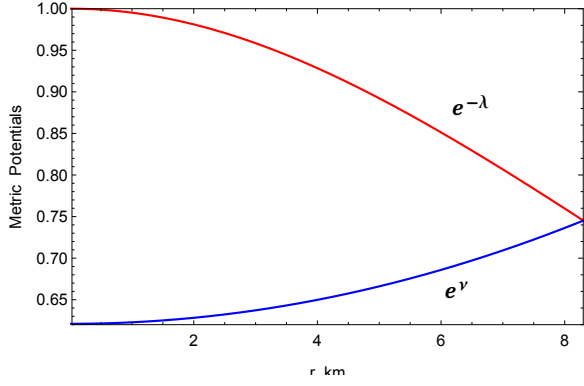


Fig. 1 Variation of metric functions with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

tensor $\mathcal{P}_{\lambda\mu\nu\sigma}$ of the constant curvature spacetime can be written as

$$\mathcal{P}_{\lambda\mu\nu\sigma} = \frac{1}{k^2} (\gamma_{\mu\nu}\gamma_{\lambda\sigma} - \gamma_{\mu\sigma}\gamma_{\lambda\nu}). \quad (5)$$

By using (4), one can also find the difference in the Ricci tensor i.e. $\mathcal{K}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \mathcal{P}_{\mu\nu}$ which satisfy the same form of Einstein's field equations.

Finally, the field equations in bigravity can be written as Rosen (1978)

$$G_{\mu\nu} = S_{\mu\nu} - 8\pi T_{\mu\nu}, \quad (6)$$

where,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} \quad (7)$$

$$S_{\mu\nu} = \frac{3}{k^2} (\gamma_{\mu\nu} - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\gamma_{\alpha\beta}) \quad (8)$$

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - p g_{\mu\nu}. \quad (9)$$

Now the interior and background de-Sitter spacetime are taken as

$$ds_-^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (10)$$

$$ds_b^2 = \left(1 - \frac{r^2}{k^2}\right) dt^2 - \left(1 - \frac{r^2}{k^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (11)$$

Here k represents the scale parameter in the de-Sitter universe.

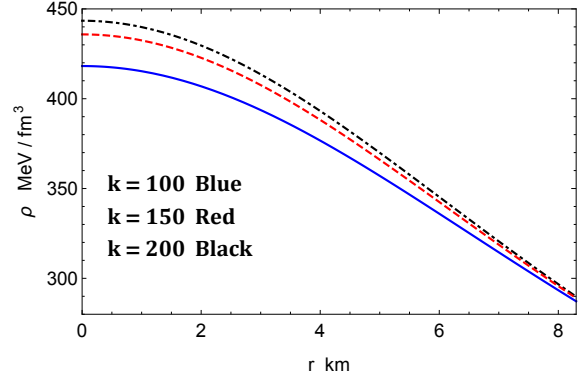


Fig. 2 Variation of density with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

Following Falik & Rosen (1980) we can write the field equations as

$$8\pi\rho = e^{-\lambda} \left(\frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} + \frac{3e^{-\nu}}{2k^2} \quad (12)$$

$$8\pi p = e^{-\lambda} \left(\frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} + \frac{3e^{-\nu}}{2k^2} \quad (13)$$

$$8\pi p = \frac{e^{-\lambda}}{2} \left[\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{r} \right] + \frac{3e^{-\nu}}{2k^2} \quad (14)$$

Due to the constant curvature of the background metric, the density and pressure are modified as

$$\rho_e(r) = \rho(r) - \frac{3e^{-\nu}}{16\pi k^2} \quad (15)$$

$$p_e(r) = p(r) - \frac{3e^{-\nu}}{16\pi k^2} \quad (16)$$

which also satisfies the TOV-equation i.e.

$$-\frac{\nu'}{2}(\rho_e + p_e) - \frac{dp_e}{dr} = 0. \quad (17)$$

To analyze in deeper aspects, we will ansatz the Tolman IV spacetime and discuss its behavior w.r.t. the scale factor k .

