

Thermo-elastic interaction with energy dissipation in an unbounded body with a spherical hole

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Abstract

The distribution of stresses due to step input of temperature at the boundary of a spherical hole in a homogeneous isotropic unbounded body is investigated by applying Laplace transform technique in the context of generalized theories of thermo-elasticity. The inverse of the transformed solution is carried out by applying a method of Bellman et al. The stresses are computed numerically and presented graphically in a number of figures for aluminum–epoxy composite material. The comparison among the theories i.e. classical thermo-elasticity (CTE), classical coupled thermo-elasticity (CCTE), temperature-rate-dependent thermo-elasticity (TRDTE(GL)) and thermo-elasticity with energy dissipation (TEWED(GN)) theory is presented graphically.

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1. Introduction

The classical theories of thermo-elasticity involving infinite speed of propagation of thermal signals, contradict physical facts. During the last three decades, non-classical theories involving finite speed of heat transportation in elastic solids have been developed to remove this paradox. In contrast to the conventional coupled thermo-elasticity theory (Lord and Shulman, 1967), which involves a parabolic-type heat transport equation, these generalized theories involving a hyperbolic-type heat transport equation are supported by experiments exhibiting the actual occurrence of wave-type heat transport in solids, called second sound effect. The extended thermo-elasticity theory (ETE) proposed by Lord and Shulman (1967), incorporates a flux-rate term into Fourier's law of heat conduction, and formulates a generalized form that involves a hyperbolic-type heat transport equation admitting finite speed of thermal signals. Green and Lindsay (1972) developed temperature-rate-dependent thermo-elasticity (TRDTE) theory by introducing relaxation time factors that does not violate the classical Fourier law of heat conduction and this theory also predicts a finite speed for heat

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propagation. The closed-form solutions for thermo-elastic problems in generalized theory of thermo-elasticity have been obtained by Hetnarski and Ignaczak (1993). Hetnarski and Ignaczak (1994) studied the response of semi-space to a short laser pulse in the context of generalized thermo-elasticity. Sinha and Sinha (1974) and Sinha and Elsibai (1996, 1997) studied the reflections of thermo-elastic waves from the free surface of a solid half-space and at the interface of two semi-infinite media in welded contact in the context of generalized thermo-elasticity. Sharma (1988) investigated the reflection of thermo-elastic waves from the stress-free insulated boundary of a transversely isotropic half-space in the light of the theory developed by Dhaliwal and Singh (1980) in which the theory developed in Lord and Shulman (1967) is extended to anisotropic (transversely isotropic) thermo-elastic materials. Bhatta and Roychoudhuri (1983) and Roychoudhuri and Chatterjee (1989) studied thermo-elastic wave propagation in an infinitely extended thin plate following the theory developed in Green and Lindsay (1972) for short-time approximation using Laplace transform technique.

Most engineering materials such as metals possess a relatively high rate of thermal damping and thus are not suitable for use in experiments concerning second sound propagation. But, given the state of recent advances in material science, it may be possible in the foreseeable future to identify (or even manufacture for laboratory purposes) an idealized material for the purpose of studying the propagation of thermal waves at finite speed. The most recent and relevant theoretical developments on the subject are due to Green and Naghdi (1991, 1992, 1993) and provide sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems, labelled as types I, II, III. The natures of these three types of constitutive equations are such that when the respective theories are linearized, type I is the same as the classical heat equation (based on Fourier's law) whereas the linearized versions of types II and III theories permit propagation of thermal waves at finite speed. The entropy flux vectors in types II and III (i.e. thermo-elasticity without energy dissipation (TEWOED) and thermo-elasticity with energy dissipation (TEWED)) models are determined in terms of the potential that also determines stresses. When Fourier conductivity is dominant the temperature equation reduces to classical Fourier law of heat conduction and when the effect of conductivity is negligible the equation has undamped thermal wave solutions without energy dissipation. Several investigations relating to thermo-elasticity without energy dissipation theory (TEWOED) have been presented by Roychoudhuri and Dutta (2005), Sharma and Chouhan (1999), Roychoudhuri and Bandyopadhyay (2004), and Chandrasekharaiyah (1996a,b).

The main object of the present paper is to study the thermo-elastic stress distribution in an isotropic homogeneous infinitely extended body containing a spherical hole due to step input of temperature at the boundary of the hole in the context of generalized thermo-elasticity. The Laplace transform technique has been applied. The inversion of Laplace transform is done following Bellman et al. (1966). The results obtained theoretically have been computed numerically and are presented graphically for aluminum–epoxy composite material. A complete and comprehensive analysis and comparison among the different theories (CTE, CCTE, TRDTE(GL), TEWED(GN)) are presented.

