

TAMING THE SCALAR MASS PROBLEM WITH A SINGLET HIGGS BOSON

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Abstract

We investigate the fine-tuning problem in the Standard Model and show that Higgs boson and top quark masses consistent with current experimental bounds cannot be obtained unless one extends the particle spectrum. A minimal extension which achieves this involves addition of a singlet real scalar and one generation of vectorlike fermions. We show that this leads to a phenomenologically viable prediction for the mass of the Standard Model Higgs boson.

October 1994

1. Introduction

Electroweak precision tests at the CERN e^+e^- collider LEP and the recently reported discovery of the top quark at Fermilab [1] have established beyond reasonable doubt the fact that the Standard Model (SM) is an excellent description of fundamental interactions at least upto the electroweak symmetry-breaking scale. Nevertheless, there is a general belief that the SM does not tell us the whole story, but merely provides an effective Lagrangian of a deeper underlying theory which is yet to be established. One of the chief reasons for such a belief is the so-called *fine-tuning* problem.

In a nutshell, the fine-tuning problem is the following. The masses of scalars — specifically the Higgs boson — receives radiative corrections which are quadratically divergent. If the SM ceases to be applicable at a scale Λ , the mass of the Higgs boson would, therefore, be driven to the same order Λ . That this cannot be so is known from the fact that this would result in a strongly-interacting scalar sector where perturbation theory would break down. One is, therefore, driven to argue that the tree-level mass of the Higgs boson must cancel with the radiatively-induced self-energy function to yield acceptable values of the physical mass of the Higgs boson (60 GeV – 1 TeV). Taking Λ to be the symmetry-breaking scale of Grand Unified Theories (GUTs), *i.e.* $\Lambda \sim 10^{16}$ GeV, this implies an unnatural cancellation of about 26 – 28 orders of magnitude.

The fine-tuning problem described above affects the masses of scalars only, since the masses of fermions and vector bosons are protected by chiral and gauge symmetries, so that their radiative corrections can have only logarithmic divergences. This can be clearly seen on computation of radiative corrections to, say, the mass of the Z -boson, where the quadratic divergences in individual diagrams will cancel in the final result. With this idea in mind, an elegant restatement of the fine-tuning solution to the problem of runaway corrections to scalar masses is the so-called *Veltman condition* [2]. Assuming that the underlying theory has some yet-to-be-discovered symmetry which protects the scalar mass, one simply sets to zero the sum of computed quadratic divergences in the radiative corrections to the scalar self-energy. Clearly this implies some relation between the physical Higgs boson mass and the masses of other particles such as the top quark and the gauge bosons. The *explanation* of such a relationship must lie, as already stated, in the underlying theory. For a phenomenological study, however, the Veltman condition is very useful, since it reduces, to some degree, the arbitrariness in the choice of top quark and Higgs boson masses.

Application of the Veltman condition to a model implies a little more than mere cancellation of the coefficients of the quadratic divergences in self-energy diagrams of

the scalars. The condition must not change with renormalisation group (RG) flow of the couplings. Thus, if $f(g_i, m_i)$ be the net coefficient of the quadratic divergence in question, then the Veltman condition is

$$f(g_i, m_i) \sim \frac{v^2}{\Lambda^2} \quad (1)$$

where $v = \langle 0 | H^0 | 0 \rangle$. Stability under RG flow requires

$$\frac{d}{dt} f(g_i, m_i) = 0 \quad (2)$$

where $t \equiv \ln(\frac{Q^2}{\mu^2})$. These two equations would lead to a unique prediction for m_t, m_H in the SM. Unfortunately one does not obtain any real solution to these equations.

One solution to the fine-tuning problem lies in banishing fundamental scalars from the theory altogether. Attempts have been made in this direction, but without conspicuous success. The other solution to this dilemma seems to lie in extension of the SM beyond its minimal particle content. Typical of such solutions is supersymmetry, where pairing of bosons and fermions occurs in such a way that contributions to $f(g_i, m_i)$ cancel pairwise for every SM particle and its superpartner. Even the minimal supersymmetric extension of the SM, however, requires the addition of 66 particles to the SM! It is desirable, therefore, to consider minimal extensions of the SM particle spectrum to see if the Veltman condition can be satisfied more economically.