3 Tolman IV solution in bigravity

The Tolman IV (Tolman 1939) spacetime is given by

$$ds_-^2 = B^2(1 + ar^2)dt^2 - \frac{2ar^2 + 1}{(ar^2 + 1)(1 - br^2)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (18)$$

The nature of the metric functions are shown in Fig. 1.

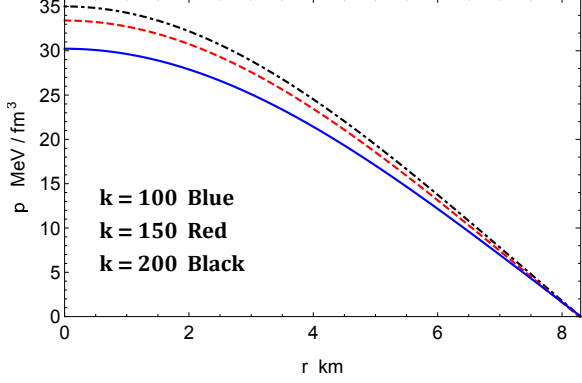


Fig. 3 Variation of pressure with respect to the radial coordinate r for for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

For the Tolman IV space-time the density and pressure of the stellar configuration can be written as

$$\rho(r) = \frac{1}{16B^2k^2(2ar^2+1)^2(\pi ar^2+\pi)} \left[4a^3B^2k^2r^4 (3br^2+1) + 2a^2\{B^2k^2r^2(13br^2+5)+6r^4\} + 2a\{B^2k^2(10br^2+3)+6r^2\} + 6bB^2k^2+3 \right] \quad (19)$$

$$p(r) = \frac{1}{8\pi} \left[\frac{3}{2B^2k^2(ar^2+1)} - \frac{(3ar^2+1)(br^2-1)}{2ar^4+r^2} - \frac{1}{r^2} \right]. \quad (20)$$

The graphical representations of our obtained density and pressure are shown in Figs. 2 and 3 with respect to the radial coordinate r for the compact star LMC X-4. Fig.

The density and pressure gradients are calculated as

$$\frac{d\rho}{dr} = -\frac{ar}{8\pi B^2k^2(ar^2+1)^2(2ar^2+1)^3} \left[8a^4B^2k^2r^6 + 4a^3\{B^2k^2r^4(br^2+9)+6r^6\} + 6a^2\{B^2k^2r^2(3br^2+8)+6r^4\} + 2a\{2B^2k^2(6br^2+5)+9r^2\} + 10bB^2k^2+3 \right] \quad (21)$$

$$\frac{dp}{dr} = -\frac{ar}{8\pi B^2k^2(ar^2+1)^2(2ar^2+1)^2} \left[4a^3B^2k^2r^4 + 2a^2\{B^2k^2r^2(br^2+4)+6r^4\} + 4a\{B^2k^2(br^2+1)+3r^2\} + 2bB^2k^2+3 \right]. \quad (22)$$

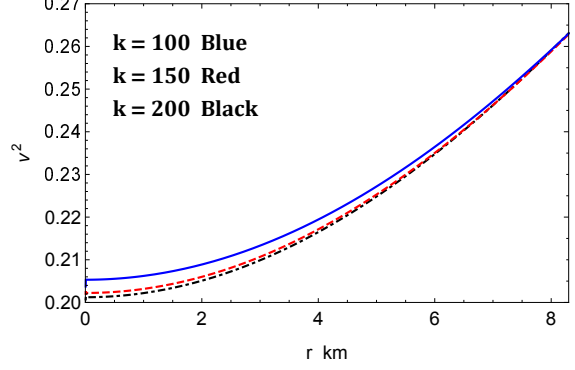


Fig. 4 Variation of sound speed with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