2. Basic equations and constitutive relations

We consider a homogeneous isotropic infinitely extended thermo-elastic body in an undisturbed state and initially at uniform temperature T_0 containing a spherical hole of radius a . We use spherical polar co-ordinate (r, θ, ϕ) with the centre of the spherical hole as the origin.

The stress–strain–temperature relations in the generalized theory of thermo-elasticity are

$$\tau_{ij} = \lambda \Delta \delta_{ij} + 2\mu e_{ij} - \gamma(T + \delta_{1k} \alpha \dot{T}) \delta_{ij} \quad (1)$$

and generalized heat conduction equation is

$$\left(\delta_{1k} + \delta_{2k} \frac{\partial}{\partial t} \right) K \nabla^2 T + \delta_{2k} K^* \nabla^2 T = \rho C_v [\delta_{1k} \dot{T} + (\delta_{1k} \alpha^* + \delta_{2k}) \ddot{T}] + \gamma T_0 \left(\zeta \delta_{1k} + \delta_{2k} \frac{\partial}{\partial t} \right) \dot{T}, \quad (2)$$

where τ_{ij} ($i = r, \theta$) is the stress tensor, Δ is the dilatation, T is the temperature increase over the absolute reference temperature T_0 , $\gamma = (3\lambda + 2\mu)\alpha_t$, λ and μ are the Lamé's constants, α_t is the coefficient of linear thermal expansion of the material, α and α^* are the relaxation times, K is the coefficient of thermal conductivity, K^* is

the additional material constant, ρ is the mass density, C_v is the specific heat of the solid at constant strain, δ_{ij} is the Kronecker delta.

In Eqs. (1) and (2):

- (i) if $\alpha = 0, \alpha^* = 0, k = 1,$ and $\zeta = 0,$ then they reduce to the equations of classical theory of thermo-elasticity (CTE);
- (ii) if $\alpha = 0, \alpha^* = 0, k = 1,$ and $\zeta = 1,$ then they reduce to the equations of classical coupled theory of thermo-elasticity (CCTE);
- (iii) if $k = 1$ and $\zeta = 1,$ then they reduce to the equations of temperature-rate-dependent thermo-elasticity theory (TRDTE);
- (iv) if $k = 2,$ then they reduce to the equations of thermo-elasticity with energy dissipation (TEWED).

The thermal relaxation times satisfy the inequalities (Green and Lindsay, 1972)

$$\alpha \geq \alpha^* > 0$$

in the case of GL theory.

In the present problem (due to spherical symmetry) the displacement and temperature are assumed to be functions of r and time t only. If $\mathbf{u} = [u(r, t), 0, 0]$ be the displacement vector, then

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\theta\theta} = e_{\phi\phi} = \frac{u}{r}. \tag{3}$$

The stress equation of motion in spherical polar co-ordinates is given by

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2}{r}(\tau_{rr} - \tau_{\theta\theta}) = \rho \frac{\partial^2 u}{\partial t^2}. \tag{4}$$

Introducing the following dimensionless quantities

$$\begin{aligned} (U, R) &= \left(\frac{u}{a}, \frac{r}{a}\right), \quad \Theta = \frac{T}{T_0}, \quad \eta = \frac{Gt}{a}, \\ (\sigma_R, \sigma_\theta) &= \frac{1}{2\mu}(\tau_{rr}, \tau_{\theta\theta}), \quad G^2 = \frac{\lambda + 2\mu}{\rho}, \\ \alpha' &= \alpha\omega', \quad \alpha^* = \alpha^*\omega^*, \quad \omega' = \frac{\gamma T_0}{a\rho G}, \quad \omega^* = \frac{G}{a}, \end{aligned}$$

Eqs. (1), (2) and (4) become in dimensionless forms

$$\sigma_R = \frac{\partial U}{\partial R} + \frac{\lambda}{2\mu} \left(\frac{\partial U}{\partial R} + \frac{2U}{R}\right) - \frac{\lambda + 2\mu}{2\mu} (a_1\Theta + \delta_{1k}\alpha'\dot{\Theta}), \tag{5}$$

$$\sigma_\theta = \frac{U}{R} + \frac{\lambda}{2\mu} \left(\frac{\partial U}{\partial R} + \frac{2U}{R}\right) - \frac{\lambda + 2\mu}{2\mu} (a_1\Theta + \delta_{1k}\alpha'\dot{\Theta}), \tag{6}$$

$$\left\{ \delta_{1k} + \left(\frac{\partial}{\partial \eta} + a_0\right) \delta_{2k} \right\} (D_1 D) \Theta = a_3 \left(\zeta \delta_{1k} \frac{\partial}{\partial \eta} + \delta_{2k} \frac{\partial^2}{\partial \eta^2} \right) (D_1 U) + a_2 \left\{ \delta_{1k} \frac{\partial \Theta}{\partial \eta} + (\alpha^* \delta_{1k} + \delta_{2k}) \frac{\partial^2 \Theta}{\partial \eta^2} \right\}, \tag{7}$$

