

Proton dripline in a new formula for nuclear binding energy

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Abstract

The location of the proton dripline in a new phenomenological mass formula is calculated. Predictions of different mass formulas for the dripline are compared. The implications of the new mass formula for rapid proton nucleosynthesis beyond ^{56}Ni are discussed. It is seen that the new formula indicates that masses up to $A = 80$ are easily synthesized in a typical X-ray burst.

1 Introduction

Prediction of binding energy is a very important task in nuclear physics. For example, binding energy of the nuclear ground state is one of the most important inputs in the study of astrophysical reactions. Experimental mass measurements are difficult to make in nuclei far from the stability valley. Thus, one has to take recourse to theoretical predictions. In a recent paper[1] (hereafter called Ref. I) a new phenomenological formula for ground state binding energies has been introduced. In the present work, we explore a few of the implications of the new formula and also compare our results with the predictions from some other formulas.

The area we concentrate on in the present work is related to the study of proton dripline. As pointed out in Ref. I, the root mean square (r.m.s.) error in the ground state binding energies is 0.376 MeV for 2140 nuclei. Near the stability valley, single nucleon separation energy (proton or neutron) is of the order of 8 MeV. Thus, a small error in binding energy does not influence the results of processes which involve nuclei near this valley. However, the separation energies of protons and neutrons near the corresponding driplines are actually very small, sometimes comparable to the errors in theoretical predictions. Thus, it should be interesting to study the performance of the formula from Ref. I in predicting the location of the proton dripline. We also investigate the effect of the mass formula on the rapid proton (rp) process nucleosynthesis beyond ^{56}Ni . This process goes along the $N = Z$ line and is dependent on the binding energy of protons near the proton dripline.

2 Calculation and Results

For details of the mass formula, we refer the readers to Ref I where it was developed. The present section is divided into two parts. In the first part, we calculate the location of the proton dripline according to the present formula and compare with experimental observations and a few other predictions. In the second part, we study the effect of the present mass formula in astrophysical rp -process in mass 60-80 region.

2.1 Proton dripline

The proton dripline for a particular neutron number is defined to be the nucleus beyond which the proton separation energy becomes zero or negative. Obviously, because of the pairing effect, the Z of the proton dripline nucleus is expected to be even. Studying the nucleon dripline is one of the major activities in nuclear physics. However, it has been possible to reach the neutron dripline in only very light nuclei. Proton dripline, on the other hand, is more accessible to experiments. We note that the location of the proton dripline is known for a number of neutron numbers, particularly in mass 100-220 region. Nuclei with protons in positive energy continuum show proton radioactivity where protons tunnel through the Coulomb barrier. A number of such nuclei are known in mass 100-200 region.

Table 1: Location of proton dripline. The columns stand for the following: Exp. - Experimental, Ref. I - Present calculation, FRDM - Finite Range Droplet Model and D-Z - Dufflo-Zuker formula.

N	Z of proton dripline nucleus				N	Z of proton dripline nucleus			
	Exp.	Ref. I	FRDM	D-Z		Exp.	Ref. I	FRDM	D-Z
18	20	20	20	20	96	78	78	78	78
32	32	32	34	32	97	78	80	78	78
34	34	34	36	34	98	78	80	78	80
36	36	36	36	36	99	80	80	78	80
54	50	50	50	50	100	80	80	80	80
56	52	52	50	52	101	80	80	82	82
58	54	54	54	54	102	82	80	82	82
78	66	68	68	68	103	82	82	82	82
79	68	68	70	68	104	82	82	82	82
82	70	70	70	70	105	82	82	82	82
83	70	72	72	70	106	82	82	82	82
84	70	72	72	72	108	84	84	82	84
85	72	72	72	72	110	84	84	86	84
86	72	72	74	72	112	86	86	86	86
87	74	74	74	72	113	86	86	86	86
88	74	74	74	74	114	86	86	88	86
90	74	74	76	74	117	88	88	88	88
91	76	76	76	76	118	88	88	88	88
92	76	76	76	76	121	90	90	90	90
93	76	78	78	76	122	90	90	90	90
94	76	78	78	76	123	90	92	92	92

