Recently, it has been shown [1-4] that the electroweak symmetry of the Standard Model may be broken dynamically by a $t\bar{t}$ condensate. This is referred to in the literature [5] as "top-mode Standard Model". The top quark, being much heavier than the other known fermions (and lying close in the mass spectrum to the electroweak scale $v = 246$ GeV), may, in this picture, be responsible for the breaking of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ to $SU(3)_c \times U(1)_{em}$. It has been shown [2] that in this model, where the presence of a four-fermion interaction of the form $G(\bar{\psi}_L t_R)(\bar{t}_R \psi_L)$ induces the symmetry breaking, the bound-state spectrum consists of three massless Nambu-Goldstone bosons, which give masses to the massless gauge bosons, and one massive neutral scalar, which may be identified as the Higgs.

This model has several attractive features. First, the naturalness problem arising in the elementary scalar sector of the Standard Model can be isolated in the coupling constant G once for all. Second, no elementary scalar is necessary for the theory, and there is no problem regarding the violation of the unitarity bound in $WW \rightarrow WW$ scattering. Third, a definite relationship can be established between the top mass and the Higgs masses reducing thereby (to some extent) the embarrasingly great laxity in the choice of parameters of the otherwise so successful Standard Model. Finally, there are physical examples of dynamical symmetry breaking at the eV scale (BCS theory of superconductivity) and the MeV scale (the breaking of chiral symmetry for nucleons) and it would be aesthetically satisfying if the mechanism should recur again at this higher scale of energy.

Although the above model is elegant and economical in the sense that it does not predict any new particle (even the Higgs scalar is a composite object), unfortunately the top-quark mass m_t in this model, as determined from the renormalization group flow of the coupling constants, appears to be untenable with the present experimental upper bound of 190 GeV. To resolve this difficulty within the same framework, it was proposed [6-8] that one can include an additional $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant term in the Lagrangian, which is of the form

$$G'(\bar{\psi}^I_{Li}(A^P_Q)_{IJ} t^J_R)(\bar{t}^K_R (A^Q_P)_{KM} \psi^M_{Li}). \tag{1}$$

Here G' is the coupling constant (of mass dimension -2) for the new interaction, i is the $SU(2)_L$ index and I, J, P, Q are the $SU(3)_c$ indices running from 1 to 3. The A matrices are the real generators of $SU(3)$ *á lá* Okubo which we find more convenient for our problem than the usual Gell-Mann matrices [9]. The four-fermionic interaction being nonrenormalizable in $3+1$ dimensions, a high energy cutoff Λ is needed for the regularization of this theory. Effectively, this means that the theory ceases to be valid beyond Λ. For simplicity, we will use the same cutoff for all four-fermionic operators.

In a theory with strong coupling, one can use the perturbative analysis in the low-energy limit by introducing auxiliary fields [10] in the action. Alternatively, one can write down a low-energy Lagrangian, which, at a high-energy scale, when all auxiliary fields are integrated out, gives back the Lagrangian with four-fermionic interaction. For this, one has to suitably define the different renormalization constants, taking account of compositeness [2]. Following the latter approach, we will define the effective potential of our theory to be

$$V = -\mu^2 \phi^i \phi_i + m^2 \chi^{iI}_J \chi^J_{iI} + a_1(\phi^i\phi_i)^2 + a_2(\chi^{iI}_J \chi^J_{iI})^2$$

$$+a_3(\chi^{iI}_J \chi^J_{iK} \chi^{jK}_M \chi^M_{jI}) + a_4(\chi^{iI}_J \chi^J_{jI} \chi^{jK}_M \chi^M_{iK})$$

$$+a_5(\phi^i\phi_i\chi_K^{jJ}\chi_{jJ}^K) + a_6(\phi^i\phi_j\chi_K^{jJ}\chi_{iJ}^K)$$

$$+g_t(\bar\psi_{Li}t_R\phi^i + \text{ h.c.}) \ + g_t'(\bar\psi_{Li}^I(A_Q^P)_{IJ}t_R^J\chi_P^{iQ} + \text{ h.c.}). \tag{2}$$

