

# Search for a ‘stable alignment limit’ in two Higgs-doublet models

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## Abstract

We study the conditions required to make the 2HDM scalar potential stable up to the Planck scale. The lightest CP-even scalar is assumed to have been found at the LHC and the *alignment limit* is imposed in view of the LHC Higgs data. We find that ensuring stability up to scales  $\gtrsim 10^{10}$  GeV necessitates the introduction of a soft breaking parameter in the theory. Even then, some interesting correlations between the nonstandard masses and the soft breaking parameter need to be satisfied. Consequently, a 2HDM becomes completely determined by only two nonstandard parameters, namely,  $\tan\beta$  and a mass parameter,  $m_0$ , with  $\tan\beta \gtrsim 3$ . These observations make a 2HDM, in the *stable alignment limit*, more predictive than ever.

## 1 Introduction

With the discovery of a Higgs-like particle at the Large Hadron Collider (LHC) [1, 2], all the parameters of the Standard Model (SM) are now known and the fate of the SM is sealed. Taking all the uncertainties of the current experimental data into account, it has been found that the SM scalar potential becomes unstable somewhere within  $10^8$ - $10^{10}$  GeV [3–6]. In fact, it has been shown that for the SM scalar potential to be stable all the way up to the Planck scale ( $M_P = 10^{19}$  GeV) the Higgs mass ( $m_h$ ) needs to be in the following range [5]:

$$m_h > 129.6 \text{ GeV} + 2.0 (M_t[\text{GeV}] - 173.34 \text{ GeV}) - 0.5 \text{ GeV} \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3 \text{ GeV}, \quad (1)$$

where,  $M_t$  denotes the top quark pole mass and  $\alpha_s(M_Z)$  is the strong coupling constant at the  $Z$ -boson mass scale. Hence, an SM Higgs boson with mass in the range 124-126 GeV certainly disfavors the possibility of having an absolutely stable vacuum up to  $M_P$ . As a way out of this vexing situation, it has been suggested that while absolute stability of the SM potential might be a tall ask, a metastable vacuum is entirely consistent with the current experimental value of the Higgs mass [4, 7]. Nevertheless, the problem of vacuum stability in the SM remains one of the most discussed topics after the Higgs discovery and often has been taken as a hint for the intervention of some new physics.

Our objective in this paper is to investigate whether the two-Higgs-doublet models (2HDMs) [8] can do better in this respect. 2HDMs extend the scalar potential of the SM by adding one extra scalar doublet and therefore, rank amongst the simplest of beyond the Standard Model (BSM) constructions. For decades, 2HDMs have attracted a lot of attention because the minimal supersymmetry relies on a 2HDM scalar structure. Another attractive feature of 2HDMs is that the value of the electroweak  $\rho$ -parameter remains unity at the tree level. But one ominous consequence of introducing one additional scalar doublet is that now we will have two Yukawa matrices for each type of fermion and diagonalization of the fermion mass matrix will not guarantee the simultaneous diagonalization of the Yukawa matrices. As a result, there will be flavor changing neutral currents(FCNC), at the tree level, mediated by neutral scalars. This problem was addressed by Glashow and Weinberg [9] and independently by Paschos [10]. According to Glashow-Weinberg-Paschos theorem, tree level FCNC can be avoided altogether if suitable arrangements are made such that fermions of a particular charge receive their

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masses from a particular scalar doublet. Usually, a  $Z_2$  symmetry, under which  $\phi_1 \rightarrow \phi_1$  and  $\phi_2 \rightarrow -\phi_2$ , is employed to achieve this. Proper assignments of the  $Z_2$  charge to different fermions then complete the purpose. Labeling as  $\phi_2$  the doublet that couples to the up-type quarks, the following four conventional variants of 2HDMs emerge from the  $Z_2$  charge assignments to the fermions:

- Type I: All quarks and leptons couple only to the doublet  $\phi_2$ ;
- Type II:  $\phi_2$  couples to the up-type quarks and  $\phi_1$  couples to down-type quarks and charged leptons;
- Type X or lepton specific: All quarks couple to  $\phi_2$ , while  $\phi_1$  couples to the charged leptons;
- Type Y or flipped: Up-type quarks and charged leptons couple to  $\phi_2$  and all down-type quarks couple to  $\phi_1$ .

Currently the measured values of Higgs signal strengths into different decay channels are consistent with the corresponding SM expectations. In anticipation that the data will continue to agree with the SM with increasing accuracy in the upcoming experiments, those BSM scenarios which can deliver an SM-like Higgs in some limiting case, will hold the upper hand in future survival race. It is quite well known that 2HDMs promise an *alignment limit* [11–13] when an SM-like Higgs can be recovered in the form of the lightest CP-even scalar in the 2HDM particle spectrum. Although vacuum stability constraints in 2HDMs have been studied previously both before [14] and after [15] the Higgs discovery, the distinct implications of the alignment limit in this context have never been emphasized before. Motivated by the LHC Higgs data, in this paper we concentrate exclusively on the alignment limit and explore the consequences of demanding the stability of the 2HDM potential all the way up to the GUT and Planck scales. In the process we have uncovered many new and interesting features. We have found that the requirement of stability of the 2HDM scalar potential compels us to introduce a soft breaking parameter and at the same time, entails a strong correlation between the soft breaking parameter and the other nonstandard masses. Accordingly, in this ‘stable alignment limit’ a 2HDM is completely determined by only two nonstandard parameters: (i)  $\tan\beta$  ( $= v_2/v_1$ ) which is the ratio of the two vacuum expectation values (vevs) and, (ii) a mass parameter ( $m_0$ ). How the requirement of high scale stability of the 2HDM scalar potential in the alignment limit leads us to this intriguing conclusion constitutes the central theme of our paper.

The paper is organized in the following way: in Section 2 we describe the model and the various constraints that we use in our study. We present our results in Section 3. In Section 4 we summarize important findings and draw our conclusions.

## 2 The scalar potential

The general 2HDM potential with  $Z_2$  symmetry under which  $\phi_1 \rightarrow \phi_1$  and  $\phi_2 \rightarrow -\phi_2$ , is usually written as

$$\begin{aligned}
V = & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - \left( m_{12}^2 \phi_1^\dagger \phi_2 + \text{h.c.} \right) + \frac{\lambda_1}{2} \left( \phi_1^\dagger \phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \phi_2^\dagger \phi_2 \right)^2 \\
& + \lambda_3 \left( \phi_1^\dagger \phi_1 \right) \left( \phi_2^\dagger \phi_2 \right) + \lambda_4 \left( \phi_1^\dagger \phi_2 \right) \left( \phi_2^\dagger \phi_1 \right) + \left\{ \frac{\lambda_5}{2} \left( \phi_1^\dagger \phi_2 \right)^2 + \text{h.c.} \right\}, \quad (2)
\end{aligned}$$

where the term proportional to  $m_{12}^2$  breaks the  $Z_2$  symmetry softly. For simplicity, we assume that all the parameters of the potential are real so that CP symmetry is not explicitly broken in the scalar sector. To minimize the potential, we express the doublets as

$$\phi_i = \left( \begin{array}{c} X_i^+ \\ \frac{1}{\sqrt{2}}(h_i + i\eta_i) \end{array} \right), \quad (3)$$

and define the minimum as follows:

$$\langle \phi_i \rangle_{\min} = \left( \begin{array}{c} 0 \\ \frac{v_i}{\sqrt{2}} \end{array} \right) \equiv \left( \begin{array}{c} 0 \\ \frac{(x_i + iy_i)}{\sqrt{2}} \end{array} \right). \quad (4)$$