The known mass values of nuclei enable us to calculate the location of the proton dripline. In the first two columns of table I, we summarize our knowledge for the neutron numbers for which the experimental proton dripline is exactly known. The mass table of Audi *et al.*[2] and the more recent references on binding energy measurements included in Ref. I have been used to determine the experimental proton dripline. We calculate the location of the proton dripline from the new binding energy formula of Ref. I and present it in Table I. The results of the Finite Range Droplet Model (FRDM)[3] and the Duflo-Zuker (D-Z) mass formula[4] are also presented. We see that the present method can accurately predict the location of the proton dripline in most of the cases and is actually better than FRDM. The Duflo-Zuker formula is even better in this respect.

We would like to point out that the conclusions of different experiments on the binding energy of ^{65}As do not agree. Schury *et al.*[5] have concluded that ^{65}As has a negative proton separation energy while a recent measurement[6] has raised the possibility of it being stable with respect to proton emission. The implication of this uncertainty in proton separation energy of ^{65}As has been discussed in the next subsection in more detail.

Even in the few cases where the present approach fails to accurately locate the dripline, the next even Z nuclei is indicated. One needs to remember that the formula has an r.m.s. error of 376 keV. Thus, if the predicted binding energy of the last proton in an odd Z nucleus is very small, it is possible that the nucleus is actually beyond the proton dripline. Conversely, prediction of a very small negative value of last proton separation energy cannot guarantee that the dripline does not actually lie beyond. For example, in $N = 93$ and 94 nuclei, the calculated binding energy of the last proton in $^{170,171}_{77}\text{Lu}$ are only 0.047 MeV and 0.012 MeV, respectively. On the other hand, the calculated separation energy of the last proton in $^{183}_{81}\text{Tl}$ ($N = 102$) is -0.030 MeV only.

2.2 Astrophysical rapid proton process

We want to look at the effect of the new mass formula on nucleosynthesis. Particularly, in view of the success of the formula to predict the proton dripline, it should be interesting to see the effect of the proton separation energy predicted by the present formula on the rp -process. In this process, which takes place in hot explosive proton-rich environment such as X-ray bursts, a nucleus may capture high energy protons. However, this process has to compete with its inverse, i.e. photodisintegration by emitting a proton at high temperature ((γ, p) reaction). A negative or a small positive value of proton separation energy implies that the inverse reaction dominates and the rp -process stalls at that point, the so-called waiting point. Two proton capture may lead to the nucleosynthesis bridging the waiting point and proceed beyond. More details of the process is available in standard text books (For example the book by Illiadis[7]).

The stellar decay constant (λ) for photodisintegration, *i.e.* (γ, p) reaction, is related to the proton capture rate by the reciprocity theorem in the following

numerical form[7]

$$\lambda = 9.86851 \times 10^9 T^{\frac{3}{2}} \left(\frac{M_p M_X}{M_Y} \right)^{\frac{3}{2}} \frac{(2J_p + 1)(2J_X + 1)}{(2J_Y + 1)} \frac{G_p G_X}{G_Y} N \langle \sigma v \rangle_{pX \rightarrow Y\gamma} \exp \left(\frac{-11.605Q}{T} \right) \quad (1)$$

for the reaction $p + X \leftrightarrow Y + \gamma$ and is expressed in sec^{-1} when the forward reaction rate, given by $N \langle \sigma v \rangle_{pX \rightarrow Y\gamma}$, is expressed in $cm^3 mol^{-1} sec^{-1}$. The temperature T is expressed in GK ($10^9 K$) and Q is the ground state Q-value of the forward reaction expressed in MeV. The normalized partition functions, G_X and G_Y , may be obtained from Rauscher *et al.*[8]. For protons, we have $G_p = 1$ and $J_p = 1/2$.