It may be noted that there are two composite doublets in V, namely, ϕ and χ. They have the same isospin properties, but under $SU(3)_c$ the former is a singlet while the latter is an octet, which is denoted by the explicit $SU(3)_c$ indices. The field ϕ arises from the $SU(3)_c$ singlet bilocal fermionic field $\bar\psi_L t_R$ while the χ field arises from the $SU(3)_c$ octet bilocal $\bar\psi_L^I(A_Q^P)_{IJ}t_R^J$. It is evident that $\phi_i^\dagger \equiv \phi^i$ and $(\chi_{iJ}^I)^\dagger \equiv \chi_I^{iJ}$, because $(A_Q^P)^\dagger = (A_Q^P)^T = (A_P^Q)$. The ϕ field behaves in the low energy limit as the Standard Model Higgs doublet with $<\phi>_0 = v$, while the χ field has zero VEV since it carries color. The absence of a VEV for the χ doublet has two important implications: first, it guarantees that in the symmetry-broken phase, the SM relation $v = \sqrt{-\mu^2/a_1}$ is preserved, and second, it allows us to introduce a gauge-invariant mass term of the form $m^2\chi^2$ in the Lagrangian. The a_i (i=1 to 6) parameters generically denote four-boson couplings; the $SU(2)$ and $SU(3)$ indices of χ can be contracted in three different ways to produce a singlet. Gauge-invariant kinetic terms for the ϕ and the χ fields are also induced.

We can now solve numerically a set of β-functions corresponding to the parameters a_i, g_t and g_t', and find the infrared quasi-fixed point solution to be [7]

$$a_1 = 0.18, \quad a_5 = 0.18, \quad a_6 = 0.42 \tag{3}$$

$$\frac{9}{4}g_t^2 + 2{g_t'}^2 = \frac{9}{4}(g_t)_{BHL}^2. \tag{4}$$

We have not shown the other couplings at the quasi-fixed point because they are not relevant for our analysis, but they turn out to be small and positive. It is to be noted that the parameters are not exactly those shown in eq. (2) but suitably renormalized ones, and we use the same symbol only for brevity. The value of a_1 gives $m_H = 209$ GeV. The suffix BHL indicates the results in ref. 2, and it is evident from eq. (4) that m_t is a free parameter of the theory, being always less than $(m_t)_{BHL}$. Therefore, it is not possible to predict m_t in this model. In the succeeding analysis, we take some phenomenologically plausible values for m_t.

The mass of χ will be an important theme in our discussion. We immediately note that though the field is an auxiliary one arising as a composite of two spinor fields in an $SU(3)_c$ octet combination, it is not possible to predict the masses as was done for the "Higgs scalar" in ref. 2, because here the strong interaction plays a nontrivial part and the $1/N$ approximation is not valid. Not being determinable from the renormalization group equations, m^2 remains a free parameter of the theory. When the symmetry is broken, two more terms of the form $\frac{1}{2}a_5v^2$ and $\frac{1}{2}a_6v^2$ contribute to m_χ^2, the second term contributing only for the neutral field, so that

$$m_{\chi^0}^2 - m_{\chi^+}^2 = \frac{1}{2}a_6v^2. \tag{5}$$

These new bosons can have profound consequences through various one-loop effects which are experimentally observable, and we can place lower bounds on their masses. One notes that the interaction in the minimal condensate scheme is confined only to the third generation of quarks. The other quark generations take part in the one-loop effects through the mixing between the mass eigenstates and the weak eigenstates of the quark wavefunctions [11]. This means that the physics of $K^0-\bar{K}^0$, $B_d^0-\bar{B}_d^0$ and $B_s^0-\bar{B}_s^0$ mixing will be affected by χ, and the same is true for the CP-violating ϵ parameter. In an earlier paper [8] we have discussed these effects in detail and showed that we can obtain a lower bound on the mass of the charged scalar χ^+, which is of the order of a few hundreds of GeV. Another bound can be extracted from the observed rate of the radiative B-decays [12], which is of the same order of magnitude, and which is free from a number of undetermined or poorly determined parameters which entered in ref. 8. We have also shown that the maximum mass splitting in the doublet cannot be greater than 47 GeV [13]. This last result is obtained from the present experimental bounds on the oblique electroweak parameters [14].

