

Statistics of Red Sites on Elastic and Full Backbone

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Abstract

We investigate the number of red sites on the elastic and real backbone when right at the percolation threshold a spanning cluster exists between two sites at opposite faces of the lattice and found that it scales in the same way as in the case of percolation between two plates. We also find out that the number of common red sites scales similarly for both kinds of backbones for percolation between pairs of sites on opposite faces of the lattice. Our statistics for several quantities show that the the exponent for the elastic backbone approaches the one of the full backbone as dimensionality is increased.

Studies on cluster structure dates back to more than two decades. Initially, de Gennes [1] and Skal and Shklovskii [2] independently postulated that the backbone at the percolation threshold consists of a network of nodes connected by effectively one-dimensional links. This picture turns out to be correct for dimensions greater than six, but is too simplistic for low dimensions. Consequently, the nodes, links and blobs picture was postulated by Stanley [3] where the nodes are connected by one dimensional links which are often separated by multiconnected pieces or blobs. In the backbone (without the dangling ends) there exist two kinds of bonds, the red (blue) bonds, which, if cut, separate (do not separate) the infinite cluster in two separate clusters. However, this definition is perfect only as long as there is only one spanning cluster; if there are more than one spanning cluster (which has a finite probability [4], especially in higher dimensions), an infinite cluster still exists if red bonds of another spanning cluster are cut. Hence, in general, we define, a red bond is that which when absent, cuts one spanning cluster in the lattice which may still have other spanning clusters; the traditional definition allows for configuration without any red bonds [5]. It was proved by Coniglio [6] that the number of red bonds scales as $(p-p_c)^{-1}$, which was supported by Pike and Stanley [7] who showed numerically for two dimensions that the number of red bonds indeed follows the above scaling.

There can be two different ways of realising a percolating lattice. Either one can set all the sites of the top and bottom surfaces of the lattice to be conducting (percolation between two conducting plates which we abbreviate as LLP - line to line percolation) or one can have a percolating path between a pair of sites on the opposite faces of the lattice (SSP or site to site percolation). In the latter, not all sites are occupied in the top and bottom surfaces. A percolating path may be regarded as occurring between site a on the top surface to site b on the bottom surface, both belonging to the infinite cluster. Therefore, there can be a number of paths connecting the top and bottom of the lattice depending on which two points are chosen at the top and bottom surfaces respectively.

Skal [8] argues that there is another class of special bonds, which apparently are similar to red bonds but follows a totally different scaling law $(p-p_c)^a$ where a is positive. These special bonds are the ones through which every path connecting the opposite faces of the lattice must pass (for site to site percolation). One can obtain several paths between the opposite faces of a lattice, each with a number of red bonds of its own, but there will be some which cannot be avoided by any path. At p_c , according to [8], there is only one such bond and its number increases away from p_c .

Herrmann et al [9] defined another fractal object which they called the elastic backbone - a cluster of the sites that lie on the union of the shortest paths between two points. The number of cutting bonds or red bonds for the elastic backbone at p_c found by them follows a different behaviour compared with that on the full backbone for two and three dimensions. The elastic backbone comprises of links and multiconnected paths where each path has the same length. The full backbone can be obtained by growing it from the elastic backbone.

In this paper, we have primarily investigated whether the scaling laws for the red sites (either on the real or elastic backbone) are affected by the above two boundary conditions in the sense, whether it depends on the fact that the percolating path is between two sites or two plates. Throughout the simulation, we have considered site percolation, and it is expected that red sites and bonds behave in the same way.

We have also shown that the quantities like the red sites, shortest path and mass of the

elastic backbone (but not the mass of the full backbone) have essentially the same exponent ($= D_{min}$ where D_{min} is the shortest path exponent at p_c [10]) in all dimensions. The exponent D_{min} for the shortest path on the elastic backbone is as expected, as by definition, the elastic backbone is the shortest path. However, the result that the mass and the number of red sites on the elastic backbone also have the same exponent is not obvious. This indicates that the elastic backbone, even at lower dimensions, corresponds more than the full backbone to the description of the node and links picture. Hence it is not surprising, that the elastic backbone exponents and the backbone ones come closer as the dimensionality is increased.