In this paper, we first consider the addition of vector singlet and doublet fermions to the SM and show that this extension also fails to yield a real solution. However, the further addition of a singlet real scalar (which has no interactions with the gauge sector but interacts with the Higgs doublet and the vectorlike fermions) not only can satisfy the Veltman condition (for both doublet and singlet), but also leads to potentially interesting predictions from a phenomenological point of view.

In Section 2, we discuss the Veltman condition in the SM and show that it has no real solution. We also show that inclusion of vector singlet or doublet fermions does not improve the situation. Section 3 is devoted to a model with a singlet real scalar and vectorlike exotic fermions. Finally, our conclusions are given in Section 4.

2. Veltman Condition in the SM

In this work, we consider the coefficients of quadratic divergences generated at the one-loop level, anticipating that contributions from higher orders will be suppressed by

powers of the coupling constants. To this order, then, the Veltman condition has the most general form

$$|m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2| \leq \frac{16\pi^2}{3\Lambda^2} v^2 m_H^2 \quad (3)$$

In the limit $\Lambda \gg v$, this leads to a simple relation

$$m_H^2 \simeq 4m_t^2 - 2m_W^2 - m_Z^2 \quad (4)$$

which yields $m_H = 182 \pm 22$ GeV for $m_t = 174 \pm 17$ GeV. In determining the above, we set $m_W = 80.2$ GeV, $m_Z = 91.2$ GeV. If we allow new physics to appear at a lower scale, say 10 TeV, in which case the right side of equation (3) is of the order of m_H^2 , the uncertainty in m_H is increased by about 5 GeV either way.

We can rewrite equation (4) using the tree-level relations between masses and coupling constants in the SM. This leads to the alternative form

$$8\lambda + g_1^2 + 3g_2^2 - 8g_t^2 \simeq 0 \quad (5)$$

where $m_H^2 = 2\lambda v^2$, $m_t = g_t v / \sqrt{2}$. If we now impose RG stability on this equation, we demand

$$\frac{d}{dt}[8\lambda + g_1^2 + 3g_2^2 - 8g_t^2] = 0. \quad (6)$$

Using the well-known β -functions of the SM, *viz.*

$$16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 6g_t^2\lambda - \frac{3}{2}g_1^2\lambda - \frac{9}{2}g_2^2\lambda - 3g_t^4 + \frac{3}{16}g_1^4 + \frac{3}{8}g_1^2g_2^2 + \frac{9}{16}g_2^4, \quad (7)$$

$$16\pi^2 \frac{dg_t}{dt} = \left(\frac{9}{4}g_t^2 - \frac{17}{24}g_1^2 - \frac{9}{8}g_2^2 - 4g_3^2\right)g_t, \quad (8)$$

$$16\pi^2 \frac{dg_1}{dt} = \frac{41}{12}g_1^3, \quad (9)$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{19}{12}g_2^3, \quad (10)$$

$$16\pi^2 \frac{dg_3}{dt} = -\frac{7}{2}g_3^3\theta(q^2 - m_t^2) - \frac{23}{6}g_3^3\theta(m_t^2 - q^2) \quad (11)$$

we obtain

$$\begin{aligned} &72\lambda^2 + 36g_t^2\lambda - 45g_t^4 - 9g_1^2\lambda - 27g_2^2\lambda \\ &+ \frac{25}{4}g_1^4 - \frac{15}{4}g_2^4 + \frac{9}{4}g_1^2g_2^2 + 48g_3^2g_t^2 + \frac{17}{2}g_1^2g_t^2 + \frac{27}{2}g_2^2g_t^2 = 0, \end{aligned} \quad (12)$$

Numerical studies show that equations (5,12) have no real solutions for m_t , m_H in the range $10 \text{ GeV} < m_t < 2 \text{ TeV}$. This tells us that even if the Veltman condition is satisfied

at a low energy scale, it is not valid when we go to high energies, where the problem of runaway scalar masses reappears.

Some authors [3] have argued that g_3 should not appear in the above analysis, since mass generation is essentially an electroweak phenomenon. We do not agree with this point of view, as g_3 appears only in the RG evolution of g_t , where its role is known to be important. In any case, exclusion of g_3 does not improve matters significantly ¹. We also note, in passing, that even if one considers a lower value of Λ one does not obtain real solutions, though, in this case, the fine-tuning problem is not so severe.