Note that, although the potential of Eq. (2) is explicitly CP conserving, the vevs can, in general, be complex [16, 17]. Now the minimization conditions will read as follows:

$$\begin{aligned} \left. \frac{\partial V}{\partial h_1} \right|_{\min} &= 2(m_{11}^2 x_1 - m_{12}^2 x_2) + \lambda_1 x_1 (x_1^2 + y_1^2) + x_1 (\lambda_3 + \lambda_4) (x_2^2 + y_2^2) \\ &\quad + \lambda_5 (x_1 x_2^2 + 2x_2 y_1 y_2 - x_1 y_2^2) = 0, \end{aligned} \quad (5a)$$

$$\begin{aligned} \left. \frac{\partial V}{\partial h_2} \right|_{\min} &= 2(m_{22}^2 x_2 - m_{12}^2 x_1) + \lambda_2 x_2 (x_2^2 + y_2^2) + x_2 (\lambda_3 + \lambda_4) (x_1^2 + y_1^2) \\ &\quad + \lambda_5 (x_2 x_1^2 + 2x_1 y_1 y_2 - x_2 y_1^2) = 0, \end{aligned} \quad (5b)$$

$$\begin{aligned} \left. \frac{\partial V}{\partial \eta_1} \right|_{\min} &= 2(m_{11}^2 y_1 - m_{12}^2 y_2) + \lambda_1 y_1 (x_1^2 + y_1^2) + y_1 (\lambda_3 + \lambda_4) (x_2^2 + y_2^2) \\ &\quad + \lambda_5 (y_1 y_2^2 + 2x_1 x_2 y_2 - x_2^2 y_1) = 0, \end{aligned} \quad (5c)$$

$$\begin{aligned} \left. \frac{\partial V}{\partial \eta_2} \right|_{\min} &= 2(m_{22}^2 y_2 - m_{12}^2 y_1) + \lambda_2 y_2 (x_2^2 + y_2^2) + y_2 (\lambda_3 + \lambda_4) (x_1^2 + y_1^2) \\ &\quad + \lambda_5 (y_2 y_1^2 + 2x_1 x_2 y_1 - x_1^2 y_2) = 0. \end{aligned} \quad (5d)$$

It can be easily checked that the choice  $y_1 = y_2 = 0$  which implies the vevs are real, satisfies Eqs. (5c) and (5d) trivially. In this case, Eqs. (5a) and (5b) will take the following simpler forms:

$$\left. \frac{\partial V}{\partial h_1} \right|_{\min} = 2(m_{11}^2 v_1 - m_{12}^2 v_2) + v_1 \{ \lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_2^2 \} = 0, \quad (6a)$$

$$\left. \frac{\partial V}{\partial h_2} \right|_{\min} = 2(m_{22}^2 v_2 - m_{12}^2 v_1) + v_2 \{ \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 \} = 0. \quad (6b)$$

Thus the assumption of real  $v_1$  and  $v_2$  is consistent with the minimization conditions of Eq. (5) and in what follows, we shall work under this assumption.

When both the doublets receive vevs, the  $Z_2$  symmetry is broken spontaneously too and we can rewrite the 2HDM potential in the following form [18]:

$$\begin{aligned} V &= \beta_1 \left( \phi_1^\dagger \phi_1 - \frac{v_1^2}{2} \right)^2 + \beta_2 \left( \phi_2^\dagger \phi_2 - \frac{v_2^2}{2} \right)^2 + \beta_3 \left( \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 - \frac{v_1^2 + v_2^2}{2} \right)^2 \\ &\quad + \beta_4 \left\{ (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right\} + \beta_5 \left( \text{Re } \phi_1^\dagger \phi_2 - \frac{v_1 v_2}{2} \right)^2 + \beta_6 \left( \text{Im } \phi_1^\dagger \phi_2 \right)^2. \end{aligned} \quad (7)$$

In comparison with the previous case, in this notation  $\beta_5$  plays the role of the soft breaking parameter. For easy understanding of the future results, we will often switch between the parametrizations of Eqs. (2) and (7). Therefore, a comparison between the parameters of Eqs. (2) and (7) will be in order. It should be emphasized that unlike Eq. (2), Eq. (7) manifestly assumes that both the doublets acquire vevs so that  $m_{11}^2$  and  $m_{22}^2$  in Eq. (2) can be traded for  $v_1$  and  $v_2$  (using Eqs. (6a) and (6b)) followed by suitable rearrangements of the quartic parameters to obtain Eq. (7). Since, in this paper, we restrict ourselves only to the *non-inert* cases where  $\tan \beta$  is nonzero and finite, above two parametrizations are equivalent to us. The connections between the two sets of parameters are given by the following relations:

$$\begin{aligned} m_{11}^2 &= -(\beta_1 v_1^2 + \beta_3 v^2); & \lambda_1 &= 2(\beta_1 + \beta_3); \\ m_{22}^2 &= -(\beta_2 v_2^2 + \beta_3 v^2); & \lambda_2 &= 2(\beta_2 + \beta_3); \\ m_{12}^2 &= \frac{\beta_5}{2} v_1 v_2; & \lambda_3 &= (2\beta_3 + \beta_4); \\ \lambda_4 &= \frac{\beta_5 + \beta_6}{2} - \beta_4; & \lambda_5 &= \frac{\beta_5 - \beta_6}{2}. \end{aligned} \quad (8)$$

At this point, it is interesting to note that in the parametrization of Eq. (2), the combination  $m_{12}^2/(\sin \beta \cos \beta)$  instead of  $m_{12}^2$  itself controls the nonstandard masses [11]. Thus the relation between  $m_{12}^2$  and  $\beta_5$  in Eq. (8)

suggests that  $\beta_5$  is a better parameter for tracking the effect of soft breaking. In passing, we note that in the limit  $\lambda_5 = 0$  (in Eq. (2)) or equivalently  $\beta_5 = \beta_6$  (in Eq. (7)), the symmetry of the 2HDM potential is enhanced from softly broken  $Z_2$  to softly broken  $U(1)$  under which  $\phi_1 \rightarrow \phi_1$  and  $\phi_2 \rightarrow e^{i\theta}\phi_2$ . This  $U(1)$  symmetry will be relevant in our future discussions.

To get the mass eigenstates, we expand the scalar doublets around their vevs as follows:

$$\phi_i = \left( \begin{array}{c} \chi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + i\eta_i) \end{array} \right). \quad (9)$$

Our assumption that CP is a good symmetry of the scalar potential actually allows us to define neutral scalar eigenstates that are also eigenstates of CP. In total, there are five physical eigenstates: two CP-even scalars ( $h, H$ ), one CP-odd scalar ( $A$ ) and a pair of charged scalars ( $H^\pm$ ) along with three Goldstones ( $G^\pm, G^0$ ) which will be absorbed to give masses to the SM gauge bosons ( $W^\pm, Z$ ). The rotations that lead us to the mass eigenstates are given below:

$$\left( \begin{array}{c} G^\pm \\ H^\pm \end{array} \right) = \left( \begin{array}{cc} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{array} \right) \left( \begin{array}{c} \chi_1^\pm \\ \chi_2^\pm \end{array} \right), \quad (10a)$$

$$\left( \begin{array}{c} G^0 \\ A \end{array} \right) = \left( \begin{array}{cc} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{array} \right) \left( \begin{array}{c} \eta_1 \\ \eta_2 \end{array} \right), \quad (10b)$$

$$\left( \begin{array}{c} h \\ H \end{array} \right) = \left( \begin{array}{cc} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{array} \right) \left( \begin{array}{c} h_1 \\ h_2 \end{array} \right). \quad (10c)$$

where,  $c_{\beta(\alpha)} \equiv \cos \beta(\alpha)$  and  $s_{\beta(\alpha)} \equiv \sin \beta(\alpha)$ . The mixing angle of the CP-even sector is defined through the following relation:

$$\tan 2\alpha = \frac{2(\beta_3 + \frac{\beta_5}{4})v_1v_2}{\beta_1v_1^2 - \beta_2v_2^2 + (\beta_3 + \frac{\beta_5}{4})(v_1^2 - v_2^2)}. \quad (11)$$

It is now instructive to count the number of free parameters in the scalar potential. Note that, Eqs. (2) and (7) both contain eight free parameters. In the notation of Eq. (7), these are  $v_1, v_2$  and six  $\beta_i$  couplings. We can trade  $v_1$  and  $v_2$  for  $v = \sqrt{v_1^2 + v_2^2}$  and  $\tan \beta$ . Except for  $\beta_5$ , all other  $\beta$  parameters may be traded for four physical scalar masses ( $m_h, m_H, m_A$  and  $m_{H^\pm}$ ) and the angle,  $\alpha$ . The equivalence of these two sets of parameters is demonstrated by the following relations:

$$\beta_1 = \frac{1}{2v^2c_\beta^2} \left[ m_H^2c_\alpha^2 + m_h^2s_\alpha^2 - \frac{s_\alpha c_\alpha}{\tan \beta} (m_H^2 - m_h^2) \right] - \frac{\beta_5}{4} (\tan^2 \beta - 1), \quad (12a)$$

$$\beta_2 = \frac{1}{2v^2s_\beta^2} \left[ m_h^2c_\alpha^2 + m_H^2s_\alpha^2 - s_\alpha c_\alpha \tan \beta (m_H^2 - m_h^2) \right] - \frac{\beta_5}{4} (\cot^2 \beta - 1), \quad (12b)$$

$$\beta_3 = \frac{1}{2v^2} \frac{s_\alpha c_\alpha}{s_\beta c_\beta} (m_H^2 - m_h^2) - \frac{\beta_5}{4}, \quad (12c)$$

$$\beta_4 = \frac{2}{v^2} m_{H^\pm}^2, \quad (12d)$$

$$\beta_6 = \frac{2}{v^2} m_A^2. \quad (12e)$$

Among the eight redefined parameters that appear on the RHS of Eq. (12), not all are unknown. We already know  $v = 246$  GeV and under the assumption that the lightest CP-even Higgs is what has been found at the LHC,  $m_h \approx 125$  GeV is also known.