and

$$DD_1 U - a_1 D \Theta - \delta_{1k} \alpha' \frac{\partial}{\partial \eta} (D \Theta) = \frac{\partial^2 U}{\partial \eta^2}, \tag{8}$$

where $D \equiv \frac{\partial}{\partial R}, D_1 \equiv \frac{\partial}{\partial R} + \frac{2}{R}$ and $a_0 = \frac{aK^*}{KG}, a_1 = \frac{\gamma T_0}{\lambda + 2\mu}$ and $a_2 = \frac{\rho C_v a G}{K}$ are dimensionless constants. Here $\frac{a_1 \times a_3}{a_2} = \frac{\gamma^2 T_0}{\rho C_v (\lambda + 2\mu)} = \epsilon$ is the thermo-elastic coupling constant.

The boundary conditions on the hole $R = 1$ are given by

$$\begin{aligned} \sigma_R &= 0 \quad \text{on } R = 1, \quad \eta \geq 0 \\ \Theta &= \chi H(\eta) \quad \text{on } R = 1, \quad \eta > 0, \end{aligned}$$

where χ is a dimensionless constant and $H(\eta)$ is the Heaviside unit step function. The above conditions indicate that for time $\eta \leq 0$ there is no temperature ($\Theta = 0$). A thermal shock is given on the surface of the cavity of the sphere ($R = 1$) immediately after time $\eta = 0$. Thermal stresses in the elastic medium due to the application of this thermal shock are calculated. We assume that the medium is at rest and undisturbed initially. The initial and regularity conditions can be written as

$$U = \Theta = 0 \quad \text{at } \eta = 0, R \geq 1, \quad \frac{\partial U}{\partial \eta} = 0 \quad \text{at } \eta = 0,$$

$$U = \Theta = 0 \quad \text{when } R \rightarrow \infty.$$

3. Method of solution

Let

$$\{\bar{U}(R, p), \bar{\Theta}(R, p)\} = \int_0^\infty \{U(R, \eta), \Theta(R, \eta)\} e^{-p\eta} d\eta$$

with $Re(p) > 0$ denotes the Laplace transform of U and Θ respectively.

On taking Laplace transform, Eqs. (7) and (8) reduce to

$$[\{\delta_{1k} + (p + a_0)\delta_{2k}\}D_1D - a_2\{(p + \alpha' p^2)\delta_{1k} + p^2\delta_{2k}\}]\bar{\Theta} = a_3(p\zeta\delta_{1k} + p^2\delta_{2k})(D_1\bar{U}), \tag{9}$$

$$(DD_1 - p^2)\bar{U} = (a_1 + \alpha' p\delta_{1k})(D\bar{\Theta}). \tag{10}$$

Operating D_1D in Eq. (9) and using Eq. (10) we get

$$[L^2 - (m_1^2 + m_2^2)L + m_1^2m_2^2]\bar{U} = 0. \tag{11}$$

Again operating DD_1 in Eq. (10) and using Eq. (9) we get

$$[M^2 - (m_1^2 + m_2^2)M + m_1^2m_2^2]\bar{\Theta} = 0, \tag{12}$$

where $L \equiv DD_1$ and $M \equiv D_1D$ and m_1^2 and m_2^2 are the roots of the quadratic in m^2 given by

$$\{\delta_{1k} + (p + a_0)\delta_{2k}\}m^4 - [(1 + \epsilon\zeta)a_2\delta_{1k}p + \{(1 + \alpha'\zeta a_3 + \alpha'a_2)\delta_{1k} + (a_0 + a_2 + a_1a_3 + p)\delta_{2k}\}p^2]m^2 + a_2\{(1 + \alpha'p)\delta_{1k} + p\delta_{2k}\}p^3 = 0 \tag{13}$$

with $\epsilon = \frac{a_1a_3}{a_2}$ being the thermo-elastic coupling constant.

As the solutions of Eqs. (11) and (12) we take (Watson, 1980)

$$\bar{U} = \sum_{i=1,2} A_i K_{3/2}(m_i R) / \sqrt{R} \tag{14}$$

and

$$\bar{\Theta} = \sum_{i=1,2} B_i K_{1/2}(m_i R) / \sqrt{R}, \tag{15}$$

where $K_{1/2}(m_i R)$ and $K_{3/2}(m_i R)$ are the modified Bessel functions of order 1/2 and 3/2 respectively; A_i and B_i are constants independent of R .

