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Power law enhancement of neutrino mixing angles in extra dimensions

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Abstract

We study the renormalization of the $llHH$-type Majorana neutrino mass operator in a scenario where there is a compactified extra dimension and the fields involved correspond to only the standard model particles and their Kaluza-Klein excitations. We observe that in a two flavour scenario, where one of the neutrinos is necessarily ν_τ, it is indeed possible to generate a large mixing at ~ 100 GeV starting from a very small mixing near the ultra-violet cutoff ~ 30 TeV. *En passant*, we also derive the Higgs mass upper and lower limits from perturbative unitarity and stability of the potential, respectively.

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If lepton number is not a good symmetry of the Lagrangian, then, without enlarging the standard model (SM) particle content, a neutrino Majorana mass operator can be written as (with i, j as generation indices)

$$-\mathcal{L}^{\rm SM} = \frac{\kappa_{ij}}{M_X} \bar{l^c}_i l_j HH + {\rm h.c.} \tag{1}$$

This dimension-5 operator can be viewed as a consequence of integrating out a superheavy right-handed neutrino of mass $\sim M_X$ which is exchanged at the tree level. Here l is the SM lepton doublet and H is the SM Higgs doublet. Eq. (1) gives a neutrino mass matrix $m_{ij} \sim \kappa_{ij}(v^2/M_X)$, where v is the vacuum expectation value of the SM Higgs. Assuming $\kappa \sim 1$, a choice of $M_X \sim 10^{15}$ GeV produces $m \sim 0.1$ eV. It has been pointed out in [1, 2, 3] that starting from a small mixing angle between two active neutrinos at a high scale, a large mixing between them can be generated at a low scale due to renormalization group (RG) evolution. In this paper we intend to investigate the renormalization of the above operator in extra-dimensional models. For simplicity, we consider only one additional space dimension which is compactified on a circle. Since both solar and atmospheric neutrino data prefer large neutrino mixing, our primary aim is to examine whether the extra-dimensional models can reproduce this feature. We restrict ourselves only to the case of oscillation between two active generations where one of the two neutrinos is necessarily ν_τ. Even though the mass scales in such models are expected to be quite close – around 1 TeV in our choice – and the energy range for RG running small, we will show that because of the power law evolution of the κ operator, the neutrino mixing angle runs rather fast once the Kaluza-Klein (KK) modes of the higher dimensional fields open up. As a result, even if the two-flavour mixing angle happens to be quite small near the ultra-violet cut-off $\Lambda \sim \mathcal{O}$ (10 TeV), where the textures are defined, near-maximal mixing can be generated at the 100 GeV scale. If the mixing is large at the high scale then it undersgoes further enhancement due to RG running.

We stick to a very simple extra-dimensional scenario in which the extra space dimension (y) is compactified on a circle of radius R, i.e., $y \leftrightarrow y + 2\pi R$. In our simple approach, all fermions are localized at the brane at $y = 0$, but the bosons can also travel in the bulk [4, 5, 6]. In the effective 4-dimensional representation, after the fifth coordinate is integrated out, the resulting Majorana mass operator looks like

$$-\mathcal{L}^{\rm eff} = \frac{\kappa_{ij}}{\pi M^2 R} \bar{l^c}_i l_j H_0 H_0 + {\rm h.c.} \tag{2}$$

Above, $H_0(x)$ is the zero mode of the KK excitations of the doublet scalar in five dimensions: $H(x,y) = (1/\sqrt{\pi R}) \sum_{n=-\infty}^{\infty} H_n(x) \exp\,(iny/R)$. M corresponds to some higher dimensional mass scale beyond which new physics sets in.