As a next level of simplification, we recall that the experimental values of the Higgs signal strengths into different decay modes are showing increasing affinity towards the SM expectations [19, 20]. This encourages us to work in the *alignment limit* [11]:

$$\beta - \alpha = \frac{\pi}{2}, \quad (13)$$

which means,  $h$  will have the exact SM-like tree-level couplings with the fermions and the vector bosons. The recent global fits of the LHC data in the 2HDM context [21–27] certifies that Eq. (13) is indeed a reasonable assumption. In what follows, we will work exclusively in the alignment limit. Therefore, in this limit, we are left with five unknown parameters ( $m_H, m_A, m_{H^\pm}, \beta_5$  and  $\tan\beta$ ) which will be constrained from the requirement of high scale stability of the 2HDM potential. We will see that this requirement in association with the constraints from electroweak  $T$ -parameter entail strong correlations between most of the remaining parameters making the 2HDM more constrained than ever.

## 2.1 Theoretical constraints from vacuum stability and unitarity

First we have to ensure that there is not any direction in the field space along which the potential becomes infinitely negative. In the parametrization of Eq. (2), the necessary and sufficient conditions for the potential to be bounded from below read [28–31]

$$\text{VSC1 : } \quad \lambda_1 > 0, \quad (14a)$$

$$\text{VSC2 : } \quad \lambda_2 > 0, \quad (14b)$$

$$\text{VSC3 : } \quad \lambda_3 + \sqrt{\lambda_1\lambda_2} > 0, \quad (14c)$$

$$\text{VSC4 : } \quad \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1\lambda_2} > 0. \quad (14d)$$

The  $S$ -matrix eigenvalues that will be constrained from unitarity of the scattering amplitudes are also listed below [32–35]:

$$a_1^\pm = \frac{3}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2}, \quad (15a)$$

$$a_2^\pm = \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}, \quad (15b)$$

$$a_3^\pm = \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}, \quad (15c)$$

$$b_1^\pm = \lambda_3 + 2\lambda_4 \pm 3\lambda_5, \quad (15d)$$

$$b_2^\pm = \lambda_3 \pm \lambda_5, \quad (15e)$$

$$b_3^\pm = \lambda_3 \pm \lambda_4. \quad (15f)$$

The requirement of tree unitarity then restricts the above eigenvalues as follows:

$$|a_i^\pm|, |b_i^\pm| \leq 16\pi. \quad (16)$$

We impose that the inequalities (14) and (16) should be satisfied at all energies between the electroweak and Planck scales. The renormalization group (RG) equations that we use to calculate the lambdas at any intermediate energy scale are given in Appendix A.

Additionally it is also important to check whether the minimum defined by  $v_1$  and  $v_2$  is a global minimum or not. The condition for the global minimum, in the notation of Eq. (2), is given by [36, 37],

$$\mathcal{D} = m_{12}^2 \left( m_{11}^2 - m_{22}^2 \sqrt{\frac{\lambda_1}{\lambda_2}} \right) \left( \tan\beta - \sqrt[4]{\frac{\lambda_1}{\lambda_2}} \right) > 0. \quad (17)$$

## 2.2 Experimental constraints

In addition to the theoretical constraints mentioned above, we also take the following experimental facts into account.

- There is a very strong lower limit on  $m_{H^+}$ , for Type II models, arising mainly due to the excellent agreement of the  $b \rightarrow s\gamma$  branching ratio with the SM prediction. Because of this we take  $m_{H^+} > 300$  GeV [38, 39] for Type II models. However, if  $\tan\beta > 1$ , there is no such bound for Type I models from flavor data [38]. Therefore, for Type I models, we only consider the direct search limit  $m_{H^+} > 80$  GeV [40]. At this stage, it should be noted that since the Yukawa couplings in the quark sector are the same for Type II and Y models and the top-Yukawa gives the dominant fermionic contribution in the RG equations, whatever we comment on Type II models in this paper will eventually hold for Type Y models too. The same is true for Type I and X models.
- The oblique  $T$ -parameter can restrict the splitting between the heavy scalar masses. In the 2HDM alignment limit, the expression for the new physics contribution to the  $T$ -parameter can be expressed as [41, 42]

$$\Delta T = \frac{1}{16\pi \sin^2 \theta_w M_W^2} [F(m_{H^+}^2, m_H^2) + F(m_{H^+}^2, m_A^2) - F(m_H^2, m_A^2)] , \quad (18)$$

$$\text{with, } F(x, y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y} \ln\left(\frac{x}{y}\right) & \text{for } x \neq y, \\ 0 & \text{for } x = y. \end{cases} \quad (19)$$

Taking the new physics contribution to the  $T$ -parameter as [43]

$$\Delta T = 0.05 \pm 0.12, \quad (20)$$

we use the  $2\sigma$  uncertainty range around the mean value for our numerical constraints. Note that the function  $F(x, y)$  is symmetric under  $x \leftrightarrow y$  and is sensitive only to the difference  $|x - y|$ . Thus  $\Delta T = 0$  when either  $m_{H^+} = m_H$  or  $m_{H^+} = m_A$ . In these cases  $\Delta T$  puts no constraints on the heavy scalar masses. But if, for some reason,  $m_H \approx m_A$  then  $\Delta T$  severely restricts the splitting between charged and neutral scalar masses.

### 3 Numerical analysis and results

For the purpose of analysis, we choose the set  $\{v, \tan\beta, \alpha, m_h, m_H, m_A, m_{H^+}, \beta_5\}$  which appears on the RHS of Eq. (12) to be our independent set of eight parameters to describe the 2HDM scalar potential. Among them, we set  $v = 246$  GeV and  $\alpha = \beta - \pi/2$ . In the case of an exact  $Z_2$  symmetry we also set  $\beta_5 = 0$  and perform a random scan for the rest of the parameters in the following ranges:

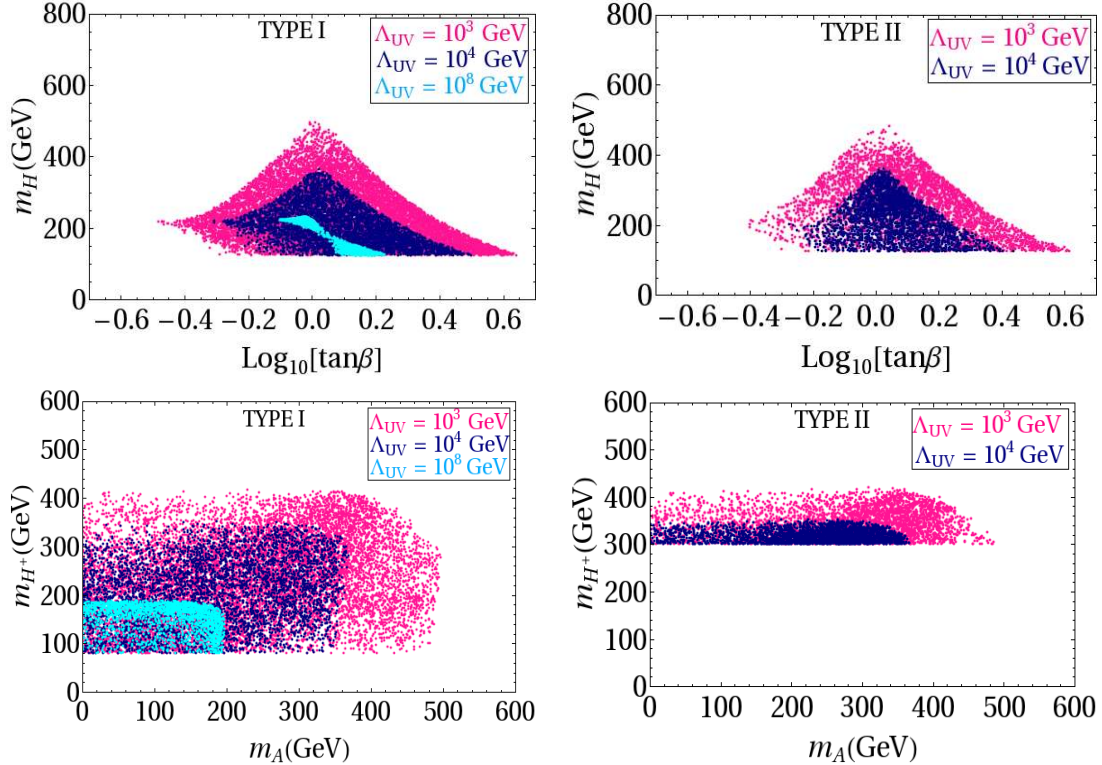
$$\begin{aligned} \tan\beta &\in [0.1, 10] , \quad m_h \in [124, 126] \text{ GeV} , \quad m_H \in [m_h, 1500] \text{ GeV} , \quad m_A \in [0, 1500] \text{ GeV} , \\ m_{H^+} &\in [80, 1500] \text{ GeV} \quad (\text{for Type I models}), \quad m_{H^+} \in [300, 1500] \text{ GeV} \quad (\text{for Type II models}). \end{aligned} \quad (21)$$