Therefore substituting the values of \bar{U} and $\bar{\Theta}$ in Eq. (10) we get

$$B_i = \frac{(p^2 - m_i^2)A_i}{(a_1 + \alpha' p\delta_{1k})m_i}, \quad i = 1, 2. \tag{16}$$

The condition $\bar{\sigma}_R = 0$ on $R = 1$ gives

$$A_1 \left[-2K_{3/2}(m_1) - \frac{\lambda + 2\mu}{2\mu m_1} p^2 K_{1/2}(m_1) \right] + A_2 \left[-2K_{3/2}(m_2) - \frac{\lambda + 2\mu}{2\mu m_2} p^2 K_{1/2}(m_2) \right] = 0. \tag{17}$$

Also $\bar{\Theta} = \frac{\zeta}{p}$ on $R = 1$ gives

$$A_1 \frac{p^2 - m_1^2}{(a_1 + \alpha' p \delta_{1k}) m_1} K_{1/2}(m_1) + A_2 \frac{p^2 - m_2^2}{(a_1 + \alpha' p \delta_{1k}) m_2} K_{1/2}(m_2) - \frac{\chi}{p} = 0. \tag{18}$$

From the above relations Eqs. (17), (18) and the recurrence relations (Watson, 1980)

$$ZK'_v(Z) = K_v(Z) - ZK_{v+1}(Z), \tag{19}$$

$$ZK'_v(Z) = -vK_v(Z) - ZK_{v-1}(Z) \tag{20}$$

we obtain

$$A_1 = \chi(a_1 + \delta_{1k} \alpha' p) m_1 \left[2m_2 K_{3/2}(m_2) + \frac{\lambda + 2\mu}{2\mu} p^2 K_{1/2}(m_2) \right] / 2p \\ \times [(p^2 - m_1^2) m_2 K_{1/2}(m_1) K_{3/2}(m_2) - (p^2 - m_2^2) m_1 K_{1/2}(m_2) K_{3/2}(m_1)], \tag{21}$$

$$A_2 = -\chi(a_1 + \delta_{1k} \alpha' p) m_2 \left[2m_1 K_{3/2}(m_1) + \frac{\lambda + 2\mu}{2\mu} p^2 K_{1/2}(m_1) \right] / 2p \\ \times [(p^2 - m_1^2) m_2 K_{1/2}(m_1) K_{3/2}(m_2) - (p^2 - m_2^2) m_1 K_{1/2}(m_2) K_{3/2}(m_1)]. \tag{22}$$

Eqs. (21), (22) with Eqs. (16), (14) and (15) constitute the solutions for displacement and temperature fields in the transformed domain and $\bar{\sigma}_R$ and $\bar{\sigma}_\theta$ are determined by

$$\bar{\sigma}_R = \sum_{i=1,2} A_i \left[-2K_{3/2}(m_i R) - \frac{\lambda + 2\mu}{2\mu m_i} p^2 R K_{1/2}(m_i R) \right] / R^{3/2}, \tag{23}$$

$$\bar{\sigma}_\theta = \sum_{i=1,2} A_i \left[K_{3/2}(m_i R) - \frac{\lambda m_i^2 + (\lambda + 2\mu)(p^2 - m_i^2)}{2\mu m_i^2} R K_{1/2}(m_i R) \right] / R^{3/2}. \tag{24}$$

We have from Eq. (13)

$$m_1^2 + m_2^2 = \frac{p^2 [\delta_{1k} + a_0 \delta_{2k} + a_2 \{ (1 + \epsilon) \delta_{2k} + \alpha' \delta_{1k} \} + a_3 \alpha' \zeta \delta_{1k}] + a_2 (1 + \epsilon \zeta) p \delta_{1k} + p^3 \delta_{2k}}{\delta_{1k} + (p + a_0) \delta_{2k}}, \tag{25}$$

$$m_1^2 m_2^2 = \frac{a_2 \{ \delta_{1k} + (\alpha' \delta_{1k} + \delta_{2k}) p \} p^3}{\delta_{1k} + (p + a_0) \delta_{2k}}. \tag{26}$$

Then

$$m_1, m_2 = \frac{\sqrt{p}(\delta_{1k} + \sqrt{p} \delta_{2k})}{2(\delta_{1k} + \sqrt{(p + a_0) \delta_{2k}})} (\sqrt{\alpha} \pm \sqrt{\beta}), \tag{27}$$

where

$$\alpha, \beta = a_0 \delta_{2k} + (1 + \epsilon \zeta) a_2 + \{ 1 + (\alpha' \zeta a_3 + \alpha' a_2) \delta_{1k} \} p \pm 2\sqrt{a_2(p + a_0 \delta_{2k} + \alpha' p^2 \delta_{1k})}. \tag{28}$$

