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Tensor coupling and vector mesons in dense nuclear matter

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Abstract

The effects of magnetic interaction between vector mesons and nucleons on the propagation (mass and width) of the ρ-meson in particular moving through very dense nuclear matter is studied and the modifications, qualitative and quantitative, due to the relevant collective modes (zero-sound and plasma frequencies) of the medium discussed. It is shown that the ρ-mesons produced in high-energy nuclear collisions will be longitudinally polarized in the region of sufficiently dense nuclear matter, in the presence of such an interaction.

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1. Introduction

The study of the properties of hadrons in hot and dense nuclear matter is of cardinal importance in the understanding of various signals that probe the dynamics of heavy ion collisions [1, 2], as also in the study of the equation of state of superdense matter. Such investigations furthermore are of relevance to various important issues of nuclear astrophysics dealing with the properties of neutron stars, cooling of supernovae, the gravitational collapse of massive stars etc [3, 4]. Despite numerous theoretical attempts to determine the properties of vector mesons in dense nuclear matter, controversy still exists regarding their in-medium effective masses and decay widths [5]; various formalisms such as the NJL model, Walecka model or QCD sum rules [6, 7, 8, 9] give different results [10, 11, 12]. Calculation of mass shifts and decay widths of mesons in nuclear matter has assumed particular importance in view of proposals to make such measurements at CEBAF [13]. The essential focus of this paper is to investigate the effect of the magnetic interaction on vector mesons propagating in dense nuclear matter. In particular we study the case of the ρ meson where this effect is expected to be more pronounced.

The paper is organised as follows: the formalism is first set forth and this is succeeded by a discussion of the results.

2. Formalism

The vector meson-nucleon interaction Lagrangian may be written as

$$\mathcal{L}_{int} = g_\alpha[\bar{N}\gamma_\mu\tau^\alpha N - \frac{\kappa_\alpha}{2M}\bar{N}\sigma_{\mu\nu}\tau^\alpha N\partial^\nu]V_\alpha^\mu \tag{1}$$

where $V_\alpha = \{\omega, \rho\}$, α running from 0 to 3, indexes quantities relevant for ω when $\alpha = 0$, while $\alpha = 1$ to 3 refers to the ρ meson; $\tau^0 = 1$ and τ^i are the isospin Pauli matrices. The coupling constants g_ρ, g_ω and the "anomalous" or tensor-coupling parameters κ_ρ and κ_ω may be estimated [7] from the Vector Meson Dominance (VMD) of nucleon form-factors or from the fitting of the nucleon-nucleon interaction data as done by the Bonn group [14]. In view of the relatively small value of the iso-scalar magnetic moment of the nucleon as compared to the iso-vector part, the tensor coupling is more important for the ρ than it is for the ω. In our calculation the handling of these two mesons runs very similarly, the only essential difference residing in the values of the coupling parameters for the two cases. From the Lagrangian the VNN vertex factor is given by

$$\Gamma_\mu^\alpha = g_\alpha[\gamma_\mu\tau^\alpha - \frac{\kappa_\alpha}{2M}\sigma_{\mu\nu}\partial^\nu\tau^\alpha] \tag{2}$$

We shall consider the propagation of vector mesons in dense nuclear matter at zero temperature. As such we shall be following the usual methods of relativistic quantum field theory, with the vacuum replaced by the ground state of nuclear matter at zero temperature, specified by the Fermi momentum (k_F) corresponding to its density. Such a field theoretic approach to the study of many body problems was developed, in the context with which we are concerned, by Matsubara, Galitskii and Migdal [15, 16, 17], and the relativistic generalization provided by Fradkin [18], and most importantly in the approach being rather closely followed in this paper, by Chin [19]. Furthermore, the Hartree or Mean-Field energy density for dense nuclear matter goes as k_F^6, while the correlation energy varies as k_F^4 and this leads us to the position that we may use the mean-field description for nuclear matter at very high densities (that being our present concern).