When the  $Z_2$  symmetry is softly broken, we find it convenient to introduce the mass parameter

$$m_0 = \sqrt{\frac{1}{2}\beta_5 v^2}, \quad (22)$$

and vary  $m_0$  in the range  $[0, 1500]$  GeV. In the case of a softly broken  $U(1)$  symmetry, we impose  $m_0 = m_A$ .

Next we use Eq. (12) to convert our set of parameters into the set of  $\beta_i$ s followed by Eq. (8) to convert the  $\beta_i$ s again into  $\lambda_i$ s. We then use the RG equations for  $\lambda_i$ s given in Appendix A to compute them at any intermediate energy scale and check that the unitarity and stability conditions hold all the way up to the Planck scale. We set the top quark pole mass at 173.3 GeV [44] for our numerical analysis. Our observations for different cases appear in the following subsections.



**Figure 1:** (Exact  $Z_2$ ) Allowed points in the  $\text{Log}_{10}(\tan\beta)$ - $m_H$  and  $m_A$ - $m_{H^+}$  planes for the 2HDM potential to be stable up to  $\Lambda_{UV}$ . The points in different colors correspond to different choices for  $\Lambda_{UV}$  which appear in the legends.

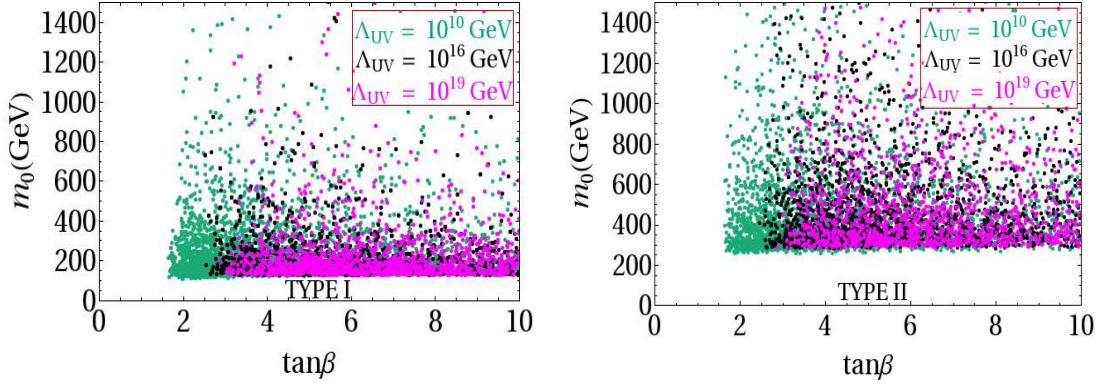
### 3.1 Exact $Z_2$

In this case setting  $\beta_5 = 0$ , we scan the rest of the parameters in the range specified previously. The result has been displayed in Fig. 1. A couple of noteworthy features that emerge from this figure are given below:

- (i) The value of  $\tan\beta$  is bounded and the bound depends on the energy scale,  $\Lambda_{UV}$ , up to which stability is demanded. As we increase  $\Lambda_{UV}$ , the allowed parameter space shrinks continuously. It should be noted that a limit on  $\tan\beta$  for exact  $Z_2$  symmetry is not surprising at all and is a direct consequence of unitarity and stability at the electroweak scale [45, 46].
- (ii) It is found that, for the non-inert cases, neither Type I nor Type II models can be absolutely stable all the way up to the Planck scale when the  $Z_2$  symmetry is exact in the potential. This result is in agreement with the previous analysis of Ref. [15] in the context of exact  $Z_2$  symmetry. We have checked for each order of magnitude in  $\Lambda_{UV}$  and noticed that Type I models can remain stable up to a maximum of  $10^8$  GeV whereas Type II models can be stable only up to  $10^4$  GeV. This difference between Type I and Type II models may be understood by noting that stability up to  $10^8$  GeV for Type I models requires a light charged scalar,  $m_{H^+} \sim 180$  GeV (see lower left panel of Fig. 1), which is not allowed for Type II models from  $b \rightarrow s\gamma$ .

Now that we know 2HDMs with exact  $Z_2$  symmetry fail to maintain stability up to the Planck scale, it is time to investigate whether the introduction of a soft symmetry breaking parameter can improve the situation.





**Figure 2:** (Softly broken  $Z_2$ ) Allowed points in  $\tan\beta$ - $m_0$  plane for the 2HDM potential to be stable up to  $\Lambda_{UV}$ . The points in different colors correspond to different choices for  $\Lambda_{UV}$  which appear in the legends.

### 3.2 Softly broken $Z_2$

Here we study the effects of nonzero  $\beta_5$ . For convenience we have encoded the information of  $\beta_5$  into the mass parameter,  $m_0$ , through Eq. (22). The same analysis as in the case of exact  $Z_2$  symmetry is then performed for three representative choices of  $\Lambda_{UV}$ , namely,  $10^{10}$ ,  $10^{16}$  and  $10^{19}$  GeV. We exhibit our results in Figs. 2 and 3. Some intriguing features that emerge from these figures are listed below.