The causality condition can be verify by computing the speed of sound as

$$v^2 = \frac{dp}{d\rho} = (2ar^2+1) \left[4a^3B^2k^2r^4 + 2a^2\{B^2k^2r^2(br^2+4)+6r^4\} + 4a\{B^2k^2(br^2+1)+3r^2\} + 2bB^2k^2+3 \right] \left[8a^4B^2k^2r^6 + 4a^3\{B^2k^2r^4(br^2+9)+6r^6\} + 6a^2\{B^2k^2r^2(3br^2+8)+6r^4\} + 2a\{2B^2k^2(6br^2+5)+9r^2\} + 10bB^2k^2+3 \right]^{-1} \quad (23)$$

To satisfy the causality condition the speed of sound v should be less than unity (in $c = 1 = G$ unit). The trend of the sound speed is shown in Fig. 4.

To analyze the energy conditions we can calculate the following parameters:

$$\rho(r) - p(r) = \frac{6a^2br^4 + 6abr^2 + a + 2b}{4\pi(2ar^2+1)^2} \quad (24)$$

$$\rho(r) - 3p(r) = \frac{1}{8B^2k^2(2ar^2+1)^2(\pi ar^2+\pi)} \left[2bB^2k^2(12a^3r^6+23a^2r^4+14ar^2+3) - 4a^3B^2k^2r^4 - 4a^2(B^2k^2r^2+3r^4) - 12ar^2 - 3 \right]. \quad (25)$$

For any physical matters, the strong, weak, dominant and null energy conditions need to be satisfy i.e.

$$\rho \geq 0; \quad \rho - p \geq 0; \quad \rho - 3p \geq 0; \quad \rho \geq |p|.$$

The results obtained in Eqs. (24) and (25) are shown graphically in Fig. 5.

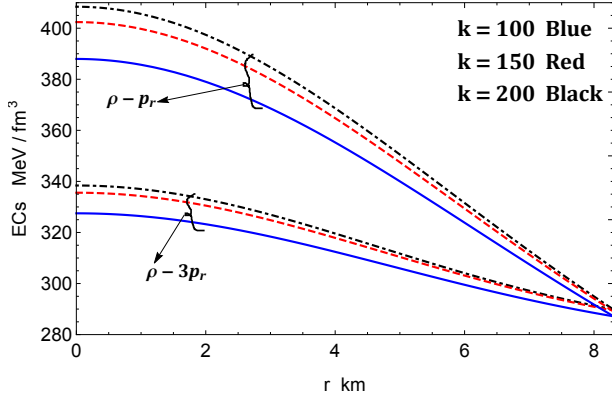


Fig. 5 Variation of energy conditions with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

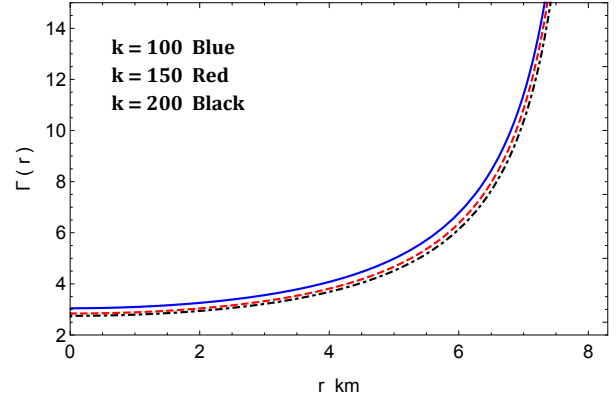


Fig. 6 Variation of adiabatic index with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