The second order polarization tensor $\Pi_{\mu\nu}$, for the vector meson arising from the nucleon loop (Fig. 1), is thus calculable from the Lagrangian and yields

$$\Pi^{\alpha\beta}_{\mu\nu} = \frac{-i}{(2\pi)^4} \int d^4k \ \mathrm{Tr}[i\Gamma^{\alpha}_{\mu} iG(k+q) i\bar{\Gamma}^{\beta}_{\nu} iG(k)] \tag{3}$$

where (α, β) are the isospin indices and $G(k)$ is the in-medium nucleon propagator

$$G(k) = G_F(k) + G_D(k) \tag{4}$$

where

$$G_F(k) = (k_\mu \gamma^\mu + M^*)[\frac{1}{k^2 - M^{*2} + i\epsilon}] \tag{5}$$

and

$$G_D(k) = (k_\mu \gamma^\mu + M^*)[\frac{i\pi}{E^*(k)} \delta(k_0 - E^*(k)) \theta(k_F - |\vec{k}|)] \tag{6}$$

with M^* denoting the effective mass of the nucleon in the medium and $E^*(|k|) = \sqrt{|\vec{k}|^2 + M^{*2}}$. The first term in $G(k)$ namely $G_F(k)$ is the same as the free propagator of a spin $\frac{1}{2}$ Fermion, except for the fact that the effective mass of the nucleon is to be used, while the second part, $G_D(k)$, involving $\theta(k_F - |\vec{k}|)$, arises from Pauli blocking, describes the modifications of the same in the nuclear matter at zero temperature [20], as it deletes the on mass-shell propagation of the nucleon in nuclear matter with momenta below the Fermi momentum.

In a similar vein the polarization insertions can also be written as sum of two parts:

$$\begin{aligned} \Pi_{\mu\nu}(q) &= \Pi^F_{\mu\nu}(q) + \Pi^D_{\mu\nu}(q), & (7) \\ \Pi^F_{\mu\nu}(q) &= \frac{-i}{(2\pi)^4} \int d^4k \ \mathrm{Tr}[\Gamma_\mu G_F(k+q) \bar{\Gamma}_\nu G_F(k)] & (8) \end{aligned}$$

$\Pi^D_{\mu\nu}(q)$ denotes the density dependent part of the polarization and $\Pi^F_{\mu\nu}(q)$ denotes the free part i.e. it contains the effect of 'Dirac sea'. The density dependent part has a natural cut off because of the $\theta(k_F - |\vec{k}|)$ function whereas the free part is regularized by adding appropriate counter-terms to the Lagrangian. As known from ordinary quantum field theory the effect of the finite part of $\Pi_F(q)$ is to render the coupling constant momentum dependent and this modifies the interaction subtantially as short distances. Here the procedure of regularization and renormalization is very similar except that the nucleon mass M would have to be replaced by the effective nucleon mass in nuclear matter (M^*); the tensor coupling, however, belongs to the genre of unrenormalizable theories and has to be tackled in the spirit of an effective theory. In the case of ordinary nuclear matter densities this short distance modification is important as discussed in Ref [7], however, in the present context where we concentrate on the long wavelength limit, i.e. low momentum transfer to study the collective modes in dense nuclear matter, the effect of vacuum polarization would in general be small as pointed out by Chin [19].

The real part of the density dependent piece of the polarization is given by

$$\Pi^D_{\mu\nu} = \frac{g_v^2\pi}{(2\pi)^4}\int \frac{d^4k}{E^*(k)}\delta(k^0 - E^*(k))\theta(k_F - |\vec{k}|) \cdot\Big[\frac{T_{\mu\nu}(k-q,k)}{(k-q)^2 - M^{*2}} + \frac{T_{\mu\nu}(k,k+q)}{(k+q)^2 - M^{*2}}\Big] \tag{9}$$

Although the form of $\Pi^D_{\mu\nu}(q)$ is the same as that of Chin [19], here, however, it has three part corresponding to vector-vector, vector-tensor and tensor-tensor terms. Hence the self energy can be written as

$$\Pi^D_{\mu\nu}(q) = \Pi^{vv}_{\mu\nu}(q) + \Pi^{vt+tv}_{\mu\nu}(q) + \Pi^{tt}_{\mu\nu}(q) \tag{10}$$

The $\Pi^D_{\mu\nu}(q)$ functions in this case are as follows

$$\Pi^{vv}_{\mu\nu} = \frac{g_v^2}{\pi^3}\int_0^{k_F} \frac{d^3k}{E^*(k)}\frac{\mathcal{K}_{\mu\nu} - Q_{\mu\nu}(k\cdot q)^2}{q^4 - 4(k\cdot q)^2} \tag{11}$$

$$\Pi^{vt+tv}_{\mu\nu} = \frac{g_v^2}{\pi^3}(\frac{kM^*}{4M})2q^4Q_{\mu\nu}\int_0^{k_F} \frac{d^3k}{E^*(k)}\frac{1}{q^4 - 4(k\cdot q)^2} \tag{12}$$

$$\Pi^{tt}_{\mu\nu} = -\frac{g_v^2}{\pi^3}(\frac{k}{4M})^2(4q^4)\int_0^{k_F} \frac{d^3k}{E^*(k)}\frac{\mathcal{K}_{\mu\nu} + Q_{\mu\nu}M^{*2}}{q^4 - 4(k\cdot q)^2} \tag{13}$$

where $\mathcal{K}_{\mu\nu} = (k_\mu - \frac{k.q}{q^2}q_\mu)(k_\nu - \frac{k.q}{q^2}q_\nu)$ and $Q_{\mu\nu} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})$. It is clear that the form for the polarization tensor conforms to the requirement of current conservation, i.e.