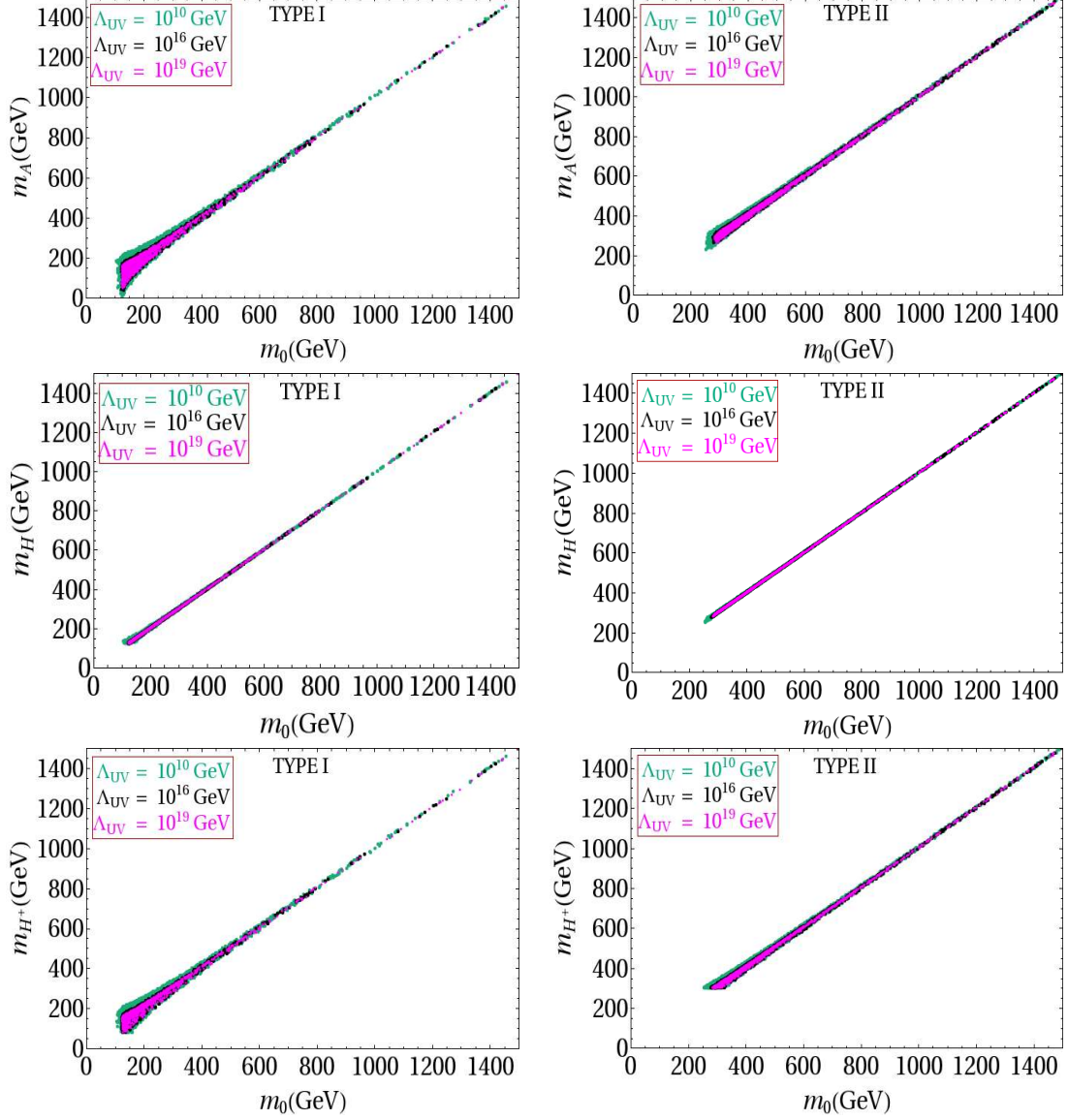
- (a) From Figs. 2 and 3, we note that it is indeed possible for a 2HDM potential to remain stable all the way up to the Planck scale in the presence of a soft symmetry breaking parameter. But certain conditions need to be satisfied for this to happen. These conditions which we describe below, together define the *stable alignment limit* for 2HDMs.
- (b) From Fig. 2 we see that for the 2HDM potential to be stable up to  $\Lambda_{UV}$ , there must exist a lower bound on  $\tan\beta$ . For example, when  $\Lambda_{UV} = 10^{19}$  GeV this bound reads  $\tan\beta \gtrsim 3$ . We also note from Fig. 2 that there is a lower limit on  $m_0$  (or equivalently  $\beta_5$ ) too. For Type I models we read  $m_0 \gtrsim 120$  GeV whereas for Type II models we obtain  $m_0 \gtrsim 280$  GeV. For the energy scales that we are concerned with, we observe that these limits on  $m_0$  for Type I and II models are essentially independent of  $\Lambda_{UV}$ . We emphasize that this is one of the important differences between our result and that of Ref. [15] where the stability of 2HDMs with softly broken  $Z_2$  symmetry was analyzed including  $\tan\beta = 2$  without realizing that such a low value of  $\tan\beta$  is forbidden when the alignment limit is exact.
- (c) From Fig. 3 we notice that the requirement of absolute stability up to  $\Lambda_{UV}$  entails a strong correlation between  $m_0$  and the other nonstandard masses. In fact, a closer scrutiny of the plots reveals that all the nonstandard scalar masses have to be nearly degenerate with  $m_0$ .
- (d) Observations (b) and (c) together lead us to the conclusion that, in the *stable alignment limit*, a 2HDM becomes completely determined by only two nonstandard parameters, namely,  $\tan\beta$  and a mass parameter ( $m_0$ ). Additionally, both of them are bounded from below:

$$\tan\beta \gtrsim 3, \quad m_0 \gtrsim 120 \text{ GeV (Type I)}, \quad m_0 \gtrsim 280 \text{ GeV (Type II)}. \quad (23)$$

- (e) We have also checked that above conclusions do not crucially depend on the input parameters, especially the top-quark mass. To be quantitative, instead of the central value of 173.3 GeV, if we consider the  $2\sigma$  lower limit of the top-quark pole mass, 171.8 GeV [44], the lower limit on  $\tan\beta$  changes to  $\tan\beta \gtrsim 2.8$ .

We feel that an *a posteriori* explanation of most of the above features is possible, at least on a qualitative level. In the following we present, one by one, the steps of our argument that helps us apprehend different characteristics of Figs. 2 and 3.





**Figure 3:** (Softly broken  $Z_2$ ) Allowed points in  $m_0$ - $m_H$ ,  $m_0$ - $m_{H^+}$ ,  $m_0$ - $m_A$  planes for the 2HDM potential to be stable up to  $\Lambda_{UV}$ . The points in different colors correspond to different choices for  $\Lambda_{UV}$  which appear in the legends.

- (1) First we note from Eq. (41) in association with Eqs. (42e), (43e), (44e) and (45e) that the evolution of  $\lambda_5$  is proportional to itself. This is not surprising because, as mentioned earlier, in the absence of  $\lambda_5$  the symmetry of the scalar potential is enhanced to a global  $U(1)$ . Thus if we start with  $\lambda_5 = 0$  at the electroweak scale, it will remain zero at all energy scales. But any initial nonzero value of  $\lambda_5$  will cause it to grow with energy. This may eventually jeopardize the vacuum stability condition (14d) at high energy scales. Therefore, it will be no wonder if, by demanding high scale stability of the 2HDM potential, we are led to the  $U(1)$  limit,  $\lambda_5 \approx 0$ . Evidently, this limit will have the following implication on the masses (using Eq. (12e) and remembering that  $\lambda_5 \approx 0$  implies  $\beta_5 \approx \beta_6$ ):

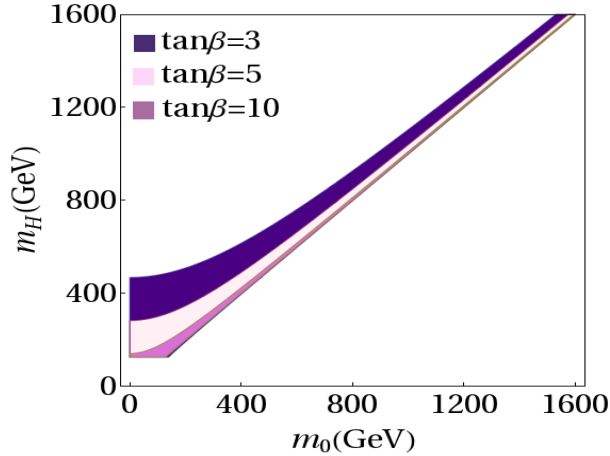
$$m_A^2 \approx m_0^2, \quad (24)$$

which has been clearly depicted by the first row of plots in Fig. 3.

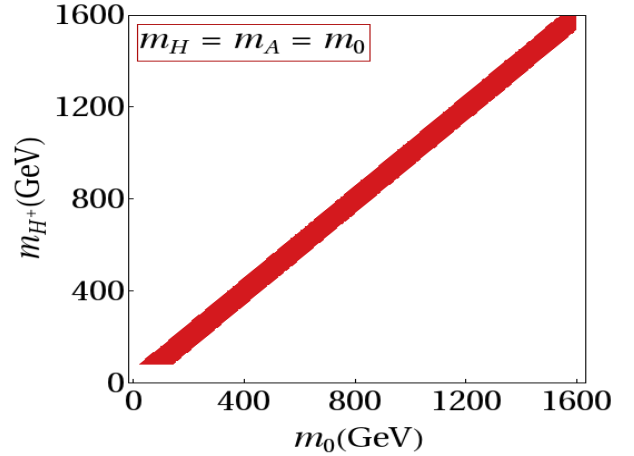
- (2) Next important thing to note is that unitarity and stability conditions at the electroweak scale imply, among other things, [46, 47]

$$0 < (m_H^2 - m_0^2)(\tan^2 \beta + \cot^2 \beta) + 2m_h^2 < \frac{32\pi v^2}{3}. \quad (25)$$

This inequality has been plotted in Fig. 4 for three different values of  $\tan \beta$ . It is obvious that for  $\tan \beta$  away from unity, the inequality (25) renders a degeneracy between  $m_H$  and  $m_0$ . This explains the second row of Fig. 3.