Now we can defined the adiabatic index, mass function and compactness parameter as

$$\Gamma = \frac{\rho + p}{p} \frac{dp}{d\rho} \quad (26)$$

$$m(r) = 4\pi \int_0^r \rho(r)r^2 dr = \frac{1}{4a^{3/2}B^2k^2(2ar^2 + 1)} \left[\sqrt{ar} \{ 2a^2B^2k^2r^2(br^2 + 1) + 2ar^2(bB^2k^2 + 3) \} - 3(2ar^2 + 1) \tan^{-1}(\sqrt{ar}) \right] \quad (27)$$

$$u(r) = \frac{2m(r)}{r} = \frac{1}{2a^{3/2}B^2k^2(2ar^3 + r)} \left[\sqrt{ar} \{ 2a^2B^2k^2r^2(br^2 + 1) + 2ar^2(bB^2k^2 + 3) \} - 3(2ar^2 + 1) \tan^{-1}(\sqrt{ar}) \right]. \quad (28)$$

The variation of the above physical quantities can be seen in Figs. 6 and 7.

4 Physical properties of the solution

The non-singular nature of the solution can be seen from the central values of density and pressure. The central values of these parameter can be written as

$$\rho_c = \frac{6aB^2k^2 + 6bB^2k^2 + 3}{16\pi B^2k^2} > 0 \quad (29)$$

$$p_c = \frac{2a - 2b + 3/B^2k^2}{16\pi} > 0. \quad (30)$$

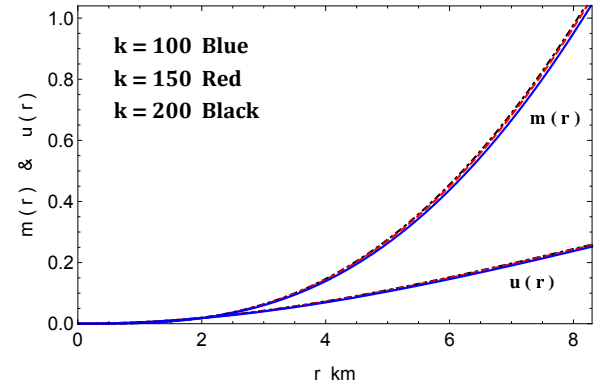


Fig. 7 Variation of mass function and compactness parameter with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

For any physical matter, the Zeldovich's condition (Zeldovich & Novikov 1971) has to be satisfy i.e. $p_c/\rho_c \leq 1$, which implies

$$0 \leq a + 2b. \quad (31)$$

The TOV-equation of hydrostatic equilibrium can be written as

$$-\frac{\nu'}{2}(\rho + p) - \frac{dp}{dr} = 0 \quad \text{or} \quad F_g + F_h = 0. \quad (32)$$

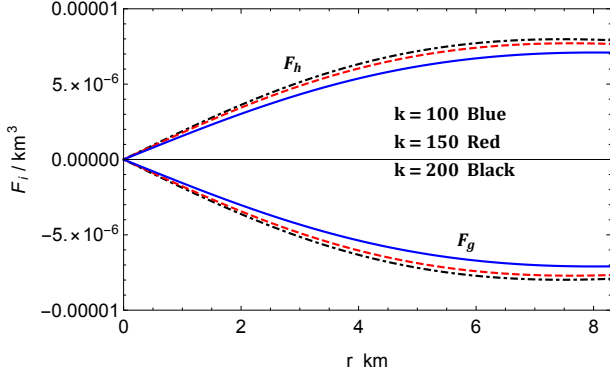


Fig. 8 Variation of various forces in TOV-equation with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_{\odot}$, $R = 8.3\text{km}$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

Here the gravitational and hydrostatic forces can be defined as

$$F_g = -\frac{\nu'}{2}(\rho + p) = \frac{-ar}{8\pi B^2 k^2 (ar^2 + 1)^2 (2ar^2 + 1)^2} \left[3 + 4a^3 B^2 k^2 r^4 + 2a^2 \{ B^2 k^2 r^2 (br^2 + 4) + 6r^4 \} + 4a \{ B^2 k^2 (br^2 + 1) + 3r^2 \} + 2bB^2 k^2 \right] \quad (33)$$

$$F_h = -\frac{dp}{dr} = \frac{ar}{8\pi B^2 k^2 (ar^2 + 1)^2 (2ar^2 + 1)^2} \left[4a^3 B^2 k^2 r^4 + 2a^2 \{ B^2 k^2 r^2 (br^2 + 4) + 6r^4 \} + 4a \{ B^2 k^2 (br^2 + 1) + 3r^2 \} + 2bB^2 k^2 + 3 \right]. \quad (34)$$