The photodisintegration rate depends exponentially on the proton separation energy. This quantity, thus, plays a very important role in the possibility of rp -process proceeding beyond the waiting point. There are significant disagreements in the predictions of proton separation energy of different mass formulas, and even between different measurements. In some other cases, experimental measurements may exist but the error in measurement is too large to draw any meaningful conclusion. As an example, we look at an example of the waiting point nucleus, ^{64}Ge . The measured and estimated Q-values of the proton capture reaction differ over a large range. Schury *et al.*[5] have measured the Q-value to be -0.255 ± 0.104 MeV. A recent measurement[6] has found the proton separation energy of ^{65}As to be $0.401(530)$ MeV, *i.e.* the nucleus may even be bound. The Duflo-Zuker formula[4] suggests the value to be -0.407 MeV. The FRDM calculation of Möller *et al.*[3] predicts the value as 0.13 MeV. The mass table[2] gives the Q-value as -0.08 MeV. A very recent measurement[9] has given a value of $-0.090(0.085)$ MeV. The new mass formula suggests a value -0.116 MeV, which is very close to the last estimation. In such cases, it becomes imperative to look at the implications of different mass formulas. We present the bridging this particular waiting point in some detail.

For the calculation of proton capture process, a small network has been designed which includes the following processes. The waiting point nucleus, which acts as a seed, may capture a proton. The resulting nucleus ^{65}As may either capture another proton to produce ^{66}Se or undergo photodisintegration emitting a proton to go back to the seed nucleus. The nucleus ^{66}Se may also undergo photodisintegration. In addition, all the three nuclei mentioned above may undergo β -decay.

We calculate the change in half life of ^{64}Ge in an explosive astrophysical environment assuming the density to be $10^6 gm/cm^3$ and the proton fraction to be 0.7 . A microscopic optical potential has been obtained by folding the DDM3Y potential[10] with nuclear densities obtained in Relativistic Mean Field model with the Lagrangian density FSU Gold[11]. The calculation follows our earlier works[12] where more details are available. The measured half life values for β -decay have been taken from the compilation by Audi *et al.*[13] except in the case of the case of ^{65}As . For this nucleus, a value of 0.128 seconds from an experimental measurement[14] has been assumed. If measured values are not available, we have adopted the values from the work by Möller *et al.*[15]

The results of the calculation on effective half life of ^{64}Ge as a function of temperature are presented for different mass estimates in Fig. 1. The different

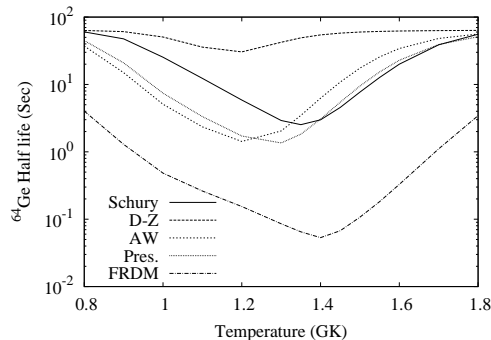


Figure 1: Effective half life values of ^{64}Ge as a function of temperature for different Q-values of $^{64}\text{Ge}(p, \gamma)$ reaction.

values have been indicated as follows: Schury– measurement by Schury *et al.*[5], D-Z– Duflo-Zucker formula[4], A-W– Mass table of Audi *et al.*[2], Pres.– Present work and FRDM– FRDM results of Möller *et al.*[3]. At low temperature proton capture rate is small and β -decay predominates. At high temperature photodisintegration dominates over capture. Thus the predominant decay mode is again β -decay. In between there is a temperature window where two proton capture leads to ^{66}Se . From the figure, it is clear that the rate of two proton capture is sensitively dependent on the masses used in the calculation. The Q-values predicted by the Duflo-Zucker formula leads to a minimum lifetime of nearly 30 seconds. The values predicted by the present mass formula and that by Audi *et al.*[2] decrease the lifetime by more than one order of magnitude. If ^{65}As is bound as suggested by some works[6, 3], the rate of the inverse process is small. In such a situation, two proton capture dominates and the waiting point is easily bridged.

As is evident from the above discussion, the bridging of waiting points, and hence the rp -process abundance crucially depends on the proton separation energy. However, this is sometimes poorly known on the $N = Z$ nucleus as the process goes along the proton dripline where information is meager. The degree of bridging of the waiting point has significant effect on the path that the rp -nucleosynthesis takes in explosive proton-rich environments such as X-ray bursters. Typical X-ray burst has a time scale of 10-100 seconds. In case a significant fraction of the population does not undergo two proton capture reaction, it traverses along a path of more stable isotopes and nucleosynthesis is delayed. Thus, it may not be possible for an X-ray burst to populate nuclei with significant mass in case a particular waiting point stalls the process substantially.