4. Numerical results and discussions

The solution in the space–time domain is obtained numerically by using Bellman et al. (1966) method for fixed value of the space variable and for $\eta = \eta_i, i = 1(1)7$, where η_i 's are computed from the roots of the shifted Legendre polynomial of degree 7 (see A). The computations for the state variables are carried out for different values of R ($R \geq 1$) and values of $\eta_i = 0.0257750, 0.138382, 0.352509, 0.693147, 1.21376, 2.04612, 3.67119$. The material chosen for numerical evaluation is aluminum–epoxy material. The physical data for such material are taken as Sharma and Chouhan (1999)

$$\begin{aligned} \rho &= 2.19 \text{ g/cm}^3, \quad \epsilon = 0.073, \quad T_0 = 20 \text{ }^\circ\text{C}, \\ \lambda &= 7.59 \times 10^{11} \text{ dy/cm}^2, \quad \mu = 1.89 \times 10^{11} \text{ dy/cm}^2, \\ C_v &= 0.23 \text{ cal/g }^\circ\text{C}, \quad K = 0.6 \times 10^{-2} \text{ cal/cm }^\circ\text{C s}, \\ \alpha &= 0.75 \times 10^{-13} \text{ s}, \quad \alpha^* = 0.5 \times 10^{-13} \text{ s}. \end{aligned}$$

In the case of GN theory K^* is an additional material constant depending on the material of the composite. For aluminum–epoxy composite material K^* is taken as $K^* = \frac{C_v(\lambda+2\mu)}{4}$ (Sharma and Chouhan, 1999).

We now present our results in the form of graphs (Figs. 1–5) to compare the thermal stresses σ_R and σ_θ in the cases of CTE, CCTE, TRDTE(GL) and TEWED(GN). The magnitudes of the variation of stresses are observed to have large values near the boundary of the hole where the step input of temperature has been applied and then become smaller and smaller with the passage of time and increase of radial distance in each of the theories, which shows the existence of wave fronts. This can also be verified from the expression of $\bar{\sigma}_R$ given in Eq. (23) involving modified Bessel functions which contain sine and cosine terms. The expression does not involve any singularities in the region under our consideration ($R \geq 1$). Figs. 1a and 1b show the radial stress σ_R and hoop stress σ_θ respectively against radial distance R for fixed time $\eta = .69$. It is observed that the magnitudes of stresses are large near the boundary of the hole and approach zero value for $R \geq 2.2$ (CTE,

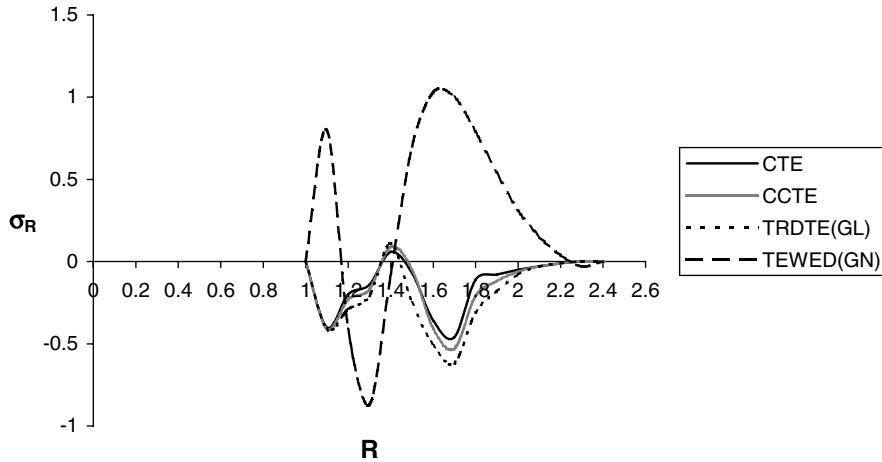


Fig. 1a. Non-dimensional radial stress with R for $\eta = .69$.

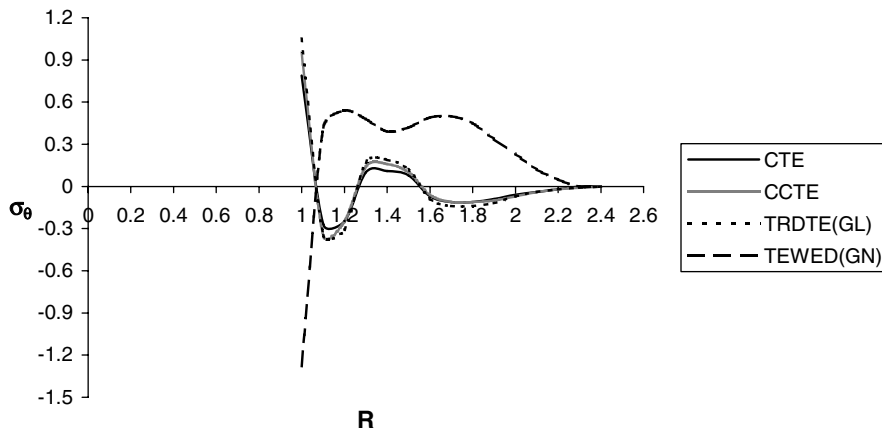


Fig. 1b. Non-dimensional hoop stress with R for $\eta = .69$.