The exact profiles of the gravitational force F_g and hydrostatics force F_h for our model are displayed in Fig. 8, which indicates that solution represents the equilibrium matter configuration.

To check the stability of the solution and how the parameter k affect it, one can adopt the static stability criterion. As per this criterion, the mass of the system must be increasing function of its central density i.e. $\partial m / \partial \rho_c > 0$ or otherwise unstable. This insures the stability of the system under radial perturbations. The

mass as a function of its central can be given as

$$m(\rho_c) = \frac{R}{4} \left[1 + br^2 - \frac{18}{2B^2 k^2 (3b - 8\pi\rho) + 3} - \frac{3(br^2 - 1)}{[2r^2(3b - 8\pi\rho_c) - 3] + 3r^2/B^2 k^2} - \frac{9B^2 k^2 \sqrt{24\pi\rho_c - 9b - 9/2B^2 k^2}}{[2B^2 k^2 (3b - 8\pi\rho_c) + 3]^2} \right] \tan^{-1} \left(r \sqrt{\frac{8\pi\rho_c}{3} - b - \frac{1}{2B^2 k^2}} \right). \quad (35)$$

The variation of mass with central density is shown in Fig. 9. It signifies that the solution gain its stability when k increases i.e. whenever the scalar curvature of the background de-Sitter spacetime ($R = 12/k^2$) decreases, the stability of the local stellar system is enhanced. This is because, as k increases the stable range of density increases thereby the change in density during radial oscillations does not trigger gravitational collapse.

5 Matching of interior and exterior boundary

Assume the exterior metric as the Schwarzschild vacuum for the region $r > R$ i.e.

$$ds_+^2 = \left(1 - \frac{2M}{r} \right) dt^2 - \left(1 - \frac{2M}{r} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (36)$$

Matching at the boundary $r = R$ we get

$$e^{\nu(R)} = e^{-\lambda(R)} = 1 - \frac{2M}{R} \quad (37)$$

$$p(R) = 0. \quad (38)$$

On using the boundary conditions (37) and (38) we get

$$a = \frac{bR^3 - 2M}{R^2(4M - bR^3 - R)} \quad (39)$$

$$b = \frac{2a^2 B^2 k^2 R^2 + 2aB^2 k^2 + 6aR^2 + 3}{2B^2 k^2 (aR^2 + 1)(3aR^2 + 1)} \quad (40)$$

$$B = \sqrt{\frac{R - 2M}{R(aR^2 + 1)}} \quad (41)$$

Here we have used M , R and k as free parameters and rest of the constants are found using the above three equations.

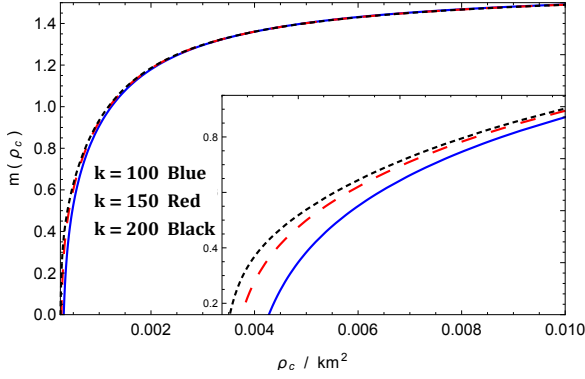


Fig. 9 Variation of mass with respect to the radial central density ρ_c for LMC X-4 ($M = 1.04M_\odot$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