The neutrino mass matrix is given by $m_{ij} \sim \kappa_{ij}(v^2/\pi M^2 R)$. For definiteness, we assume $\mu_0 \equiv R^{-1} = \mathcal{O}$ (1 TeV). μ_0 determines the mass splittings of the KK excitations. The appearance of M may be interpreted as a consequence of integrating out some physical states around $\sim M$ (e.g., a right-handed neutrino N with a mass M that couples like LHN) which leads to the effective operator in Eq. (2). Thus below the scale M the theory is essentially non-renormalizable in the sense that a heavy state is integerated out leading to effective Lagrangian in Eq. (2). Since we are basically interested in the evolution of κ, which in turn requires the running of gauge, Yukawa, and Higgs self-couplings, it seems quite reasonable to associate the cut-off parameter Λ with M.

Now we attempt to briefly address the issue of a second kind of non-renormalisibility which stems mainly from the presence of an *infinite* tower of KK states after compactification to 4 dimensions (for an extensive discussion see Appendix B of [5]). In fact, as stressed in [5], the couplings do not strictly run in a non-renormalizable theory. Instead they receive finite quantum corrections whose magnitudes explicitly depend upon the cut-off $\Lambda \sim M$. It also turns out that very often the mathematical dependence of a coupling on Λ is identical to its scale-dependence that follows from a naive calculation assuming a renormalizable theory. Since the root of this non-renormalisibility lies in having an *infinite* KK tower, the remedy, as suggested in [5], is to consider a *truncated* KK series which has been shown to serve as an excellent approximation for calculating the scale dependence of couplings. Under the above guideline, we continue to describe the quantum corrections of couplings as their RG running. Indeed, all the couplings have to remain perturbative throughout the energy interval $M_Z < \mu < M$, and a rough estimate of the hierarchy [7], namely $(M/\mu_0)^\delta \sim \ln\,(M_{\rm GUT}/M_W)$, with $M_{\rm GUT}$ as the 4-dimensional GUT scale and δ as the number of extra dimensions, yields $M \sim 30\mu_0$ for $\delta = 1$.

Here, for the sake of clarity, we stress that M should not, in general, be equated to the 5-dimensional Planck scale $M_\star$. In fact, it follows from the relation $M_P^2 = M_\star^2 (M_\star R)^\delta$, where M_P is the 4-dimensional Planck scale, that $M_\star \sim 10^{10} - 10^{11}$ TeV for $\delta = 1$ and $R^{-1} = 1$ TeV. Thus $M_\star \gg M$ and hence quantum gravity effects on the effective Majorana mass operator at the scale M or below are insignificant.

Assuming $\kappa \sim 1$, a further suppression of 9 orders of magnitude is required to produce a neutrino mass of order 0.1 eV. Such a suppression may come from a distant brane where lepton number (L) is violated and the effect at the brane under consideration is damped by the distance between the two branes [6].

Since quark mixing angles are small, our main curiosity in this paper will be to check whether a small ν_τ-ν_e or ν_τ-ν_μ mixing near $\Lambda \sim M$ can indeed become large at M_Z due to power law RG running. The mixing angle depends not on the absolute value of κ_{ij}, but on the degree of degeneracy of κ_{11} and κ_{22}. We will need to tune this difference at $\sim M$ to obtain a large mixing angle at $\sim M_Z$. In fact, we have found that this tuning ensures the mixing at M_Z to be large for essentially *any* initial mixing, small or large.

The presence of extra dimension modifies the running of κ (matrix) in the region $\mu > \mu_0$ as follows:

$$16\pi^2 \frac{d\kappa}{d\ln\mu} = \left(-3g_2^2 + 2\lambda + 2S\right) t_\delta \kappa - \frac{3}{2} t_\delta \left[\kappa(Y_l^\dagger Y_l) + (Y_l^\dagger Y_l)\kappa\right], \tag{3}$$

where $S = {\rm Tr}\,(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_l^\dagger Y_l)$, and $t_\delta = (\mu/\mu_0)^\delta X_\delta$. In Eq. (3), t_δ controls the power law behaviour, where X_δ can be expressed in terms of the Euler Gamma function as $X_\delta = 2\pi^{\delta/2}/\delta\ \Gamma(\delta/2)$. For $\delta = 0(1)$, $X_\delta = 1(2)$. It is important, for later discussions, to bear in mind that Eq. (3) is homogenous in κ.