The chief obstacle to putting phenomenological constraints on the model from low-energy data such as $B_d^0-\bar{B}_d^0$ mixing arises, as usual, from uncertainties in the hadronic parameters such as the decay constants f_K, f_B and the bag parameters B_K, B_B. In this note we investigate a different observable, *viz*, the ratio R_b, defined as

$$R_b = \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to \text{ hadrons})}. \tag{6}$$

R_b is relatively free from uncertainties in hadronic parameters which tend to cancel out of numerator and denominator. It is also relatively insensitive to m_t and QCD corrections. For this reason, the effects of new physics can show up in R_b *without being masked* by uncertainties in m_t etc., as in the case with a number of other phenomenologically interesting parameters. An analysis of the model using R_b is also facilitated by the fact that the experimental error in its determination has come down drastically with the LEP measurements and the advent of microvertex detectors, and now stands at [15]

$$R_b\ (expt.) = 0.2201 \pm 0.0031 \tag{7}$$

at 95% C.L., which is remarkably precise.

To fix ideas and notations, let us briefly discuss the features of $\Gamma(Z\to b\bar{b})$ and R_b in the Standard Model [16-17]. The tree-level contribution to $\Gamma(Z\to b\bar{b})$ is

$$\Gamma^0(Z\to b\bar{b}) = \frac{G_\mu m_Z^3}{8\pi\sqrt{2}}\sqrt{1-4\mu_b}\Big[1-4\mu_b+(1-\tfrac{4}{3}\sin^2\theta_W)^2(1+2\mu_b)\Big] \tag{8}$$

where $\mu_b = m_b^2/m_Z^2$, G_μ is the Fermi coupling constant as obtained from muon decay and θ_W is the weak mixing angle.

The electroweak radiative corrections appear in the form of two form factors κ_b and ρ_b, respectively for effective mixing angle and the overall renormalization. Thus, the decay width, calculated to one-loop, is given by

$$\Gamma^1(Z\to b\bar{b}) = \frac{G_\mu m_Z^3}{8\pi\sqrt{2}}\rho_b\sqrt{1-4\mu_b}\Big[1-4\mu_b+(1-\tfrac{4}{3}\sin^2\theta_W\kappa_b)^2(1+2\mu_b)\Big] \tag{9}$$

where $\sin^2\theta_W$ is determined from

$$\sin^2\theta_W\cos^2\theta_W = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1-\Delta r)}, \tag{10}$$

Δr being the electroweak correction to $\mu^{\pm}$ decay.

The one-loop correction is dominated by the top quark contribution. The vacuum polarization effect, which is common to all fermionic final states, is denoted by $\Delta\rho_t$, which is given by

$$\Delta\rho_t = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} = \frac{3G_\mu m_t^2}{8\pi^2\sqrt{2}} \approx \frac{\alpha}{\pi}\frac{m_t^2}{m_Z^2}, \tag{11}$$

the b-mass having been neglected. The Π functions are the standard ones used to denote the vacuum polarization of the gauge bosons. For $Z \to b\bar{b}$, the vertex corrections give

$$\Delta\rho_b = -\frac{4}{3}\Delta\rho_t \tag{12a}$$

$$\sin^2\theta_W \Delta\kappa_b = \frac{2}{3}\sin^2\theta_W \Delta\rho_t. \tag{12b}$$

Taking both these factors into account, we can write

$$\rho_b = 1 + \Delta\rho_t + \Delta\rho_b + \cdots \tag{13a}$$

$$\kappa_b \sin^2\theta_W = \sin^2\theta_W + \cos^2\theta_W \Delta\rho_t + \frac{2}{3}\sin^2\theta_W \Delta\rho_t + \cdots \tag{13b}$$