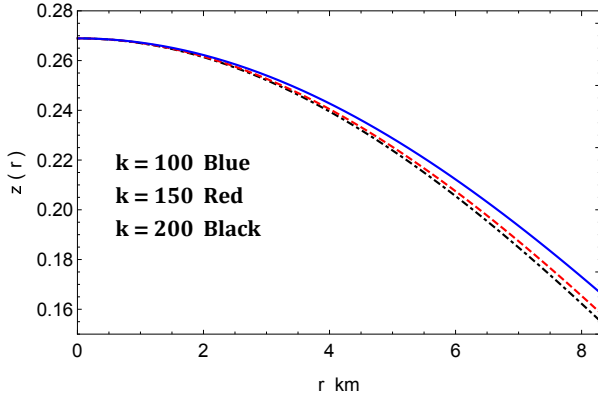


Fig. 10 Variation of redshift with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_\odot$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

Now the red-shift and equation of state parameter of compact stars can be determined as

$$z(r) = e^{-\nu/2} - 1 = \frac{1}{B\sqrt{1+ar^2}} - 1 \quad (42)$$

$$\omega = \frac{p}{\rho} \leq 1. \quad (43)$$

Fig. 10 and 11 clarifies the behavior of redshift function and the equation of state parameter with respect to the radial coordinate function r respectively.

6 Results and discussions

In this article, we have explored the behavior of the Tolman IV solution in bimetric gravity describing relativistic fluid sphere. To demonstrate the effect of the scale parameter k on the solution we used graphical methods for a specific compact star i.e. LMC X-4. In the

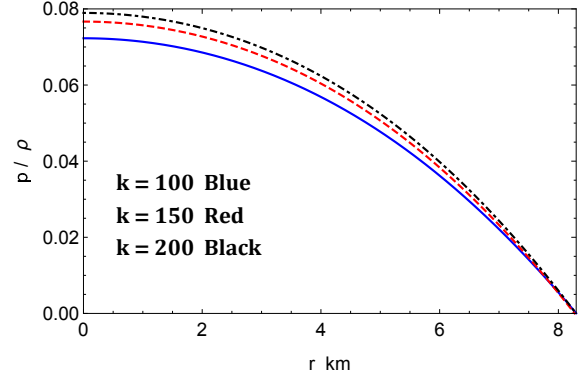


Fig. 11 Variation of equation of state parameter with respect to the radial coordinate r for LMC X-4 ($M = 1.04M_\odot$, $R = 8.3km$ and $b = 0.001896$, $B = 0.788065$) with ($a = 0.00266$, $k = 100$), ($a = 0.0029$, $k = 150$) and ($a = 0.003$, $k = 200$).

framework of bigravity as well, the Tolman IV spacetime behaves in good agreement as in GR.

The obtained physical parameters, density and pressure are as expected positive, maximum at the centre and then monotonically decreasing in nature towards the surface, Figs. 1 (Right) and 2 (Left). The mass and the compactness parameter are monotonically increasing in nature toward the surface, shown in Fig. 4 (Left). Therefore, all the physical parameters are well-behaved and physically acceptable. satisfies the TOV-equation even in the presence of the background spacetime, (see Fig. 4, Right). Hence, the solution can represent static and equilibrium relativistic fluid spheres. The nature of the equation of state parameter (EoS) i.e. $p/\rho < 1$ indicates the solution represents the physically acceptable matter distribution (Rahaman *et al.* 2010).

The scale parameter k can affect the solution and thus affect the corresponding equation of state (EoS) or the internal compositions of the compact fluid object. We have provided the central values of some physical quantities in Table 2, whereas the Table-1 display the compatible constant parameter to model SMC X-4, LMC X-1 and PSR J1614-2230. As we can see, smaller value of k corresponds to stiffer EoS i.e. $k = 33$ along with the given parameters, corresponds to mass and radius of PSR J1614-2230 ($1.97M_\odot$, $9.69km$) with compactness parameter of 0.407 and for $k = 200$ corresponds to LMC X-4 ($1.04M_\odot$, $8.3km$) with $u = 0.251$. This claim is further supported by the graphical representations. The velocity of sound and central values adiabatic index are more for $k = 100$ than $k = 200$ (see Figs. 2 Right and 3 Right). This suggest that as k increases the stiffness of the solution decreases.