The running of the Yukawa couplings (Y_u, Y_d) and Higgs self-coupling (λ) for $\mu > \mu_0$ are given by

$$\begin{aligned} 16\pi^2 \frac{dY_u}{d\ln\mu} &= \frac{3}{2} t_\delta \left(Y_u Y_u^\dagger Y_u - Y_d^\dagger Y_d Y_u\right) + t_\delta S Y_u - t_\delta \left(8g_3^2 + \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2\right) Y_u, \\ 16\pi^2 \frac{dY_d}{d\ln\mu} &= \frac{3}{2} t_\delta \left(Y_d Y_d^\dagger Y_d - Y_d Y_u Y_u^\dagger\right) + t_\delta S Y_d - t_\delta \left(8g_3^2 + \frac{1}{4} g_1^2 + \frac{9}{4} g_2^2\right) Y_d, \end{aligned} \tag{4}$$

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = 12 t_\delta \lambda^2 - \left(\frac{9}{5}g_1^2 + 9 g_2^2\right) t_\delta \lambda + \frac{9}{4} t_\delta \left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right)$$
$$+ \quad 4S\lambda - 4\mathrm{Tr}\,\left[(Y_l^\dagger Y_l)^2 + 3(Y_d^\dagger Y_d)^2 + 3(Y_u^\dagger Y_u)^2\right] .$$

It should be noted that in the limit $\delta = 0$ (i.e., $t_\delta = 1$) one reproduces the SM expressions [1, 2, 8, 9] which control the evolution in the interval $M_Z < \mu < \mu_0$. We stress here that our calculation of κ evolution agrees with that of [8] who have pointed out a small error in the original calculations of [1, 2]: more specifically, the numerical factor in front of the leptonic Yukawa contribution in Eq. (3) is indeed 3/2 rather than 1/2.

The evolution of the gauge couplings in an extra-dimensional scenario have been worked out in [5], and for $\mu > \mu_0$ are given by

$$\begin{aligned} 16\pi^2 \frac{dg_i}{d\ln\mu} &= b_i g_i^3, \quad \text{where} \\ b_1 &= 41/10 + (t_\delta - 1)(1/10), \\ b_2 &= -(19/6) - (t_\delta - 1)(41/6), \\ b_3 &= -7 - (t_\delta - 1)(21/2). \end{aligned} \tag{5}$$

In the interval $M_Z < \mu < \mu_0$, the gauge couplings run as in the SM and the corresponding beta functions are obtained by putting $t_\delta = 1$ in Eq. (5).

The computational procedure behind the power law running behaviour is simple [5, 10]. In the scenario under consideration, gauge bosons and scalars have KK excitations, but fermions are localised at a brane, which is a fixed point. The external boson legs in any diagram are their KK zero modes which represent their SM states. In the loop diagrams there can be either one or two internal KK modes. If there is only one, then each time a KK threshold is crossed, the diagram contributes the same as in the SM regardless of the KK number of the internal line. Such a situation may arise only when an internal boson meets a fermion at the brane where KK number is not conserved due to the breakdown of the fifth-dimensional translational invariance at the fixed point. If there are two internal KK modes, then both should have the same KK number as the latter is assumed to be conserved at the vertex, hence a single summation. As before, each time such a KK threshold is crossed, the diagram contributes an amount identical to the SM. Then after summing over an infinite tower of KK modes, as shown in [5], one obtains the following simple working rule: identify the diagram which contains internal KK modes and multiply its SM contribution by t_δ. In fact, t_δ represents the volume of a δ-dimensional sphere of radius μ where the unit of volume is μ_0^δ – it counts the number of KK modes excited upto an energy scale μ. So, in a sense, t_δ is a measure of the density of KK modes which accelerates the running by inducing an explicit μ^δ dependence on the right hand side of Eqs. (3), (4) and (5). Clearly, in the limit $\delta = 0$, one recovers the usual logarithmic running. Intuitively, the power law behaviour stems from the fact that couplings which are dimensionless in 4 dimensions become dimensionful in higher dimensions.