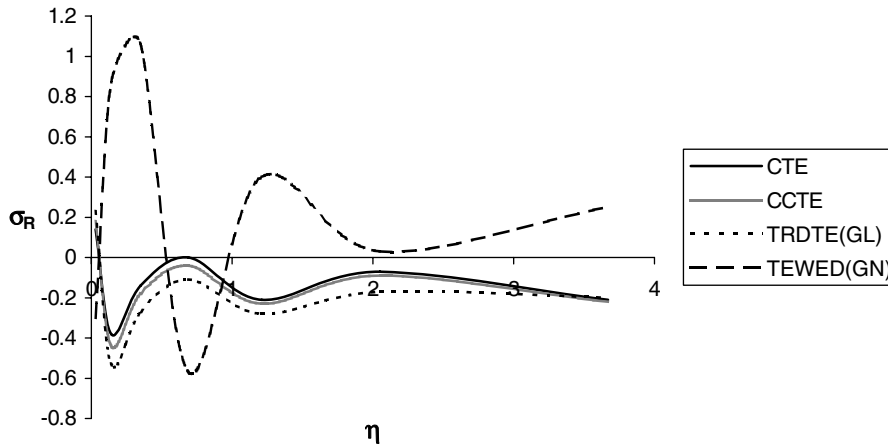


Fig. 2a. Non-dimensional radial stress with η for $R = 1.3$.

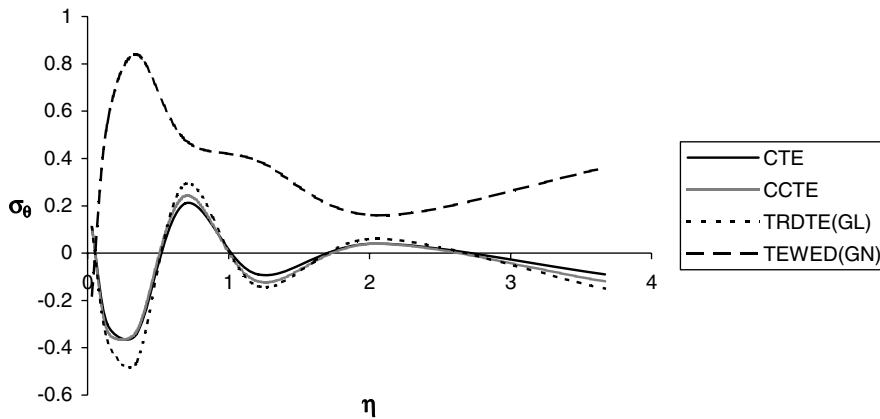


Fig. 2b. Non-dimensional hoop stress with η for $R = 1.3$.

CCTE, TRDTE(GL)) and $R \geq 2.3$ (TEWED(GN)) for $\eta = .69$. This is because the thermal wave front is positioned at $R = 2.2$ (at the instant $\eta = .69$) for CTE, CCTE, TRDTE(GL) and $R = 2.3$ (at the instant $\eta = .69$) for TEWED(GN) and beyond this wave front, the disturbance vanishes. This is physically plausible. Figs. 2a and 2b are plotted for radial stress and hoop stress respectively against time η for $R = 1.3$. Here we observe that as time increases the magnitudes of thermal stresses decrease. Also it is observed that the magnitudes of the thermo-elastic radial and hoop stress waves are large in the case of TEWED(GN) theory in comparison with the rest of the theories, though we have chosen energy dissipation theory in all cases. For small value of thermal conductivity of aluminum–epoxy material TEWED(GN) model coincides with without energy dissipation model (TEWOED). The boundary conditions on the boundary of the hole are found to be satisfied numerically (Fig. 1a). This is consistent with the theoretical results.

Figs. 3a and 3b depict radial stress σ_R and hoop stress σ_θ respectively against time for $R = 1.02$, i.e. very near to the cavity surface. In all the cases thermal stress distributions are large for small time ($\eta = .026$) which is consistent with the boundary conditions.