As an example, let us compare the effects of the Duflo-Zucker and the new mass formulas on the population at different masses compared to the waiting points with $A < 80$. We have calculated the rp -process rates for mass 60-80 region. We plot the difference of the calculated binding energy using Ref I from the experimental values in Fig. 2. The mass formulas have been used for masses which have not been experimentally measured or have considerable experimental uncertainty as in the case of ^{65}As . For consistency, even if one of the masses involved in the reaction is known, we have obtained the Q-values from theoretical predictions only. The waiting points occur at $A=64, 68, 72$

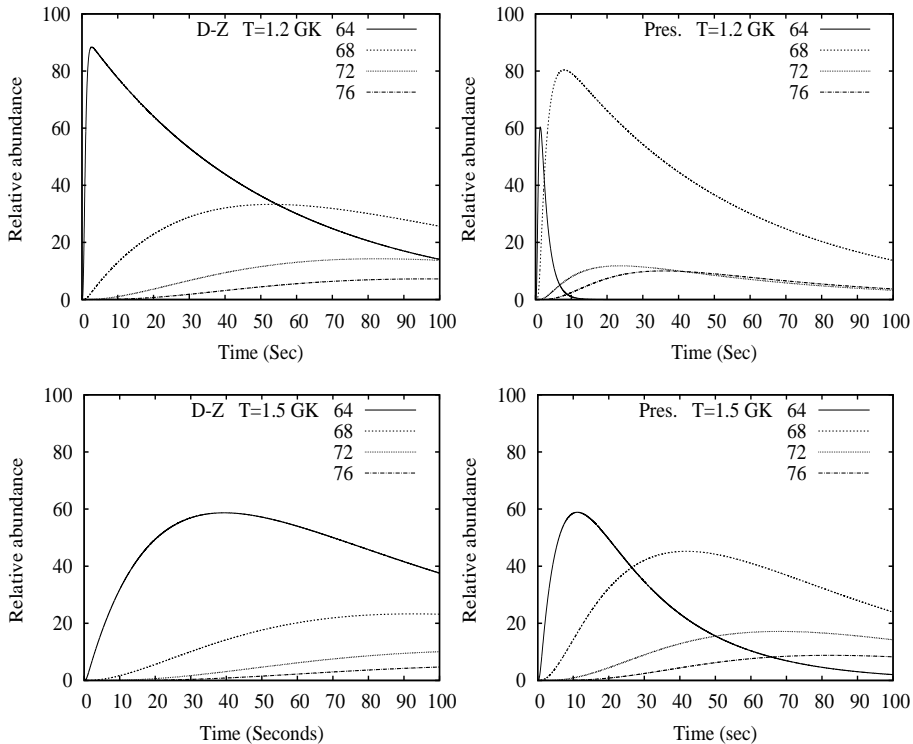


Figure 2: Evolution of abundance of mass at the waiting points in explosive proton-rich astrophysical environments. See text for details.

and 76 in the nuclei with $N = Z$. We assume a constant proton fraction of 0.7. We have designed a network which includes the proton-capture reactions and β -decay starting from the seed nucleus ^{56}Ni and goes beyond $A = 80$. Astrophysical rates have been calculated as discussed in our previous works[12]. The network is run for two constant temperatures, 1.2 GK and 1.5 GK. In Fig 2, we plot the evolution of abundance of nuclear mass as a function of time for $A = 64, 68, 72$ and 76 . The top (bottom) two panels are the results for $T = 1.2$ GK (1.5 GK). The left (right) panels indicate the results obtained when we use the Duflo-Zuker formula (Ref I) for unknown mass values as described above.

It is clear the Duflo-Zuker mass formula predicts a significant stalling at mass $A = 64$ so much so that a burst of ten second duration fails to proceed significantly beyond this mass. It is consistent with the result on half life discussed earlier as at 1.5 GK, the half life of ^{64}Ge is not expected to change significantly. Thus the rp -process can continue only after β -decay of ^{64}Ge . The process thus shifts toward more stable nuclei and gets stalled. We see that the abundance is dominated by $A = 64$ nuclei the rapid proton nucleosynthesis even in an X-ray burst of 100 seconds for $T = 1.5$ GK.