**Figure 4:** Allowed regions in the  $m_0$ - $m_H$  plane from the inequality (25) for three different values of  $\tan \beta$ .



**Figure 5:** Allowed region at 95% C.L. from the experimental limit on  $\Delta T$  under the assumption  $m_H = m_A = m_0$ .

- (3) Collecting (1) and (2) together we get  $m_H^2 \approx m_A^2 \approx m_0^2$ . Feeding this information to Eq. (19), we obtain the following expression for  $\Delta T$ :

$$\Delta T = \frac{1}{8\pi \sin^2 \theta_w M_W^2} F(m_{H^+}^2, m_0^2). \quad (26)$$

Since  $F(m_{H^+}^2, m_0^2)$  restricts the splitting  $|m_{H^+}^2 - m_0^2|$ , the experimental limit on  $\Delta T$  will impart the degeneracy between  $m_{H^+}$  and  $m_0$  as shown in Fig. 5. This explains the third row of Fig. 3.

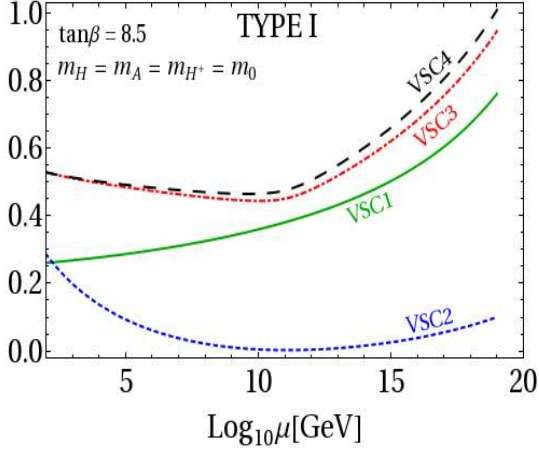
Thus, up to this point, we have

$$m_0^2 \approx m_A^2 \approx m_H^2 \approx m_{H^+}^2, \quad (27)$$

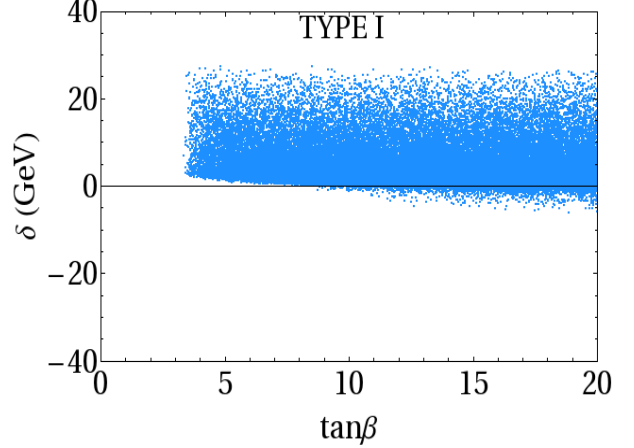
which summarizes the essential features of Fig. 3 that the 2HDM can be described by a single nonstandard mass parameter. Eq. (27) also explains the lower bounds on  $m_0$  in Fig. 2. Since  $m_{H^+} > 300$  GeV for Type II 2HDMs, the degeneracy of Eq. (27) destroys the possibility of having a light pseudoscalar for Type II models. This result is apparent from the upper right panel of Fig. 3.

- (4) To understand the lower limit on  $\tan \beta$  we turn our attention to the RG evolution equation of  $\lambda_2$  which is given by Eq. (41) in association with Eqs. (42b), (43b), (44b) and (45b). Since we have assumed  $\phi_2$  to be the doublet that gives masses to the up-type quarks,  $\lambda_2$  will face the negative pull of the top-Yukawa ( $h_t$ ). Note that in the limit of exact degeneracy,  $m_0^2 = m_A^2 = m_H^2 = m_{H^+}^2$ , we have at the electroweak scale, (using Eq. (12) followed by Eq. (8) in the alignment limit)

$$\lambda_1 = \lambda_2 = \lambda_3 = \frac{m_h^2}{v^2}, \quad \text{and,} \quad \lambda_4 = \lambda_5 = 0. \quad (28)$$



**Figure 6:** (Softly broken  $Z_2$ ) Running of stability conditions of Eq. (14) in the exactly degenerate scenario assuming  $m_h = 126$  GeV. In this scenario, these runnings are independent of  $m_0$ .



**Figure 7:** (Softly broken  $Z_2$ ) Allowed points in  $\tan\beta$ - $\delta$  plane for  $\Lambda_{UV} = 10^{19}$  GeV in a Type I 2HDM with  $m_H = m_A = m_0$ , and  $m_{H^+} = m_0 + \delta$ .

Using these, we may simplify the RG equation for  $\lambda_2$ , at the electroweak scale, as follows:

$$\mathcal{D}\lambda_2 = 16\lambda_2^2 - 3(3g^2 + g'^2)\lambda_2 + \frac{3}{4}(3g^4 + g'^4 + 2g^2g'^2) + 12h_t^2\lambda_2 - 12h_t^4. \quad (29)$$

Notice the striking similarity between the above equation and the SM running of  $\lambda$  that appears in Eq. (35). One difference is that  $\mathcal{D}\lambda_2$  receives additional contributions on the positive side (compare  $16\lambda_2^2$  with  $12\lambda^2$  in the SM case) due to the presence of extra quartic couplings. On the contrary,  $h_t$  in the 2HDM case is  $\sim \sqrt{2}m_t/(v \sin\beta)$  which is larger than the SM value of  $h_t \sim \sqrt{2}m_t/v$ . Therefore, compared to the SM case the negative drag of  $h_t$  is enhanced in 2HDMs. This effect drives  $\lambda_2$  to negative values violating vacuum stability condition (14b) at high energies unless  $\sin\beta$  is large enough to dilute the effect of the term,  $-12h_t^4$ , sufficiently. This is the origin of the lower limit on  $\tan\beta$ . Choosing  $\tan\beta = 8.5$  for illustration, we have displayed this effect in Fig. 6. There we see how the evolution of the dotted (blue) line marked as VSC2 (which is nothing but  $\lambda_2$ ) marginally survives becoming negative at an intermediate energy scale. Take  $\tan\beta$  too low and this line goes below zero spoiling the stability condition,  $\lambda_2 > 0$ . It is worth noting that when all the nonstandard masses are exactly degenerate, evolutions of the  $\lambda_i$ s depend only on  $\tan\beta$  and not on the mass parameter,  $m_0$ .

- (5) With exact degeneracy, the lower bound on  $\tan\beta$  turns out to be  $\tan\beta \gtrsim 8$  for  $\Lambda_{UV} = 10^{19}$  GeV. But from Fig. 2 we see that even lower values of  $\tan\beta$  are allowed. To understand this, we need to investigate the evolution of  $\lambda_2$  in greater detail. Instead of exact degeneracy, we now consider the following limit:

$$m_0 = m_A = m_H, \quad \text{and,} \quad m_{H^+}^2 = m_0^2 + \Delta. \quad (30)$$

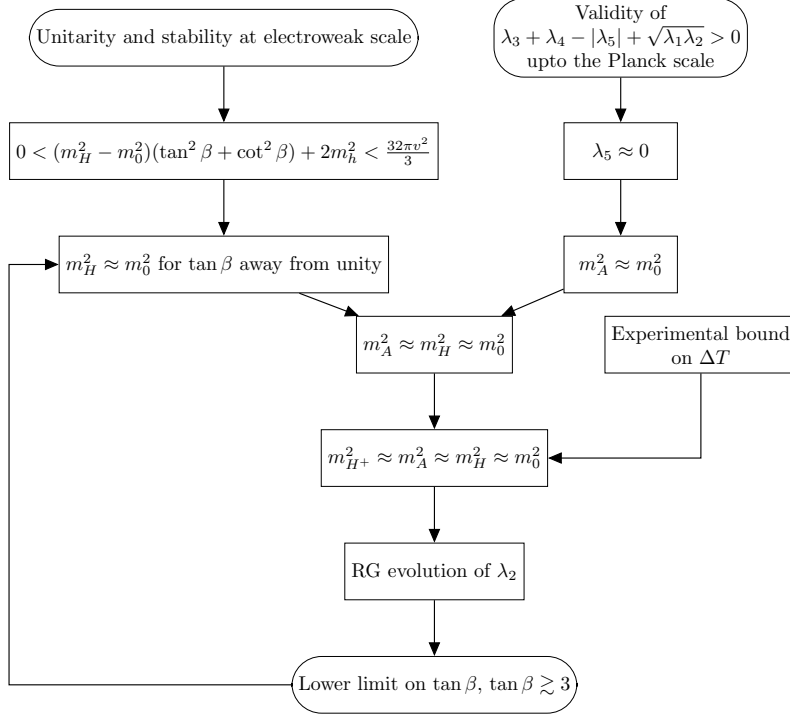
In this limit,

$$\lambda_1 = \lambda_2 = \frac{m_h^2}{v^2}, \quad \lambda_3 = \lambda_2 + \frac{2\Delta}{v^2}, \quad \lambda_4 = -\frac{2\Delta}{v^2}, \quad \lambda_5 = 0. \quad (31)$$