Fig. 4 shows the variation of hoop stress on the boundary $R = 1$. Here $\sigma_R = 0$ on $R = 1$ implies that the body is free to expand or contract along the radial distance. The thermal shock applied at $\eta > 0$, will generate some kind of discontinuous effect on the stress distribution in the medium near the inner boundary. But as the inner surface is free, the radial component of the stress will remain unaffected, it will continue to be zero, while σ_θ will be affected by the thermal shock. From Figs. 5a and 5b it is clear that the affects of thermal shock of the

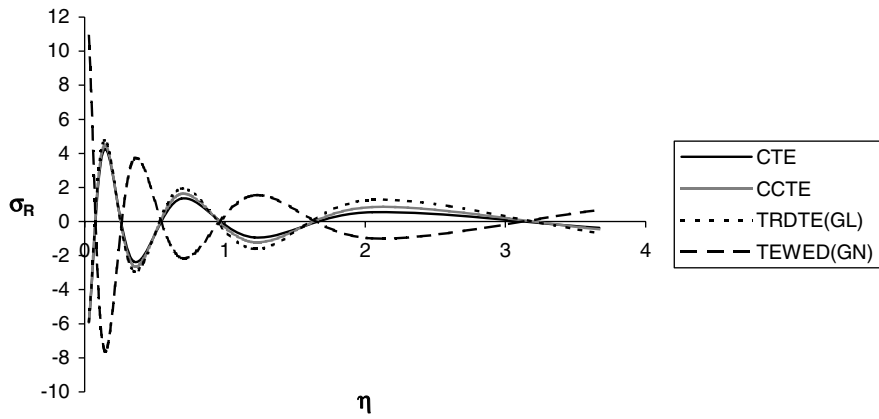


Fig. 3a. Non-dimensional radial stress with η for $R = 1.02$.

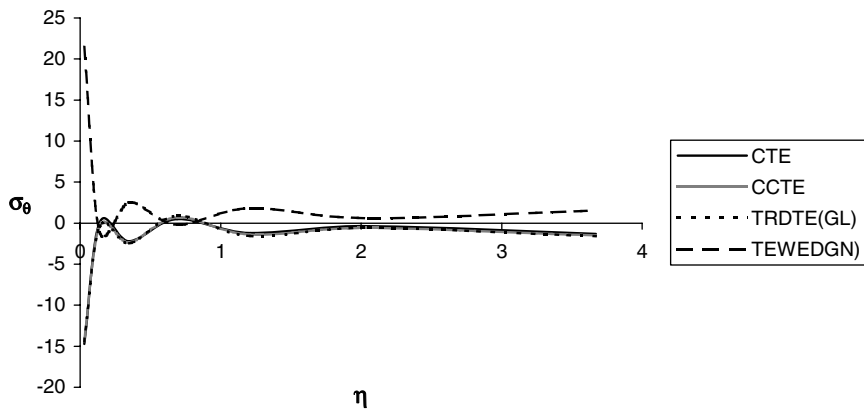


Fig. 3b. Non-dimensional hoop stress with η for $R = 1.02$.

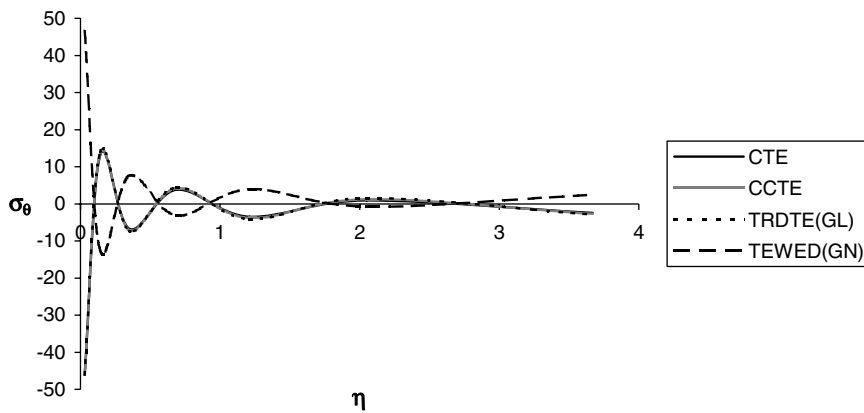


Fig. 4. Non-dimensional hoop stress with η for $R = 1.0$.

stress components are very prominent near the cavity surface for small time η . For all above numerical calculations FORTRAN-77 programming Language has been used.

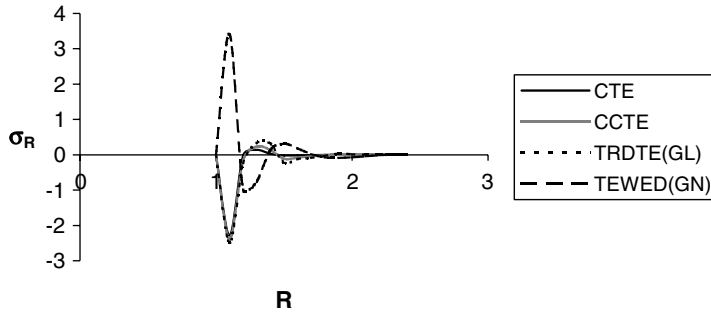


Fig. 5a. Non-dimensional radial stress with R for $\eta = .026$.