The present mass formula, on the other hand, indicates that most of the ^{64}Ge is converted to ^{66}Se very shortly. This leads to significant differences in the abundance pattern. Of course, in an actual X-ray burst, the temperature and the proton mass fraction are not constant but change with time. Thus

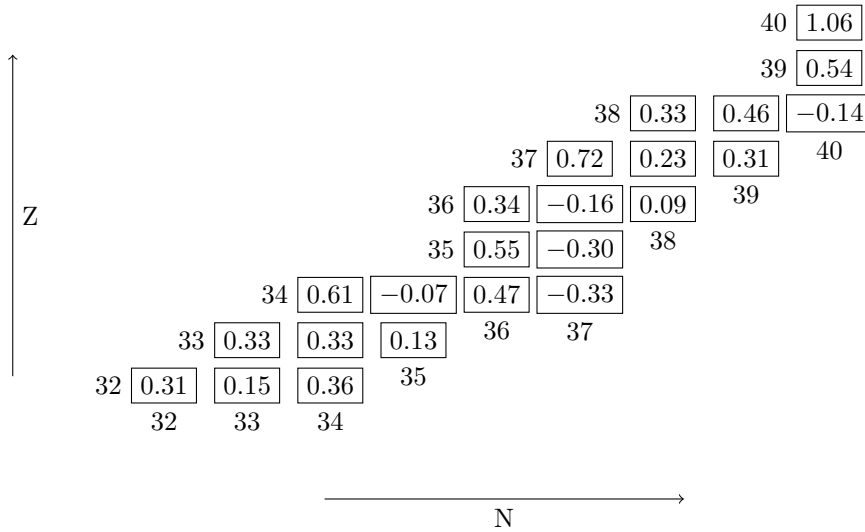


Figure 3: Differences between experimental[2] and calculated mass values near the $N = Z$ line in mass 60-80 region.

we do not expect this simple calculation to reproduce the observed abundance pattern. The aim of the present study is to point out the effect the different mass formulas can have on the final abundance pattern.

In Fig. 3, we plot the path followed by the rp -process between $A = 56$ and $A = 80$ following the mass formula of Ref. I for $T = 1.2$ GK and 1.5 GK. The dark boxes indicate the waiting points. The Z values are indicated at the left of the diagram while the N values are shown at the bottom. The lines indicate the path followed by nucleosynthesis. The paths through which more than 10% of the total flux flows are indicated by black lines while gray lines show the corresponding paths for flux between 1% - 10%. The two waiting points, where Q-values from the present formula have been used, are ^{64}Ge and ^{76}Sr . The former has been chosen as experimental values differ widely. Experimental measurement is not available in ^{76}Sr . One can see that ^{64}Ge is easily bridged by two proton capture at 1.2 GK. At 1.5 GK temperature, this waiting point delays the process so that β -decay of ^{64}Ge contributes significantly. At the next waiting point, ^{68}Se , the photodisintegration is sufficiently strong so that, independent of the temperature, β -decay is practically the only available path. This delays the nucleosynthesis significantly. At ^{72}Kr , inverse process predominates in higher temperature driving the flux through β -decay. Thus, here also, lower temperature helps nucleosynthesis speed up. The waiting point at ^{76}Sr presents a different picture where the path essentially does not depend on the temperature and principally flows along decay. It is clear that the actual process is significantly dependent on the model of the burst process where the temperature and the proton fraction are functions of time. We find that at 1.2 GK, at the end of 100 seconds, the population that reaches $A = 80$ or beyond is more than 1.5 times than the corresponding quantity at the higher temperature of 1.5 GK. In both the cases, the population beyond $A = 76$ is significant.