Let us now consider an extension of the SM particle spectrum by a single generation of exotic vectorlike singlet or doublet fermions. We have not discussed an extra sequential generation, or a generation of mirror fermions, since these are severely constrained by electroweak precision tests at LEP [4]. Vectorlike singlets are not at all constrained by these data, while doublets are merely constrained by the oblique parameter T to be nearly mass-degenerate. A lower bound on the masses of vectorlike fermions from LEP data is 45 GeV, while an analysis of CDF data tells us that vectorlike quarks must be heavier than 90 GeV [5]. However, these masses play no role in the subsequent discussion.

To study the Veltman condition taking these fermions into account, one notes that they can have gauge-invariant mass terms and can also couple to the SM gauge bosons according to their quantum number assignments. Taking these into account, one now obtains β -functions

$$16\pi^2 \frac{dg_1}{dt} = \frac{187}{36} g_1^3, \quad (13)$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{19}{12} g_2^3, \quad (14)$$

$$16\pi^2 \frac{dg_3}{dt} = -\frac{17}{6} g_3^3 \theta(q^2 - m_t^2) - \frac{19}{6} g_3^3 \theta(m_t^2 - q^2) \quad (15)$$

for vector singlets and

$$16\pi^2 \frac{dg_1}{dt} = \frac{139}{36} g_1^3, \quad (16)$$

$$16\pi^2 \frac{dg_2}{dt} = -\frac{1}{4} g_2^3, \quad (17)$$

$$16\pi^2 \frac{dg_3}{dt} = -\frac{17}{6} g_3^3 \theta(q^2 - m_t^2) - \frac{19}{6} g_3^3 \theta(m_t^2 - q^2) \quad (18)$$

for vector doublets. Consequently, equation (12) gets modified to

$$72\lambda^2 + 36g_t^2\lambda - 45g_t^4 - 9g_1^2\lambda - 27g_2^2\lambda$$

¹One gets a real solution $m_t = 117$ GeV, a value more or less ruled out by the CDF data [1].

$$+\frac{107}{12}g_1^4 - \frac{15}{4}g_2^4 + \frac{9}{4}g_1^2g_2^2 + 48g_3^2g_t^2 + \frac{17}{2}g_1^2g_t^2 + \frac{27}{2}g_2^2g_t^2 = 0, \quad (19)$$

and

$$\begin{aligned} &72\lambda^2 + 36g_t^2\lambda - 45g_t^4 - 9g_1^2\lambda - 27g_2^2\lambda \\ &+\frac{83}{12}g_1^4 + \frac{9}{4}g_2^4 + \frac{9}{4}g_1^2g_2^2 + 48g_3^2g_t^2 + \frac{17}{2}g_1^2g_t^2 + \frac{27}{2}g_2^2g_t^2 = 0, \end{aligned} \quad (20)$$

respectively.

Numerical studies of the above equation again reveal that there is no real solution for m_t, m_H as in the SM. This is true even if we consider more than one extra generation since the system of equations changes little unless the number of extra generations is very large. We conclude, therefore, that mere inclusion of vectorlike fermions does not provide a solution to the fine-tuning problem.

3. The Singlet Higgs Boson Option

Let us now consider the minimal extension of the SM scalar sector by a singlet real scalar, h^0 , which has all $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers equal to zero and hence does not couple with any of the gauge bosons of the SM. Thus the presence of h^0 does not change eqs. (9-11).

We have made three assumptions about the scalar potential. First, the potential is bounded from below, which is, strictly speaking, a necessary requirement and not an assumption. Second, h^0 and H^0 , the SM Higgs boson, do not mix with each other ². Third, h^0 does not have a vacuum expectation value (VEV). As a VEV of h^0 will not affect the masses of the sequential fermions and the gauge bosons, by keeping it equal to zero we are not losing any generality. The last condition allows us to write a term of the form $\tilde{m}^2 h^2$ in the scalar potential. Thus, we can write the full potential for doublet Φ and singlet h as

$$\mathcal{V}_{scalar} = -m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 + \tilde{m}^2h^2 + \tilde{\lambda}h^4 + a(\Phi^\dagger\Phi)h^2 \quad (21)$$

which immediately gives

$$m_h^2 = 2\tilde{m}^2 + av^2. \quad (22)$$

²If they do, some quantitative results may change but no qualitative change of what we will discuss takes place.