After some straightforward algebra, it can be shown that

$$R_b = \frac{13}{59}\Big(1 + \frac{46}{59}\delta_b^{(t)}(m_t) + \frac{24}{767}\frac{g_{Vl}}{g_{Al}} + \frac{0.1\alpha_s(m_Z^2)}{\pi}\Big) \tag{14}$$

with $\sin^2\theta_W = 0.2324$, and $\delta_b^{(t)}(m_t)$ and g_{Vl}/g_{Al} are given by

$$\delta_b^{(t)}(m_t) \simeq -\frac{20\alpha}{13\pi}\frac{m_t^2}{m_Z^2} - \frac{10\alpha}{3\pi}\ln\frac{m_t^2}{m_Z^2} \tag{15}$$

$$\frac{g_{Vl}}{g_{Al}} = 1 - 4\sin^2\theta_W. \tag{16}$$

The top-dependent contribution is of the order of 10^{-3}, the same as the order of experimental errors, and thus the variation of R_b with m_t over the allowed range of the latter is rather flat. As stated above, this is one of the reasons for which R_b is phenomenologically interesting. In our model, another term of the form

$$\delta^\chi(R_b) = \frac{13}{59}\frac{46}{59}(\delta_b^\chi(m_t) - \delta_b^\chi(0)) \tag{17}$$

gets added to the above contribution. We take only the non-oblique part as it is known [13] that the oblique part has negligible contribution. It is noteworthy that the effective Lagrangian only favors the production of left-handed b quarks, but since the same is true for the tree-level case, it will not cause any significant change in the electroweak asymmetries.

In the limit $m_b \to 0$, we can introduce the effects of the new physics through a change in the vertex factors for the $Z \to b\bar{b}$ coupling:

$$v'_L = v_L + \frac{8}{3}\frac{\alpha}{4\pi \sin^2\theta_W} F_L(P^2, m_t) \tag{18a}$$

$$v'_R = v_R \tag{18b}$$

where P is the four-momentum of the external Z, and the color factor of 8/3 comes from the octet nature of χ under $SU(3)_c$. The right-handed coupling is not changed as no term of the form $\bar{t}_L b_R \chi$ is allowed in the Lagrangian. The function F_L represents the total of all one-loop correction effects, depicted in Fig. 1. It can be written as the sum of three terms,

$$F_L = F_L^a + F_L^b + F_L^c, \tag{19}$$

where F_L^a, F_L^b and F_L^c denote the contributions from the figures 1a, 1b and 1c respectively. The correction works out to be

$$\delta_b^\chi(m_t) - \delta_b^\chi(0) = \frac{8}{3}\frac{\alpha}{4\pi\sin^2\theta_W}\frac{2v_L}{v_L^2 + v_R^2} F_L(m_Z^2, m_t) \tag{20}$$

with

$$v_L = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W, \quad v_R = \frac{1}{3}\sin^2\theta_W. \tag{21}$$

The F_L functions are

$$F_L^a = b_1(m_\chi, m_t, m_b^2) v_L \lambda_L^2, \tag{22a}$$

$$F_L^b = \Bigg[\Big[\frac{P^2}{\mu_R^2} c_6(m_\chi, m_t, m_t) - \frac{1}{2} - c_0(m_\chi, m_t, m_t)\Big] v_R^{(t)}$$

$$+ \frac{m_t^2}{\mu_R^2} c_2(m_\chi, m_t, m_t) v_L^{(t)}\Bigg]\lambda_L^2, \tag{22b}$$

$$F_L^c = c_0(m_t, m_\chi, m_\chi)(\tfrac{1}{2} - s^2)\lambda_L^2 \tag{22c}$$

where $\lambda_L = g'_t/g$, g being the usual $SU(2)_L$ coupling constant, m_χ is the mass of the charged χ, μ_R is the mass scale arising in dimensional regularization, and

$$v_R^{(t)} = -\frac{2}{3}\sin^2\theta_W, \quad v_L^{(t)} = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W. \tag{23}$$