Using these, we can rearrange the terms that appear on the RHS in the RG equation for  $\lambda_2$ , to obtain

$$\mathcal{D}\lambda_2 = 14\lambda_2^2 + 2\left(\lambda_2 + \frac{2\Delta}{v^2}\right)^2 - 3(3g^2 + g'^2)\lambda_2 + \frac{3}{4}(3g^4 + g'^4 + 2g^2g'^2) + 12h_t^2\lambda_2 - 12h_t^4. \quad (32)$$

Comparing this with Eq. (29) we see that a positive value of  $\Delta$  aids in the positive terms allowing lower values of  $\tan\beta$ . But the fact that only very small values of  $\Delta$  can be permitted from unitarity [46] and  $T$ -parameter



**Figure 8:** Chain of arguments that helps us make sense of the main features of Figs. 2 and 3.

puts a lower limit on  $\tan \beta$  anyway. In Fig. 7 we have illustrated how the difference  $\delta \equiv m_{H^+} - m_0 \approx \Delta/(2m_0)$  depends on  $\tan \beta$ . There we can see that to allow  $\tan \beta \lesssim 8$  we will need  $\delta > 0$ , *i.e.*,  $m_{H^+} > m_0$ .

We have summarized our arguments above compactly in the form of a flowchart in Fig. 8. Before moving on, we intend to make some remarks on the structure of the potential in the *stable alignment limit*. We note that in the limit,  $m_0^2 = m_A^2 = m_H^2 = m_{H^+}^2$ , the potential of Eq. (2) takes the following simple form:<sup>3</sup>

$$V = m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 (\phi_1^\dagger \phi_2 + \text{h.c.}) + \frac{\lambda_1}{2} (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2)^2. \quad (33)$$

This potential now contains four parameters among which two, disguised as  $v$  and  $m_h$ , have been measured. Two other parameters, in the form of  $\tan \beta$  and  $m_0$ , remain to be determined to fix the model completely. Note that the symmetry in the quartic terms of Eq. (33) is more than a mere  $U(1)$ , in fact, the symmetry is enhanced to a global  $U(2)$  under which  $(\phi_1, \phi_2)^T \rightarrow U(2)(\phi_1, \phi_2)^T$ . But this  $U(2)$  symmetry is explicitly broken in the Yukawa terms. Due to this, the structure of the potential in Eq. (33) is not stable under RG. Neither the correlation  $\lambda_1 = \lambda_2 = \lambda_3$  nor the equality  $\lambda_4 = 0$  is maintained at higher energies. What remains valid at any scale is the equality  $\lambda_5 = 0$  because the  $U(1)$  symmetry that prevails in the quartic part of the scalar potential in the absence of  $\lambda_5$  is also preserved in the Yukawa sector by construction. Therefore, we conclude that when high scale stability of the 2HDM potential needs to be ensured, softly broken  $U(1)$  symmetry should be a more natural choice, to tackle tree-level FCNC, than the conventional  $Z_2$  symmetry.

Now we want to check whether the condition (17) for the global minimum holds in the stable alignment limit or not. First we remember that, in this limit,  $m_0$  is positive (see Fig. 2) which implies  $m_{12}^2 > 0$ . We have also found  $\lambda_1 \approx \lambda_2 \approx \lambda_3$  with  $\tan \beta \gtrsim 3$ . These imply

$$\left( \tan \beta - \sqrt[4]{\frac{\lambda_1}{\lambda_2}} \right) > 0. \quad (34a)$$

<sup>3</sup>Similar potential can also be motivated from an  $SO(5)$  symmetry [48].

The remaining factor in Eq. (17) can be evaluated, in the stable alignment limit, as follows:

$$m_{11}^2 - m_{22}^2 \sqrt{\frac{\lambda_1}{\lambda_2}} \approx m_{11}^2 - m_{22}^2 = \beta_2 v_2^2 - \beta_1 v_1^2 \quad [\text{Using Eq. (8)}] \quad (34b)$$

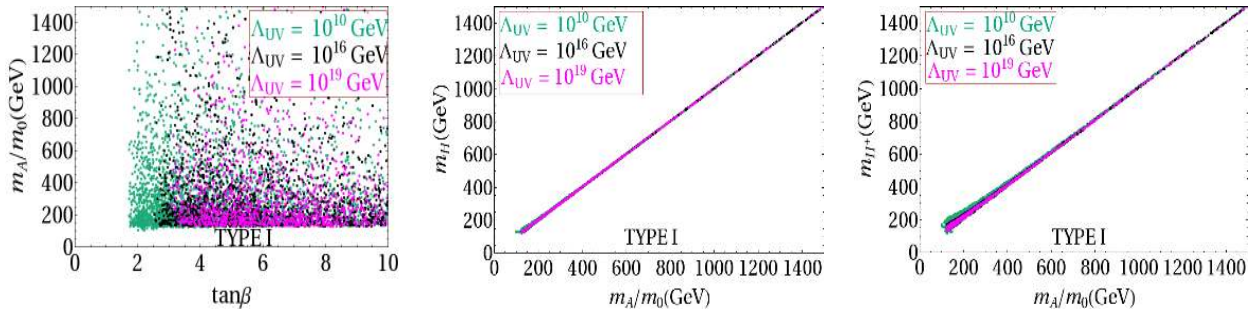
$$= m_A^2 (\sin^2 \beta - \cos^2 \beta) \quad [\text{Using Eq. (12) with } m_0 \approx m_A] \quad (34c)$$

$$= m_A^2 \cos^2 \beta (\tan^2 \beta - 1) > 0 \quad [\cdot \tan \beta \gtrsim 3] \quad (34d)$$

Therefore we conclude that, in the stable alignment limit, condition (17) is satisfied automatically and hence, existence of the global minimum is guaranteed.

### 3.3 Softly broken $U(1)$

Here we will have  $m_A = m_0$  from the beginning. Evidently, when the  $U(1)$  symmetry is exact in the potential, *i.e.*,  $m_0 = 0$ ,  $A$  will become the Goldstone boson. We present the result of our analysis, for Type I models, in Fig. 9. Although these plots do not provide any new insights, we include them just for completeness. The interpretation of Fig. 9 will be similar to that of the corresponding plots in the case of softly broken  $Z_2$  symmetry.



**Figure 9:** (Softly broken  $U(1)$ ) Allowed points in  $\tan\beta$ - $m_A$ ,  $m_A$ - $m_H$ ,  $m_A$ - $m_{H+}$  planes for the 2HDM potential to be stable up to  $\Lambda_{UV}$ . Three different colors correspond to three different choices of  $\Lambda_{UV}$  which appear in the legends.

## 4 Summary and conclusions

In this paper we have studied the consequences of demanding absolute stability of the 2HDM potential all the way up to the Planck scale. In view of the fact that the LHC Higgs data are in conformity with the SM expectations, we decided to work in the *alignment limit* where the lightest CP-even scalar ( $h$ ) possess SM-like tree-level couplings with the *standard* particles. Although we have explicitly demonstrated our results for Type I and II models only, they are equally applicable to Type X and Y models also. We have found that, in the alignment limit, requirement of high scale stability puts some remarkable restrictions on the 2HDM parameter space. This set of restrictions defines the *stable alignment limit* for 2HDMs. Some of our most important observations are summarized below:

- 2HDM scalar potential with an exact  $Z_2$  symmetry is unable to maintain stability after  $10^8$  GeV ( $10^4$  GeV) in the Type I (II) case. To ensure stability up to the Planck scale,  $Z_2$  symmetry needs to be broken softly.
- By demanding high scale stability in the presence of a soft breaking, we are led to a situation where the symmetry of the potential is enhanced from softly broken  $Z_2$  to softly broken  $U(1)$ .