Before embarking on the main theme of the running of the neutrino mixing angle, we touch upon a related issue which concerns the allowed range of the Higgs mass. In the SM, where Higgs constitutes the only scalar, the requirement that the scalar potential remains bounded from below (i.e., $\lambda > 0$) throughout the energy thoroughfare $M_Z < \mu < M_{\rm GUT}$ restricts the Higgs mass to lie above ~ 145 GeV. The crucial controlling factor is, in fact, the splitting between the top and the Higgs masses. Supposing the Higgs to weigh ~ 115 GeV, where a preliminary hint was claimed by the LEP Collaborations, the one-loop RG running in the SM drives the λ parameter towards negative values near a scale as close as $\sim 10^4 - 10^5$ GeV, which prompted the authors of Ref. [11] to invoke the case for supersymmetry which prevents the occurence of a negative λ. In our case, which deals with only SM and its bosonic KK excitations, the energy interval, as we discussed before, is $M_Z < \mu < M$, where $M \sim 30$ TeV with $R^{-1} = 1$ TeV. We have found, with the RG running given by Eq. (4), that (i) the stability of the potential ($\lambda > 0$) requires a lower limit $M_H > 98$ GeV, and (ii) the requirement of perturbativity demands an upper limit $M_H < 153$ GeV. Admittedly, these limits are merely indicative as they are based on only one loop RG evolution.

Now let us take a stock of the parameters which control the running of κ_{ij} and the neutrino mixing angle (θ). The values of all the gauge and the relevant Yukawa couplings at the weak scale are input parameters.

Similarly, a choice of the Higgs mass is necessary to fix the quartic coupling, λ, at the weak scale. We have checked that the mixing angle evolution is insensitive to the choice of the Higgs mass as long as the latter respects the stability and the perturbativity conditions of the potential. As a reference point, we have chosen $M_H = 115$ GeV. Then a two-step running determines the values of all these couplings at the scale M. In the interval $M_Z < \mu < \mu_0$, the running is logarithmic, controlled by the SM beta functions (putting $\delta = 0$), while in the range $\mu_0 < \mu < M$, power law running takes over with $\delta = 1$. We choose $\mu_0 = 10^3$ GeV and $M = 10^{4.5}$ GeV $= 30$ TeV to be our reference scales. Variations of μ and M around these reference values do not throw much insight into our agenda, and hence, for the sake of brevity and concise illustration, we stick to these values throughout this paper. The κ matrix is defined and parametrized at the scale M for the two-flavour case as $d\kappa \equiv (\kappa_{11} - \kappa_{22})/\kappa_{22}$. The other parameter to be fixed at M is the neutrino mixing angle given by $\tan 2\theta = 2\kappa_{12}/(\kappa_{22} - \kappa_{11})$, in a basis in which the charged lepton mass matrix is diagonal. The mixing angle runs according to

$$16\pi^2 \frac{d\sin^2 2\theta}{d\ln\mu} = \sin^2 2\theta(1 - \sin^2 2\theta)(y_2^2 - y_1^2)\frac{\kappa_{22} + \kappa_{11}}{\kappa_{22} - \kappa_{11}}, \qquad (6)$$

where y_2 and y_1 are the charged lepton Yukawa couplings. In our case, y_2 is Y_τ and y_1 is either Y_e or Y_μ. It is important to note, as emphasized by Chankowski et al. in [3], that although $\theta = 0$ is a fixed point, $\theta = \pi/4$ is not. In fact, the evolution of $d\kappa$ does not have a fixed point at $d\kappa = 0$.

Our goal is to choose small but non-zero values of $\sin^2 2\theta|_M$ and then investigate whether appropriate values of $d\kappa|_M$ exist which would magnify $\sin^2 2\theta|_{M_Z}$ following a two-step running. An inspection of Eq. (6) reveals that this running would be significant only when $d\kappa$ is less than or close to Y_τ^2. This requires $\kappa_{22} < \kappa_{11}$ at M. In fact, during the process of running, κ_{11} and κ_{22} cross each other at some energy scale leading to a resonance in the mixing angle at that scale. This happens due to the appearance of $d\kappa$ in the denominator of the right hand side of Eq. (6). Indeed, the scale at which this resonance occurs depends crucially on the interplay between $d\kappa|_M$ and the distinct lengths of the logarithmic and power law running determined by the choices of μ_0 and M. Our purpose is to attribute a very small mixing angle at M and probe the appropriate parameter range that generates a large mixing angle near M_Z.