The two- and three-point functions b_1, c_0, c_2 and c_6 in terms of the well-known Passarino-Veltman functions [18] are [16]

$$b_1(m_1, m_2) = B_1(m_2, m_1) + \frac{1}{2}(\Delta - \ln\mu_R^2), \tag{24a}$$

$$c_0(m_1, m_2, m_3) = -2C_{24}(m_2, m_1, m_3) + \frac{1}{2}(\Delta - \ln \mu_R^2), \quad (24b)$$

$$c_2(m_1, m_2, m_3) = \mu_R^2 C_0(m_2, m_1, m_3), \quad (24c)$$

$$c_6(m_1, m_2, m_3) = -\mu_R^2[C_{23} + C_{11}](m_2, m_1, m_3) \quad (24d)$$

where $\Delta = 2/(4-d) - \gamma - \ln \pi$ in d dimensions, and this divergence cancels out in the final formula for F_L.

In Fig. 2 we show the plot of $\delta^{\chi}(R_b)$ with m_χ for the top mass ranging from 110 GeV to 200 GeV. The corresponding g_t' values can be obtained from eq. (4). We have taken $\sin^2\theta_W = 0.2324$, $m_b = 4.7$ GeV and $\alpha_s(m_Z^2) = 0.117$. We have checked that very little change in the final results occur if we take into account the errors in $\alpha_s(m_Z^2)$ and $\sin^2\theta_W$.

In Fig. 3 we plot the lower bound on m_χ for different m_t, ranging from 100 to 180 GeV. It may be noted that this bound goes as m_t^2 and $m_t = 190$ GeV is the maximum allowed limit. For $m_t = 150$ GeV, we get $m_\chi = 380$ GeV as the lower limit. It is to be noted that the χ-contribution, being negative, makes the bound very stringent in nature. The new physics decouples in the limit $m_\chi \to \infty$; this result is in conformity with those obtained earlier [8, 12-13]. Of course, this is just a technical point since χ is a composite object and it is meaningless to carry m_χ beyond the compositeness scale. So it can be claimed that *within the framework of this model*, m_χ has both an upper as well as a lower bound, and these come closer and finally coincide as the cutoff Λ is decreased. This behaviour can be easily explained; if we decrease Λ, $(m_t)_{BHL}$ will increase, so there will be a corresponding increase in g_t', and δ_b^{χ} functions are proportional to ${g_t'}^2$. The coincidence occurs at about $\Lambda = 1$ TeV.

In this work, therefore, we have investigated the effects of isodoublet color-octet composite scalars arising in a realistic model with dynamical breaking of electroweak symmetry. The specific process focussed on is the decay $Z \to b\bar{b}$, since the ratio R_b is precisely determined and well-known to be relatively free from uncertainties in the Standard Model parameters such as m_t. We find that a stringent lower bound can be placed on the masses of these composite colored scalars, which is around 400 GeV for a top mass of 150 GeV, and increases quadratically with m_t. At the present moment, one of the main issues confronting Z decay experiments is to determine the difference between the Standard Model prediction of R_b and the experimental number, because this is one more possible gateway to look into the new physics. The Minimal Supersymmetric Standard Model predicts a positive (but small) δR_b, and nearly all extensions and modifications in the scalar sector (whether elementary or composite) predict δR_b to be slightly negative. None of the alternatives can be ruled out at the present moment, and further precision experiments could help discriminate between models.

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Figure Captions

1. The one-loop diagrams involving the colored scalars $\chi^{\pm}$ which contribute to $\Gamma(Z \to b\bar{b})$.
2. The contribution of the $\chi^{\pm}$ to the parameter R_b as a function of m_{χ^+}. The shaded region depicts the phenomenologically allowed region for the top mass.
3. The lower bound $\bar{m}_\chi$ on the mass of $\chi^{\pm}$ as a function of m_t for $R_b = 0.2170$.