The RG equations for λ , $\tilde{\lambda}$ and a are

$$16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 6g_t^2\lambda - \frac{3}{2}g_1^2\lambda - \frac{9}{2}g_2^2\lambda - 3g_t^4 + 6a^2 + \frac{3}{16}g_1^4 + \frac{3}{8}g_1^2g_2^2 + \frac{9}{16}g_2^4, \quad (23)$$

$$16\pi^2 \frac{d\tilde{\lambda}}{dt} = 36\tilde{\lambda}^2 + a^2, \quad (24)$$

$$16\pi^2 \frac{da}{dt} = \left(36\lambda + 72\tilde{\lambda} + 6g_t^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right)a. \quad (25)$$

Using this equation in the Veltman condition for the singlet field

$$3\tilde{\lambda} + a = 0 \quad (26)$$

we get the RG stability condition

$$a \left[a - \left(4\lambda + \frac{2}{3}g_t^2 - \frac{1}{6}g_1^2 - \frac{1}{2}g_2^2\right) \right] = 0 \quad (27)$$

Eqs. (26) and (27) have no nontrivial solution for $m_t > 102$ GeV, when the quantity in parentheses becomes positive. When coupled also with the Veltman condition and the RG stability condition of the SM Higgs, the set of equations have no real solutions. Thus, the inclusion of just a singlet scalar field cannot make the fine-tuning problem vanish at all scales upto Λ .

Let us now introduce the singlet real scalar field in conjunction with vectorlike exotic fermions (F). A phenomenological study [6] of such models reveals that there is no real bound from LEP-1 data on the masses and couplings of the scalar and exotic fermions. The Yukawa Lagrangian is modified to

$$\mathcal{L}_Y^{exotic} = -\zeta_F h \bar{F} F. \quad (28)$$

This introduces four new parameters in our analysis, *viz.* ζ_N , ζ_E , ζ_U and ζ_D where the suffixes are self-explanatory. For simplicity, we will take $\zeta_N = \zeta_E = \zeta_U = \zeta_D = \zeta$. We have checked that the predictions do not change if we relax this assumption.

As the vector fermions do not couple with the SM Higgs boson, the Veltman condition for H^0 will read

$$6\lambda + a + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2 = 0, \quad (29)$$

while eq. (26) will be modified to

$$3\tilde{\lambda} + a - b\zeta^2 = 0 \quad (30)$$

where $b = 8$ under the assumption that all ζ 's are equal. Equations (24) and (25) will be modified to

$$16\pi^2 \frac{d\tilde{\lambda}}{dt} = 36\tilde{\lambda}^2 + a^2 + 4b\tilde{\lambda}\zeta^2 - b\zeta^4, \quad (31)$$

$$16\pi^2 \frac{da}{dt} = (36\lambda + 72\tilde{\lambda} + 6g_t^2 + 4b\zeta^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2)a. \quad (32)$$

The RG stability equations are

$$72\lambda^2 + 36g_t^2\lambda - 45g_t^4 + 36a^2 + 36a\lambda + 72a\tilde{\lambda} + 6g_t^2a - 9g_1^2\lambda - 27g_2^2\lambda + \alpha_1g_1^4 + \alpha_2g_2^4 + \frac{9}{4}g_1^2g_2^2 + 48g_3^2g_t^2 + \frac{17}{2}g_1^2g_t^2 + \frac{27}{2}g_2^2g_t^2 - \frac{3}{2}ag_1^2 - \frac{9}{2}ag_2^2 + 4b\zeta^2a = 0, \quad (33)$$

$$12(36\tilde{\lambda}^2 + a^2 + 4b\tilde{\lambda}\zeta^2) + 4(36a\lambda + 72a\tilde{\lambda} + 6g_t^2a + 4b\zeta^2a) - 30b\zeta^4 - \frac{3}{2}ag_1^2 - \frac{9}{2}ag_2^2 = 0, \quad (34)$$

where $\alpha_1 = 107/12(83/12)$ and $\alpha_2 = -15/4(9/4)$ for vector singlet (doublet) exotic fermions.