Furhter, the solution also satisfy the causality and all the energy conditions (see Figs. 2 Right, 3 Left).

Table 1 All the parameters corresponds to well-behaved solution representing few well-known compact stars.

Objects	$a \text{ km}^{-2}$	$b \text{ km}^{-2}$	B	k	M/M_{\odot}	$R \text{ km}$	M_{obs}/M_{\odot}	$R_{obs} \text{ km}$
SMC X-4	0.003	0.0020484	0.757390	80	1.29	8.831	1.29 ± 0.05	8.831 ± 0.09
LMC X-4	0.003	0.0018955	0.788065	200	1.04	8.3	1.04 ± 0.09	8.301 ± 0.2
PSR J1614-2230	0.002	0.00332169	0.706809	33	1.97	9.69	1.97 ± 0.04	9.69 ± 0.2

Table 2 All the parameters corresponds to well-behaved solution representing few well-known compact stars.

Objects	u	z_c	z_s	ρ_c g/cm^3	ρ_s g/cm^3	p_c $dyne/cm^2$	Γ_c
SMC X-4	0.292	0.32	0.19	8.34×10^{14}	5.30×10^{14}	6.54×10^{34}	2.592
LMC X-4	0.251	0.269	0.155	7.89×10^{14}	5.16×10^{14}	5.59×10^{34}	2.726
PSR J1614-2230	0.407	0.415	0.298	10.03×10^{14}	7.40×10^{14}	6.99×10^{34}	3.601

Consequently, the solution representing matter distribution is physical. The stability of compact star is one of the most vital requirement, for that, we have focused to discuss the stability with respect to the variation of adiabatic index Γ inside the compact star. For a Newtonian fluid, stable configuration can be achieved if $\Gamma \geq 4/3$ (Bondi 1964). In our model, the stable nature of the configuration can be convinced from the graphical representation of the Γ , provided in Fig. 3 (Right). Further, the solution also satisfies the static stability criterion showing that the stability is enhance when the curvature of the background metric decreases (Fig. 5). Therefore, the stability will be maximum when assumed a flat or Minkowski's spacetime.

7 Conclusion

Since the local spacetime interior to the compact star is part of the global structure in the universal scale, the physics of such object can be influence by the scale parameter of the universe. Even though the effect of the scale parameter may be small, however, the physics of such small perturbations can be interesting and worth complete analysis. Harpaz & Rosen (1985) have shown that there is possible to exist a configuration in hydrostatic equilibrium for a collapsing star filling its Schwarzschild sphere when no such configuration exist in Einstein's gravity. It is also mentioned that in bigravity even leads a cosmology without a "Big Bang" (Rosen 1980).

Rosen (1980) also solved the field equations with background metric near the Schwarzschild sphere and found that the field differs from that of the Einstein's GTR. Instead of a black hole they have obtained an impenetrable sphere. It is also found that the bigravity is identical with the ordinary general relativity at large distances (scale of solar system), however, at very large

distances (scale of clusters of galaxies) the scale parameter affect the field equations and thus the dynamics.

As mentioned by Harpaz & Rosen (1985), all the solutions of field equations in general relativity does satisfies the field equations with a background and the same is also supported by the current work with more details analysis. The internal structure of compact stars can also be influence by the background metric. We have shown that the local structures can also be influence by the background spacetime. Hence, the bimetric GTR can inspire many researchers and also can contribute to new physics in future.