$$q_\mu \Pi^D_{\mu\nu} = \Pi^D_{\mu\nu} q_\nu = 0 \tag{14}$$

In order to evaluate $\Pi^D_{\mu\nu}$ conveniently, we choose $\vec{k}$ to be along the x axis i.e. $q = (q_0, \mid \vec{q} \mid, 0, 0)$, and $k.q = \mid k \mid\mid q \mid \chi - E^*(k)q_0$, where χ is the cosine of the angle between $\vec{k}$ and $\vec{q}$. After ϕ integration the non-vanishing components $\Pi^D_{\mu\nu}$ are as shown below

$$\begin{pmatrix} \Pi_{00} & \Pi_{01} & 0 & 0 \\ \Pi_{10} & \Pi_{11} & 0 & 0 \\ 0 & 0 & \Pi_{22} & 0 \\ 0 & 0 & 0 & \Pi_{33} \end{pmatrix} \tag{15}$$

Also for isotropic nuclear matter we have $\Pi^D_{22} = \Pi^D_{33}$ and $\Pi^D_{01} = \Pi^D_{10}$, and hence taking all this into account we have only two non-vanishing independent component of $\Pi^D_{\mu\nu}$, linear combinations of which gives us the longitudinal and transverse components of $\Pi^D_{\mu\nu}$, namely, $\Pi^D_L(q) = -\Pi^D_{00} + \Pi^D_{11}(q)$ and $\Pi^D_T(q) = \Pi^D_{22} = \Pi^D_{33}$.

For collective excitations, the wavelength of the oscillations must be greater than the interparticle spacing. Thus for super-dense matter $q << k_F$ and we can simplify the denominators of the above integrations by neglecting q^4 compared to $4(k.q)^4$[19]. In this approximation the integrations can easily be performed to give results in a closed form, viz.,

$$\begin{aligned} \Pi^D_L(q_0, \mathbf{q}) &= -\frac{g_v^2}{\pi^2} k_F \epsilon_F \beta (1 - c_0^2) \Phi(c_0/v_F) \\ \Pi^D_T(q_0, \mathbf{q}) &= \frac{1}{2}\frac{g_v^2}{\pi^2}\frac{k_F^3}{\epsilon_F}[1 + \beta(1 - \frac{c_0^2}{v_F^2})\Phi(c_0/v_F)] \\ \beta &= 1 - (\frac{\kappa}{2M})^2 q^2 \end{aligned} \tag{16}$$

where

$$\Phi(x) = -1 + \frac{1}{2}x \ \ln|\frac{(x+1)}{(x-1)}| \tag{17}$$

and $c_0 = \frac{q_0}{|\vec{q}|}$, $v_F = \frac{k_F}{\epsilon_F}$ and ϵ_F is the Fermi energy $\epsilon_F = \sqrt{M^{*2} + k_F^2}$. In the limit $\kappa \to 0$, of course, the results of Ref[19] ensue.

The effective mass of the ρ mesons in hadronic matter is described by the poles of the in-medium ρ meson propagator. For short wavelengths the oscillation of meson fields can be regarded as the usual meson propagation, as in that regime the effect of Pauli blocking is not appreciable, but in the long wavelength limit medium effects can be substantial and these can be beautifully interpreted as arising from the meson 'picking up' the collective modes of the nucleonic Fermi fluid [21]. The vector meson propagator is calculated by summing over ring diagrams, a diagrammatic equivalent of the random phase approximation (RPA), which consist of repeated insertions of the lowest order polarization, as illustrated in Fig. 2. We make use of Dyson's equation to carry out the summation

$$D_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\alpha}(q)\Pi^{\alpha\beta}(q)D_{\beta\nu}(q) \tag{18}$$

The poles are determinable from the equation

$$det[\delta^\nu_\mu - D^0_{\mu\alpha}\Pi_{\alpha\nu}] = 0 \tag{19}$$

The bracketed term is nothing but the dielectric tensor of the system

$$\epsilon^\nu_\mu = \delta^\nu_\mu - D^0_{\mu\alpha}\Pi^{\alpha\nu} \tag{20}$$

the determinant of which is the dielectric function of the system . The eigen-condition can now be expressed as

$$\epsilon(q) = 0. \tag{21}$$

As $\Pi_{\mu\nu}(q)$ is already known, $\epsilon(q)$ can be calculated immediately . The relevance of the set of ring diagrams and the origin of such an eigen-condition can be understood from linear response theory [22], where the fluctuation of the current density, the source term for the meson field in nuclear matter, given in terms of the polarisation tensor, is 'picked up' by the vector field.