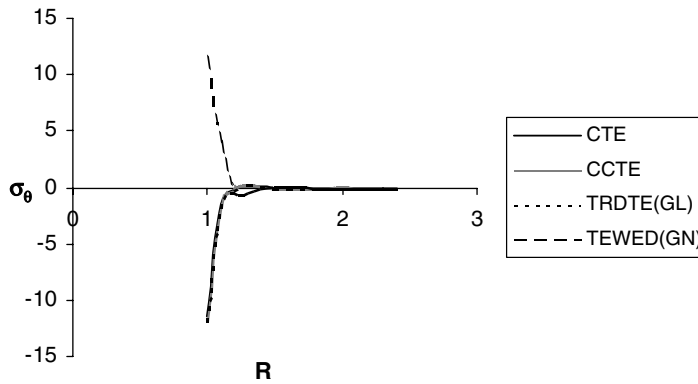


Fig. 5b. Non-dimensional hoop stress with R for $\eta = .026$.

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Appendix A

Let the Laplace transform of $\sigma_j(R, \eta)$ be given by

$$\bar{\sigma}_j(R, p) = \int_0^\infty e^{-p\eta} \sigma_j(R, \eta) d\eta. \tag{A.1}$$

We assume that $\sigma_j(R, \eta)$ is sufficiently smooth to permit the use of the approximate method we apply.

Putting $x = e^{-\eta}$ in Eq. (A.1) we obtain

$$\bar{\sigma}_j(R, p) = \int_0^1 x^{p-1} g_j(R, x) dx, \tag{A.2}$$

where

$$g_j(R, x) = \sigma_j(R, -\log x). \tag{A.3}$$

Applying the Gaussian quadrature rule to Eq. (A.2) we obtain the approximate relation

$$\sum_{i=1}^n W_i x_i^{p-1} g_j(R, x_i) = \bar{\sigma}_j(R, p), \tag{A.4}$$

where x_i 's ($i = 1, 2, \dots, n$) are the roots of the shifted Legendre polynomial and W_i 's ($i = 1, 2, \dots, n$) are the corresponding weights (Bellman et al., 1966).

Eq. (A.4) can be written as

$$\begin{aligned} W_1 g_j(R, x_1) + W_2 g_j(R, x_2) + \dots + W_n g_j(R, x_n) &= \bar{\sigma}_j(R, 1) \\ W_1 x_1 g_j(R, x_1) + W_2 x_2 g_j(R, x_2) + \dots + W_n x_n g_j(R, x_n) &= \bar{\sigma}_j(R, 2) \\ \dots & \\ \dots & \\ W_1 x_1^{n-1} g_j(R, x_1) + W_2 x_2^{n-1} g_j(R, x_2) + \dots + W_n x_n^{n-1} g_j(R, x_n) &= \bar{\sigma}_j(R, n) \end{aligned}$$

or

$$\begin{pmatrix} g_j(R, x_1) \\ g_j(R, x_2) \\ \dots \\ \dots \\ g_j(R, x_n) \end{pmatrix} = \begin{pmatrix} W_1 & W_2 & \dots & W_n \\ W_1 x_1 & W_2 x_2 & \dots & W_n x_n \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ W_1 x_1^{n-1} & W_2 x_2^{n-1} & \dots & W_n x_n^{n-1} \end{pmatrix}^{-1} \begin{pmatrix} \bar{\sigma}_j(R, 1) \\ \bar{\sigma}_j(R, 2) \\ \cdot \\ \cdot \\ \bar{\sigma}_j(R, n) \end{pmatrix}. \tag{A.5}$$

Hence $g_j(R, x_1), g_j(R, x_2), \dots, g_j(R, x_n)$ are known.

For $n = 7$ we have

Roots of the shifted Legendre polynomial	Corresponding weights
2.5446043828620886E-2	6.4742483084434816E-2
1.2923440720030282E-1	1.3985269574463828E-1
2.9707742431130145E-1	1.9091502525255938E-1
5.0000000000000000E-1	2.0897959183673466E-1
7.0292257568869853E-1	1.9091502525255938E-1
8.7076559279969706E-1	1.3985269574463828E-1
9.7455395617137909E-1	6.4742483084434816E-2

From equations in (A.5) we can calculate the discrete values of $g_j(R, x_i)$ i.e. $\sigma_j(R, \eta_i); (i = 1, 2, \dots, 7)$ and finally using interpolation we obtain the stress components $\sigma_j(R, \eta); (i = R, \theta)$.

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