- To have stability up to very high energies ( $\gtrsim 10^{10}$  GeV), all the nonstandard masses need to be nearly degenerate:  $m_0 \approx m_A \approx m_H \approx m_{H^+}$ . Thus, there is only one nonstandard mass parameter that governs the 2HDM in the *stable alignment limit*.
- The value of  $\tan \beta$  is bounded from below.

Therefore, in the *stable alignment limit*, a 2HDM can be fully described by only two new parameters, namely,  $\tan \beta$  and  $m_0$  with  $\tan \beta \gtrsim 3$  and  $m_0 \gtrsim 120$  GeV (280 GeV) for Type I (II) models.

Finally, we end the paper on a pessimistic note. One should remember that, in the alignment limit, even the self couplings of  $h$  are identical to the SM at the tree-level. This means, in this limit, any measurement involving the *standard* particles can smell the presence of nonstandard physics only through loop effects. But then the nonstandard particles can be made heavy enough to dilute these effects sufficiently. Thus, our only hope to detect the presence of nonstandard scalars beyond the reaches of the direct search experiments is via some nondecoupling effects in certain loop induced processes like Higgs to diphoton decay [49]. But with the degeneracy,  $m_0^2 \approx m_{H^+}^2$ , this possibility is also eliminated. Therefore, it appears that even if the LHC data continues to agree with the SM predictions, a 2HDM in the *stable alignment limit* which is very difficult, if not impossible, to probe experimentally, can serve as an alternative to the SM for many years to come.

**Acknowledgments:** DD thanks the Department of Atomic Energy (DAE), India for financial support.

## Appendix

### A One loop RG equations

Let us first write down the one loop RG equation (RGE) of the SM quartic coupling ( $\lambda$ ) [50, 51] as follows:

$$\mathcal{D}\lambda = 12\lambda^2 - 3(3g^2 + g'^2)\lambda + \frac{3}{4}(3g^4 + g'^4 + 2g^2g'^2) + 12h_t^2\lambda - 12h_t^4, \quad (35)$$

where,  $h_t$  denotes the top-Yukawa which dominates over the other Yukawa couplings. For convenience, we have introduced the shorthand  $\mathcal{D} \equiv 16\pi^2 \frac{d}{d(\ln\mu)}$ .

Now we will present the one loop RGEs of all the relevant couplings (gauge, Yukawa and scalar quartic couplings) of 2HDM [8, 52–54].

**Gauge couplings:** RGE for the gauge couplings,

$$\mathcal{D}g_s = -7g_s^3, \quad (36a)$$

$$\mathcal{D}g = -3g^3, \quad (36b)$$

$$\mathcal{D}g' = 7g'^3. \quad (36c)$$

**Yukawa couplings:** The initial values of top ( $h_t$ ), bottom ( $h_b$ ) and tau ( $h_\tau$ ) Yukawa couplings at electroweak scale ( $M_t$ ) for the different Yukawa structures are given by,

$$\text{Type I: } \begin{cases} h_t(M_t) &= \frac{\sqrt{2}M_t}{v \sin \beta} \left\{ 1 - \frac{4}{3\pi} \alpha_s(M_t) \right\}, \\ h_{b,\tau}(M_t) &= \frac{\sqrt{2}m_{b,\tau}}{v \sin \beta}; \end{cases} \quad (37a)$$



$$\text{Type II : } \begin{cases} h_t(M_t) &= \frac{\sqrt{2}M_t}{v \sin \beta} \left\{ 1 - \frac{4}{3\pi} \alpha_s(M_t) \right\}, \\ h_{b,\tau}(M_t) &= \frac{\sqrt{2}m_{b,\tau}}{v \cos \beta}; \end{cases} \quad (37b)$$

where,  $\alpha_s(M_t)$  denotes the strong coupling constant at top quark pole mass. The corresponding RGEs for Type I Yukawa structure are given by

$$\mathcal{D}h_t = h_t \left( a_u + \frac{9}{2}h_t^2 + \frac{3}{2}h_b^2 + h_\tau^2 \right), \quad (38a)$$

$$\mathcal{D}h_b = h_b \left( a_d + \frac{3}{2}h_t^2 + \frac{9}{2}h_b^2 + h_\tau^2 \right), \quad (38b)$$

$$\mathcal{D}h_\tau = h_\tau \left( a_e + 3h_t^2 + 3h_b^2 + \frac{5}{2}h_\tau^2 \right). \quad (38c)$$

For Type II Yukawa structure, the RGEs take the following form:

$$\mathcal{D}h_t = h_t \left( a_u + \frac{9}{2}h_t^2 + \frac{1}{2}h_b^2 \right), \quad (39a)$$

$$\mathcal{D}h_b = h_b \left( a_d + \frac{1}{2}h_t^2 + \frac{9}{2}h_b^2 + h_\tau^2 \right), \quad (39b)$$

$$\mathcal{D}h_\tau = h_\tau \left( a_e + 3h_b^2 + \frac{5}{2}h_\tau^2 \right), \quad (39c)$$

where,

$$a_u = \left( -8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 \right), \quad (40a)$$

$$a_d = \left( -8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2 \right), \quad (40b)$$

$$a_e = \left( -\frac{9}{4}g^2 - \frac{15}{4}g'^2 \right), \quad (40c)$$

for both Type I and II models.

**Scalar quartic couplings:** The RGEs for the five quartic couplings that appear in Eq. (2) are given by

$$\mathcal{D}\lambda_i = \beta_{\lambda_i} + G_i + H_i, \quad (i = 1, 2, 3, 4, 5), \quad (41)$$

where,  $\beta_{\lambda_i}$  and  $G_i$  are independent of the Yukawa structure of the model and are as follows:

$$\beta_{\lambda_1} = 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2, \quad (42a)$$

$$\beta_{\lambda_2} = 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2, \quad (42b)$$

$$\beta_{\lambda_3} = (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2, \quad (42c)$$

$$\beta_{\lambda_4} = 2\lambda_4(\lambda_1 + \lambda_2) + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 8\lambda_5^2, \quad (42d)$$

$$\beta_{\lambda_5} = \lambda_5(2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 12\lambda_4), \quad (42e)$$

and,

$$G_1 = \frac{3}{4} \left( 3g^4 + g'^4 + 2g^2g'^2 \right) - 3\lambda_1 \left( 3g^2 + g'^2 \right), \quad (43a)$$

$$G_2 = \frac{3}{4} \left( 3g^4 + g'^4 + 2g^2g'^2 \right) - 3\lambda_2 \left( 3g^2 + g'^2 \right), \quad (43b)$$

$$G_3 = \frac{3}{4} \left( 3g^4 + g'^4 - 2g^2g'^2 \right) - 3\lambda_3 \left( 3g^2 + g'^2 \right), \quad (43c)$$

$$G_4 = 3g^2 g'^2 - 3\lambda_4 (3g^2 + g'^2), \quad (43d)$$

$$G_5 = -3\lambda_5 (3g^2 + g'^2). \quad (43e)$$

The expressions for  $H_{iS}$ , however, depend on the Yukawa structure of the model. For Type I models, these are given by,

$$H_1 = 0, \quad (44a)$$

$$H_2 = 4\lambda_2 (3h_t^2 + 3h_b^2 + h_\tau^2) - (12h_t^4 + 12h_b^4 + 4h_\tau^4), \quad (44b)$$

$$H_3 = 2\lambda_3 (3h_t^2 + 3h_b^2 + h_\tau^2), \quad (44c)$$

$$H_4 = 2\lambda_4 (3h_t^2 + 3h_b^2 + h_\tau^2), \quad (44d)$$

$$H_5 = 2\lambda_5 (3h_t^2 + 3h_b^2 + h_\tau^2). \quad (44e)$$

For the Type II Yukawa structure, we have

$$H_1 = 4\lambda_1 (3h_b^2 + h_\tau^2) - (12h_b^4 + 4h_\tau^4), \quad (45a)$$

$$H_2 = 12\lambda_2 h_t^2 - 12h_t^4, \quad (45b)$$

$$H_3 = 2\lambda_3 (3h_t^2 + 3h_b^2 + h_\tau^2) - 12h_t^2 h_b^2, \quad (45c)$$

$$H_4 = 2\lambda_4 (3h_t^2 + 3h_b^2 + h_\tau^2) + 12h_t^2 h_b^2, \quad (45d)$$

$$H_5 = 2\lambda_5 (3h_t^2 + 3h_b^2 + h_\tau^2). \quad (45e)$$

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