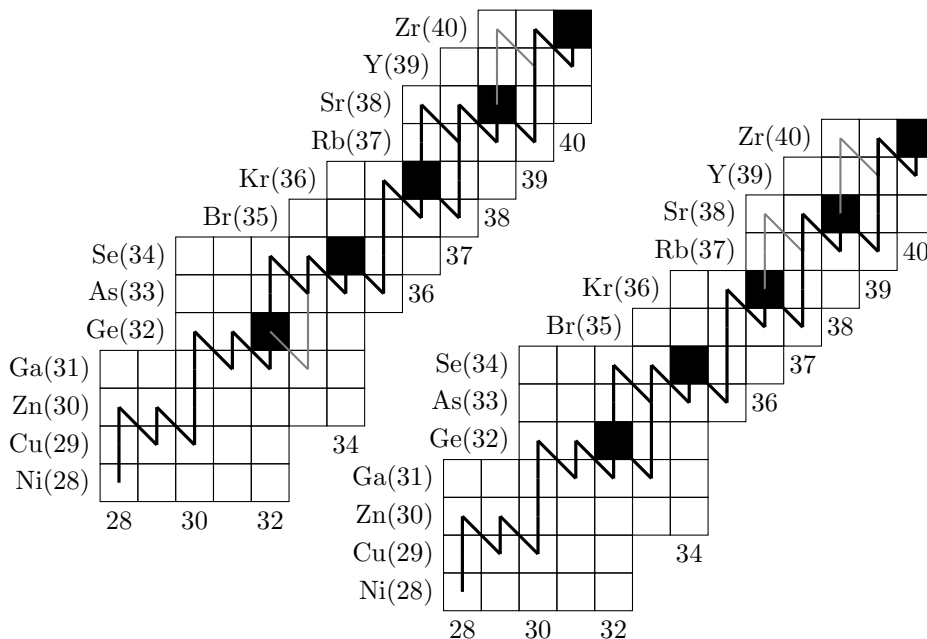


Figure 4: rp-process path for 1.2 GK (left panel) and 1.5 GK (right panel).

3 Summary

The location of the proton dripline has been calculated using a new mass formula of Ref. I and compared with several other results as well as experiment. The new formula gives a good description of the dripline. The implication of the mass prediction using the new formula on *rp*-process nucleosynthesis beyond ^{56}Ni has been investigated. The masses predicted by the present formula indicate that nucleosynthesis proceeds easily up to mass 80 in a typical X-ray burst.

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References

- [1] G. Gangopadhyay, *Int. J. Mod. Phys. E* **20** (2011) 179.
- [2] G. Audi, A.H. Wapstra, and C. Thibault, *Nucl. Phys.* **A729** (2003) 337.
- [3] P. Möller, J. R. Nix, and K. L. Kratz, *Atom. Data Nucl. Data Tabl.* **66** (1997) 131.

- [4] J. Duflo and A.P. Zuker, *Phys. Rev. C* **52** (1995) R23.
- [5] P. Schury *et al.*, *Phys. Rev. C* **75** (2007) 055801.
- [6] H.S. Xu *et al.*, *Chinese Sci. Bull.*, **54** (2009) 4749.
- [7] C. Illiadis, *Nuclear Physics of the Stars* (Wiley-VCH Verlag GmbH, Weinheim, 2007).
- [8] T. Rauscher and F-K Thielemann, *At. Data. Nucl. Data. Tables* **74** (2000) 1.
- [9] *Phys. Rev. Lett.*
- [10] A.M. Kobos, B.A. Brown, R. Lindsay, and G. R. Satchler, *Nucl. Phys.* **A425** (1984) 205; A.K. Chaudhuri, *Nucl. Phys.* **A449**(1986) 243; **A459** (1986) 417; D. N. Basu, *J. Phys. G: Nucl. Part. Phys.* **30** (2004) B7.
- [11] B.G. Todd-Rutel and J. Piekarewicz, *Phys. Rev. Lett.***95** (2005) 122501.
- [12] G. Gangopadhyay, *Phys. Rev. C* **82** (2010) 027603; C. Lahiri and G. Gangopadhyay, communicated.
- [13] G. Audi, O. Bersillon, J. Blachot and A.H. Wapstra, *Nucl. Phys.* **A729** (2003) 3.
- [14] M.J. López Jiménez *et al.*, *Phys. Rev. C* **66** (2002) 025803.
- [15] P. Möller, J. R. Nix and K. L. Kratz, *At. Data. Nucl. Data. Tables* **66** (1997) 131.