Our results are shown in Table 1 for singlet fermions and Table 2 for doublet fermions. One notes that here we can simultaneously solve four equations at some particular point of the $\lambda, \tilde{\lambda}$ space for a given m_t . m_H lies in the range currently favoured by the electroweak precision tests. a comes out to be negative but the potential still remains bounded from below. It is also noteworthy that the solution comes out in the perturbative domain of the couplings, *i.e.*, $|\lambda|, |\tilde{\lambda}|, |a|, |\zeta^2| \leq 4\pi$.

An easy way to see whether any change occurs in the result if we relax the assumption on the equality of all the ζ 's, we take (i) $\zeta_N = \zeta_E = 2\zeta_U = 2\zeta_D$ and (ii) $\zeta_U = \zeta_D = 2\zeta_N = 2\zeta_U$. For case (i), we have to put $b = 10$ in all the abovementioned equations, and for case (ii), the required value is $b = 14$. The solutions are observed to be unchanged. One can carry out other checks in a similar way, with the same result.

4. Conclusions

We have shown that the Veltman condition together with its RG stability fails to produce any acceptable solution in the SM. This leads us to extend the SM, first in the fermionic sector by introducing vectorlike exotic fermions, and then in the scalar sector by introducing a singlet real scalar. However, both of these extensions individually fail to produce any real solution to the Veltman condition. When we consider both exotic fermions as well as a singlet scalar in the particle spectrum, not only do we get solutions

to the Veltman conditions for the two scalars, but we also get a prediction of m_H , which is in the experimentally favoured range. The couplings also come out to be perturbative in nature, which is essential for the self-consistency of the entire scheme. This appears to be an encouraging result which should motivate searches for singlet Higgs bosons and exotic vectorlike fermions at the upcoming colliders.

Acknowledgements

This work was partially supported by the Department of Science and Technology, India (DO No. SR/SY/P-08/92). AK acknowledges the hospitality of the Theoretical Physics Group of the Tata Institute of Fundamental Research, Bombay.

References

- [1] F. Abe *et al* (CDF collaboration) Phys. Rev. **D50**, 2966 (1994).
- [2] M. Veltman, Acta Phys. Polon. **B12**, 437 (1981).
- [3] E. Ma, Mod. Phys. Lett. **A7**, 2741 (1992).
- [4] G. Bhattacharyya *et al*, Phys. Rev. Lett. **64**, 2870 (1990).
- [5] B. Mukhopadhyaya and D.P. Roy, Phys. Rev. **D48**, 2105 (1993).
- [6] S. Raychaudhuri and A. Raychaudhuri, Phys. Rev. **D44**, 2663 (1991).

Table Captions

Table 1

The predicted parameters for vector singlet fermions with m_t as input. All masses are in GeV.

Table 2

The predicted parameters for vector doublet fermions with m_t as input. All masses are in GeV.

Table 1

m_t	λ	$\tilde{\lambda}$	a	ζ	m_H
150	0.78	1.06	-1.26	0.49	307
155	0.84	1.11	-1.33	0.50	319
160	0.90	1.16	-1.37	0.51	330
165	0.96	1.22	-1.41	0.53	341
170	1.03	1.27	-1.50	0.54	353
175	1.10	1.33	-1.58	0.55	365
180	1.16	1.39	-1.59	0.57	375
185	1.24	1.45	-1.70	0.58	387
190	1.31	1.51	-1.75	0.59	398
195	1.38	1.58	-1.79	0.61	409
200	1.46	1.64	-1.88	0.62	420

Table 2

m_t	λ	$\tilde{\lambda}$	a	ζ	m_H
150	0.78	1.07	-1.27	0.49	307
155	0.84	1.12	-1.33	0.50	319
160	0.90	1.17	-1.37	0.52	330
165	0.96	1.23	-1.41	0.53	341
170	1.03	1.28	-1.50	0.54	353
175	1.10	1.34	-1.58	0.55	365
180	1.16	1.40	-1.59	0.57	375
185	1.24	1.46	-1.70	0.58	387
190	1.31	1.52	-1.75	0.59	398
195	1.38	1.59	-1.79	0.61	409
200	1.46	1.65	-1.88	0.62	420