Acknowledgement

Farook Rahaman would like to thank the authorities of the Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing research facilities. Susmita Sarkar is grateful to UGC (Grant No.: 1162/(sc)(CSIR-UGC NET , DEC 2016)), Govt. of India, for financial support.

Conflict of interest

The authors declare no conflict of interest.

References

- Akrami, Y., et al.: Phys. Lett. B **748**, 37 (2015)
- Avakian, M. R.: Astrophys. **35**, 318 (1991)
- Babichev, E., Crisostomi, M.: Phys. Rev. D **88**, 084002 (2013)
- Babichev, E., Brito, R.: arXiv 1503.0752
- Babichev, E., Fabbri, A.: J. High Energy Phys. **1407**, 016 (2014)
- Baccetti, V., et al.: Classical Quant. Grav. **30**, 015004 (2012)
- Bondi, H.: Proc. R. Soc. Lond. A **281**, 39 (1964)
- Boulware, D. G., Deser, S.: Phys. Rev. D **6**, 3368 (1972)
- Brito, R., et al.: Phys. Rev. D **88**, 064006 (2013)
- Comelli, D., et al.: J. High Energy Phys. **03**, 067 (2012)
- Comelli, D., et al.: Phys. Rev. D **85**, 024044 (2012)
- Damour, T., et al.: Phys. Rev. D **66** 104025 (2002)
- Enander, J., Mörtsella, E.: arxiv.org 1507.00912v3
- Falik, D., Rosen, N.: Gen. Relativ. Gravit. **13** 599 (1981)
- Falik, D., Rosen, N.: Ap. J. **239**, 1024 (1980)
- Goldman, I., Rosen, N.: Astrophysical J. **212**, 602 (1977)
- Grigorian, L. Sh.: Particles **1**, 203 (2018)
- Harpaz, A., Rosen, N.: Astrophys. J. **291**, 417 (1985)
- Hassan, S. F., Rosen, R. A.: J. High Energy Phys. **1202**, 126 (2012)
- Hassan, S. F., Rosen, R. A.: J. High Energy Phys. **1204**, 123 (2012a)
- Khadekar, G. S., Karade, T. M.: Czechoslov. J. Phys. B **39**, 962 (1989)
- Khadekar, G. S., Kandalkar, P.: Astrophys. Space Sci. **293**, 415 (2004)
- Kandalkar, S. P., Gawande, S. P.: Astrophys. Space Sci. **326** 45 (2010)
- Piyali, P.: Astrophys. Space Sci. **361**, 284 (2016)
- Rahaman, F., et al.: Phys. Rev. D **82**, 104055 (2010)
- Reddy, D. R. K., Venkateswarlu, R.: Astrophys. Space Sci. **158**, 169 (1989)
- Rosen, N.: Gen. Relativ. Gravit. **4**, 435 (1973)
- Rosen, N.: Ann. Phys. **84**, 455 (1974)
- Rosen, N.: Gen. Relativ. Gravit. **6**, 259 (1975)
- Rosen, N.: Gen. Relativ. Gravit. **9**, 339 (1978)
- Rosen, N.: Found. Phys. **10**, 673 (1980)
- Rosen, N., Rotbart, F.: Gen. Relativ. Gravit. **10**, 489 (1979)
- Rosen, N.: Found. Phys. **19** 339 (1989)
- Rosen, N.: Phys. Rev. **57**, 147 (1940)
- Rotbart, F.: Gen. Relativ. Gravit. **10**, 497 (1979)
- Strauss, M. V., et al.: JCAP **03**, 042 (2012)
- Tolman, R. C.: Phys. Rev. **55**, 364 (1939)
- Volkov, M. S.: Phys. Rev. D **85**, 124043 (2012)
- Volkov, M. S.: J. High Energy Phys. **101**, 035 (2012)
- Zeldovich, Ya. B., Novikov, I. D.: Relativistic Astrophysics Vol. 1: Stars and Relativity. University of Chicago Press, Chicago 1971)