We have already shown that $\Pi^D_{\mu\nu}(q)$ manifestly satisfies the current conservation conditions, and with the choice of q_μ already discussed, assumes a particularly simple structure eq. (15). The longitudinal and tranverse dielectric functions are defined as

$$\epsilon_T(q) = 1 - D^0\Pi^D_{22} = 1 - D^0\Pi_{33} = 1 - D^0\Pi^D_T \tag{22}$$

$$\epsilon_L(q) = (1 - D^0\Pi^D_{00})(1 - D^0\Pi_{11}) - D^0\Pi^D_{01}D^0\Pi^D_{10} \tag{23}$$

The explicit form of the longitudinal and transverse dielectric functions can be obtained with the help of eqs. (16) and (17). to yield,

$$\epsilon_T = 1 + \frac{1}{q^2 - m^2}\frac{1}{2}\frac{g_v^2}{\pi^2}\frac{k_F^3}{\epsilon_F}[1 + \beta(1 - (\frac{c_0}{v_F})^2)\Phi(c_0/v_F)] \tag{24}$$

$$\epsilon_L = 1 + \frac{1}{q^2 - m^2}\frac{g_v^2}{\pi^2}k_F^3\epsilon_F\beta(1 - (\frac{c_0}{v_F})^2)\Phi(c_0/v_F) \tag{25}$$

The eigenmodes of the vector meson are given by

$$\epsilon(q) = \epsilon^2_T(q)\epsilon_L(q) \tag{26}$$

corresponding to the three degrees of fredom of a massive vector particle. The two identical (or degenerate) transverse collective modes are each given by

$$\epsilon_T(q) = 0 \tag{27}$$

and the single longitudinal mode by

$$\epsilon_L(q) = 0 \tag{28}$$

which yield the relevant dispersion curves. Solutions to be sought may be classified as space-like ($|\vec{q}| > q_0$) or time-like ($|\vec{q}| < q_0$), the latter being the 'particle' modes, while the former corresponds, as we shall see, to zero sound.

3. Results and discussions

As already stated earlier, for collective excitations the wavelengths of the collective oscillations must be greater than the interparticle spacings, to wit, $|\vec{q}| << k_F$. Furthermore, the stability of the collective modes requires on the one hand that the dispersion curves be such that q_0^2 be non-negative or else q_0 would be imaginary and accordingly correspond to physically unacceptable exponentially growing fluctuations; on the other hand these modes must furthermore be such that they are not dissipated through nucleon-antinucleon decay (anticipating that the mass of the rho-meson, for instance, in dense nuclear matter would make this energetically possible) for the time-like solutions, or the creation of a nucleon-nucleon-hole pair (that is the absorption of the rho to lift a nucleon from below the Fermi sea to one above above it, creating thereby a hole-particle pair) in the case of a space-like mode. Accordingly, as has already been discussed in Ref [19, 20], for the collective modes to be undamped the following condition must be fulfilled $\frac{q_0}{|\vec{q}|} > v_F$ thus we can expand Φ for $c_0/v_F >> 1$. In this case we can retain only first few terms of the expansion which gives $\Phi(c_0/v_F) = \frac{1}{3c_0^2}$ (as $v_F \rightarrow 0$). In this limit for the longitudinal modes one arrives at the dispersion relation

$$q_0^4(1 + \frac{\Omega^2\kappa^2}{4M^2}) - q_0^2(\Omega^2 + |\vec{q}|^2 + m^2 + \frac{\Omega^2\kappa^2}{2M^2}|\vec{q}|^2) + \Omega^2|\vec{q}|^2(1 + \frac{\kappa^2|\vec{q}|^2}{4M^2}) = 0 \quad (29)$$

from the equation $\epsilon_L = 0$, while for the transverse modes one obtains

$$q_0^4(1 - \frac{\Omega^2\kappa^2}{8M^2}) - q_0^2(\Omega^2 + |\vec{q}|^2 + m^2 - \frac{\Omega^2\kappa^2}{4M^2}|\vec{q}|^2) - \frac{1}{2}\Omega^2|\vec{q}|^2(1 + \frac{\kappa^2|\vec{q}|^2}{4M^2}) = 0. \quad (30)$$

Here Ω, given by $\Omega^2 = \frac{1}{3}\frac{g_V^2}{\pi^2}\frac{k_F^3}{\epsilon_F}$, can be interpreted as the relativistic generalisation of the familiar plasma frequency [19] with e^2 replaced by $g_V^2/4\pi$.