In Fig. 1 we have plotted $\sin^2 2\theta$ as a function of the renormalization scale for different values of $d\kappa|_M$ and $\sin^2 2\theta|_M$. The graphs labelled by (a), (b) and (c) correspond to the choices of the initial mixing angle $\sin^2 2\theta|_M = 0.05, 0.1$ and 0.01, respectively, for a fixed $d\kappa|_M = 1.5 \times 10^{-4}$. We observe that for the plots (a) and (b) $\sin^2 2\theta|_{M_Z}$ reaches near maximal values, while for (c) it is still quite large. For the other two cases (d) and (e), $\sin^2 2\theta|_M$ has been fixed to 0.05, only that for (d) $d\kappa|_M = 1.3 \times 10^{-4}$ while for (e) $d\kappa|_M = 1.7 \times 10^{-4}$. We make two observations: (i) for smaller values of $d\kappa|_M$, the mixing angle resonance occurs at a higher scale as a result of $\kappa_{22} - \kappa_{11}$ approaching zero with less running from above, and (ii) with smaller values of $\sin^2 2\theta|_M$, the values of $\sin^2 2\theta|_{M_Z}$ are smaller, as expected. Thus, with the ultimate goal of generating a large mixing angle at M_Z in mind, a significantly large fine-tuning is admittedly involved in the selection of $d\kappa|_M$, but the situation is not at all fine-tuned when it comes to the choice of the initial mixing angle.

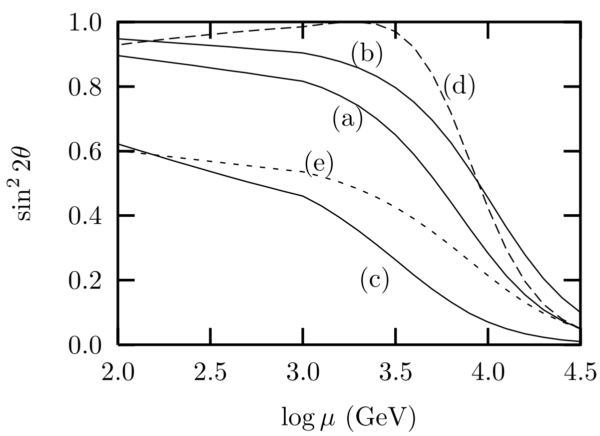

Figure 1: $\sin^2 2\theta$ has been plotted against the renormalization scale. The values of M_H, μ_0 and M are 115 GeV, 1 TeV and 30 TeV, respectively. The different plots correspond to different combinations of ($d\kappa|_M$, $\sin^2 2\theta|_M$) given by: (a) $(1.5 \times 10^{-4}, 0.05)$, (b) $(1.5 \times 10^{-4}, 0.1)$, (c) $(1.5 \times 10^{-4}, 0.01)$, (d) $(1.3 \times 10^{-4}, 0.05)$, and (e) $(1.7 \times 10^{-4}, 0.05)$.

Since $d\kappa \sim 0.5\Delta m^2/m^2$, where $m = (m_{11} + m_{22})/2$, the requirement of the mixing angle resonance near M_Z almost pins down the associated mass splitting. For the reference case $d\kappa|_M = 1.5 \times 10^{-4}$, we obtain $(\Delta m^2/m^2)_M = 3 \times 10^{-4}$. Now, we have observed that $d\kappa$ decreases by one order of magnitude during the RG evolution from M to M_Z, the bulk of the effect coming from the power law region $M > \mu > \mu_0$. This means