Next we investigate the scaling law for the number of common red sites which lie in all the different connecting paths in SSP on both kinds of backbones. If a common site is taken away, there can be no spanning cluster at all in case we had before one spanning cluster only. These are related to what Skal [8] calls the special bonds. In case there are several spanning clusters, obviously there cannot be any common red bond between them. We, however, study an average picture here, rather than a distribution of the common red sites. Apparently, it should be zero when there are more than one cluster as mentioned earlier or in the extremely unlikely cases where the different paths connecting the top and bottom surfaces do not share a red site at all.

We have also detected the presence of what can be called a "breakthrough" site. Starting from a rough estimate of p_c , say p_0 , we can find out whether a cluster percolates from top to bottom, when sites are present with the probability p_0 . The probability p_0 is subsequently decreased (increased) if there exists (does not exist) a percolating cluster in the same manner as in [8]. It is observed that very close to p_c , where this process is continued, there can occur a spanning cluster depending on whether or not *one* particular site is present. This was found to be true for all dimensions and sizes. Defined in this way, there is exactly one "breakthrough" site for each percolating cluster which appears or disappears to enable percolation and is in some sense reminiscent of the "Starry sky model" as in [8].

We have carried out simulations at the percolation threshold in dimensions $d = 2$ to 5 for lattices of size L^d where L is the linear dimension. In case of SSP, randomly some connected paths were chosen to find out the number of common red sites. The results should not depend on the number of random paths chosen as long as it is greater than 1. To be on the safe side, we have taken typically 10 to 30 such paths. It has been checked that indeed the results are independent of the number of paths chosen in two dimensions.

We have generated the elastic backbone by a forward and backward burning process [7]. In the case of site to site percolation, not all the sites of the top and bottom surfaces are occupied. One can choose any site on the bottom surface belonging to the spanning cluster and start the backward burning. Backward burning essentially traces the shortest path to the upper surface. Therefore we have control in selecting one of the sites through which site to site percolation is occurring, the other is fixed by the fact that it has to be the shortest path. The backbone has been obtained by adding those blobs in the incipient cluster which are joined to the elastic backbone at two different points. The path length, mass, red sites and common red sites for elastic backbones in different dimensions are shown in Fig. 1 for SSP and in Fig. 2 for LLP. The numerical values are different in the two cases but they all have the same slope in these log-log plots. However, in SSP, the path length and mass almost converge indicating the presence of fewer blobs in the elastic backbone. That this is not the case in the second case is obvious, there being a larger mass contribution due to the

fully occupied top and bottom surfaces. Here also, the finite size effects are more prominent due to the same reason.

We have been able to study the number of red sites and the common red sites in the full backbone only for very small sizes due to the large amount of time involved to do this. However, the data still indicate that the number of red sites and common red sites follow the Coniglio law for dimensions 2 to 4. The LLP data is expected to be more inaccurate due to the small sizes and this is indeed the case as seen in Fig. 3 (especially in case of $d = 4$).

Finally, we investigate the ratio of the elastic backbone mass (M_e) to the real backbone mass (M_b) and find that $M_e \sim M_b^\alpha$. We found that α approaches unity as the dimensionality is increased as shown in Fig. 4. This again supports the idea that the properties of the elastic backbone and the full backbone converge in higher dimensions.

Hence we conclude that there are three independent exponent for red sites (a) D_{min} , for red sites and common red sites on elastic backbone. (b) $1/\nu$, for red sites and common red sites on full backbone. (c) 0, for the breakthrough site which decides whether there will be percolation or not when the concentration is varied.

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FIGURES

FIG. 1. The number of red sites (\diamond), shortest path (+) and mass (\square) of the elastic backbone are shown for percolation between two plates in (a) two (b)three (c) four and (d) five dimensions. The dotted line has a slope $D_{min} =$ (a) 1.13, (b) 1.34, (c) 1.50 and (d) 1.80.

FIG. 2. The number of red sites (\diamond), shortest path (+), mass (\square) and common red sites (\times) of the elastic backbone are shown for percolation between two sites in (a) two (b)three (c) four and (d) five dimensions. The dashed line has a slope $D_{min} =$ (a) 1.13, (b) 1.34, (c) 1.50 and (d) 1.80.

FIG. 3. The number of backbone red sites (\diamond for LLP, + for SSP) and common red sites for SSP (\square) are shown for (a) 2 (b) 3 and (c) 4 dimensions. The dotted line has a slope $1/\nu =$ (a) 0.75, (b) 1.13 and (c) 1.47.

FIG. 4. The mass of the full backbone vs. the mass of the elastic backbone are shown for 2 (\diamond), 3(+), 4(\square) and 5 (\times) dimensions. The dashed line has slope 1.