One of the solutions of the dispersion relations for the longitudinal mode is such that as $|\vec{q}| \rightarrow 0$, $q_0 \rightarrow 0$. For such low lying excitations, we have

$$q_0^2 = \frac{\Omega^2|\vec{q}|^2}{\Omega^2 + m^2 + (1 + \frac{\Omega^2\kappa^2}{2M^2})|\vec{q}|^2} \quad (31)$$

when $|\vec{q}| \rightarrow 0$, $c_0 = \frac{q_0}{|\vec{q}|}$ approaches a constant. This form of the dispersion relation is characteristic of acoustic (sound) propagation. At zero temperature ordinary sound propagation is not possible in a Fermi fluid. Following Landau, one therefore identifies this branch of collective modes as zero sound branch which differ in nature from ordinary sound in that what is involved is not a breathing of the Fermi sphere, but its shape change without alteration of the volume, the limiting Fermi surface having the form of a surface of revolution elongated in the forward direction of the propagation of the wave [21].

On the other hand branches corresponding to high lying excitations are obtained for both the longitudinal and the transverse modes. In the static limit, these yield

$$q_{0T}^2 = \frac{m^2 + \Omega^2}{1 - \frac{\Omega^2 \kappa^2}{8M^2}} \tag{32}$$

$$q_{0L}^2 = \frac{m^2 + \Omega^2}{1 + \frac{\Omega^2 \kappa^2}{4M^2}}. \tag{33}$$

These modes correspond to collective oscillations 'picked up' by the meson and the branches (transverse and longitudinal) are known as the particle or mesonic branches. Here again Ω is the plasma frequency, but unlike the case discussed by Chin [19] where in the static limit $q_{0\mathrm{T}}^2 = q_{0\mathrm{L}}^2 = m^2 + \Omega^2$, the two modes are split. Though similar in form, our results are different from those of Chin [19] because of the inclusion of tensor coupling.

The main difference resides in the fact that in the presence of the 'magnetic' coupling the transverse mode of the mesonic branches not only lies higher (as is also the case in Chin's discussion) but is split from the longitudinal branch even in the static limit ($|\vec{q}| = 0$). This is because the interaction between two 'magnetic dipoles' does not possess azimuthal symmetry. Furthermore the longitudinal branch is substantially lowered and the transverse mode elevated in high density nuclear matter compared to the pure vector-coupling case. The non-monotonic nature of the dispersion curve for the transverse mode is an interesting feature which appears in the presence of the magnetic coupling. Nevertheless some of these amusing aspects may not be of much physical relevance for two reasons, namely, for large values of q_0 and $|\vec{q}|$ the collective mode ceases to have any physical meaning (the wavelength is less than the interparticle spacing), and for values of q_0 and $|\vec{q}|$ not so large (but larger than $2k_F$) the mode is unstable, and the meson decays into a nucleon-antinucleon pair. For high nuclear matter density, the transverse mode, even in the static limit, is unstable, as is evident from Fig. 3. This shows that in the presence of magnetic coupling, in the regime of high density nuclear matter, the produced ρ will be entirely longitudinally polarized. On the other hand, the observed flattening of the zero-sound branch indicates a reduction of the repulsive force between nucleons due to the magnetic interaction, as is evident from eq. (31). Of course, the whole analysis has been done for symmetric nuclear matter; the case where the nuclear matter is asymmetric is under study.

In summary, one can say that in the presence of the magnetic interaction, leads, apart from other effects, to a suppression and even the absence of the transverse mode of ρ in nuclear matter which is dense enough and this feature should be observable in high-energy nuclear collisions.

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Figure Captions

1. Self-energy diagram of the ρ-meson.

2. Diagrammatical representation of the Dyson equation.

3. Different collective modes with and without the magnetic interaction. The solid lines represent the dispersion curves with magnetic coupling κ taken to be 6, while the dotted curves correspond to the case without magnetic coupling. The density of nuclear matter used for illustration is that corresponding to $k_F = 3.2$ fm^{-1}. The value of g_V taken is 2.6 and the units used are MeV.