$(\Delta m^2/m^2)_{M_Z} \simeq 3 \times 10^{-5}$. According to the recently claimed evidence of neutrinoless double beta decay [12], m is expected to lie in the range 0.05 to 0.84 eV at 95% C.L. Now with ν_τ-ν_e oscillation in mind, with m towards the higher end of the above range, corresponding to $\kappa \sim 10^{-8}$, we find a mass splitting appropriate for a MSW solar neutrino oscillation in the LMA region, while with m sitting in the lower end of that range, which arises when $\kappa \sim 10^{-9}$, we may expect a MSW solar neutrino oscillation in the LOW region [13]. We make two observations at this point. First, it is not possible to produce a Δm^2 large enough to explain the atmospheric neutrino data. Second, if we consider ν_μ-ν_e oscillation, i.e., leave out ν_τ from consideration, then $d\kappa$ would have to be $\sim Y_\mu^2$ to ensure mixing angle resonance, but the corresponding Δm^2 would be too small to fit any experimental data.

If we take the neutrinoless double beta decay upper and lower limits on the absolute Majorana mass seriously, then from one stand-point our prediction can be contrasted with that of the usual 4-dimensional model. While in our extra-dimensional case, as we pointed out, both LMA and LOW solutions can be obtained, in the 4-dimensional scenario only the LOW solution can be easily achieved. Interestingly, the LOW solution is only marginally allowed after the incorporation of the SNO neutral current data [14].

Evidently, the large mixing angle which is being sought will be in the so-called 'dark side' if $\kappa_{22} - \kappa_{11}$ is negative and κ_{12} positive or *vice versa*. Only the magnitude of κ_{12} is fixed by $\sin^2 2\theta$, while its sign is arbitrary. If we take κ_{12} to be negative (positive) then the reference boundary value chosen, namely, $d\kappa|_M = 1.5 \times 10^{-4}$, puts this solution in the bright (dark) side at both M and M_Z ($\sin^2 2\theta$ has not crossed unity in curve (a) of Fig. 1). It is also possible to have small mixing in the dark (bright) side at M become large mixing in the bright (dark) side at M_Z by choosing, for example, $d\kappa|_M = 1.3 \times 10^{-4}$ (see curve (d) in Fig. 1) and κ_{12} positive (negative).

The main thrust of the paper has been on the magnification of a small mixing angle at M to a large one at the scale M_Z. For the examples that have been presented, we have verified that for the chosen parameters essentially *any* initial mixing results in a large mixing at the low scale.

Our main focus of attention has been the RG running of the neutrino mixing angles in the extra-dimensional scenario. In the process, we have also examined the evolution of the other SM parameters and we summarize the essential features now. Till the scale μ_0 no KK modes are excited and all couplings evolve as in the SM. The running is different only in the $\mu_0 < \mu < M$ range. The gauge couplings, g_i ($i = 1, 2, 3$) achieve a near equality at a scale of 1.4×10^4 GeV, as noted already in [5]. The quark Yukawa couplings run much faster than in the SM and $m_b = m_\tau$ is achieved at around 1.6×10^4 GeV. The evolution of the quartic scalar coupling λ is critical for limiting the range of the allowed Higgs masses and has already been discussed earlier. Beyond μ_0 it initially falls faster but then there is a slowing down and eventually even a slight increase. This is a major departure from the SM.

In summary, we have considered the effect on the RG evolution of the Majorana neutrino mass operator and the different SM (gauge, Yukawa, and quartic scalar) couplings due to the KK excitations arising from the compactification of one extra dimension. In the scenario under consideration, the fermions are restricted to a fixed brane and have no KK excitations, while the bosons can travel in the bulk and have higher KK modes. Our main conclusion is that in a two flavour picture, due to power law acceleration, the mixing between the ν_τ and another active neutrino can achieve near maximal values at M_Z even if it is only a few per cent at the $\mathcal{O}$ (10 TeV) scale. It is worth extending our analysis to the cases which concern fermionic KK excitations and promoting the analysis to the study of three flavour oscillation. Furthermore, all these questions can be addressed in the context of supersymmetry.

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