PION AND KAON DISSOCIATION IN HOT QUARK MEDIUM

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Abstract

Pion and kaon dissociation in a medium of hot quark matter is studied in the Nambu Jona-Lasinio model. The decay width of pion and kaon are found to be large but finite at temperatures much higher than the so called critical temperature of chiral or deconfinement transition, kaon decay width being larger. Consequently, pions and even kaons (with a lower density compared to pions) should coexist with quarks and gluons at such high temperatures. The implication of the above result in the study of Quark-Gluon plasma is discussed.

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A strong prediction of Quantum Chromodynamics (QCD), *the underlying theory of strong interaction*, is that at very high temperature and/or density, the bulk properties of strongly interacting matter would be governed by the quarks and gluons, rather than the usual hadrons. Such a phase is called quark gluon plasma (QGP) [1] in the literature and the search for such a novel phase of matter constitutes a major area of current research in the field of high energy physics.

The properties and dynamics of QGP are obviously governed by QCD. This conceptually straight forward task is, however, quite formidable in practice, particularly because of the failure of perturbative QCD already in the temperature range in the vicinity of Λ_{QCD} ($\sim$ few hundred MeV) [2]. Analytical non-perturbative methods are not yet sufficiently developed to be of much use in this context and as such, the lattice formulation of QCD has developed into the primary vehicle for the study of QGP [3]. In addition to the intensive computation, both in terms of CPU time and numerical complexities, one can only address static properties in the lattice. As a result, the space - time evolution of the system formed in the ultrarelativistic heavy ion collisions remains unapproachable in the framework of the lattice; thus the alternate, classical picture of hydrodynamic evolution, which accounts for the overall energy - momentum conservation in a collective manner and not much else, has been used quite extensively to study the evolution of the QGP [4]. QCD inputs enter into such a picture through the equation of state of the QGP, preferably evaluated on the lattice (but more often, through a phenomenological bag model [5]).

An inescapable feature of the collision process is that the quarks and gluons must, at some epoch, turn into hadrons which would ultimately be detected, never the individual quarks and gluons. The actual process of hadronisation, however, continues to elude us. It has been widely postulated that there could be an actual phase transition (the order of which is an open issue), separating the QGP phase from the hadronic phase [6]. The recent results, showing the lack of thermodynamic equilibrium [7] in the quark-gluon phase in ultrareletivistic heavy ion collisions, indicate that such an ideal situation is unlikely. It should also be noted at this juncture that although the persistence of non-perturbative effects till very high temperatures was suggested in the literature quite early on [8], it is only recently that the lattice results have confirmed that non-perturbative hadron like excitations could survive at temperatures far above the chiral phase transition temperature [9]. The lattice result for pion screening mass has been studied in ref. [10]. The analysis of [10] has been contradicted by Boyd *et al.* in ref. [11]. The conclusion of these authors [11] is consistent with the existence of free quarks at high temperatures. On the other hand, Shuryak [12] argued in a subsequent work that the non-perturbative modes, especially pion- like excitations, could indeed survive till temperatures above T_c. Furthermore, similar results for pion screening masses are obtained in σ- model as well [11]. It is thus imperative to understand the behaviour of such hadronic resonances, their formation, stability and so on, in a quark gluon medium at high temperature. In this work we confine our attention to the case of pions and kaons only; these, being lighter than other hadrons, account for the bulk of the multiplicity.

Formation of light mesons like pions and kaons, a bound state of light relativistic quarks, is an extremely difficult problem to handle in QCD. This is where all the troublesome features of non-perturbative QCD would make their presence felt. We therefore employ the usual practice of looking at the pion and kaons as a Goldstone boson arising from the spontaneous breaking of the chiral symmetry. In the present work we have tried to understand the behaviour of pseudoscalar mesons (π and K) using Nambu Jona-Lasinio model [13]. The decay width of pions and kaons have also been explored using the same model.

The Nambu Jona-Lasinio (NJL) model, in its original form, was constructed as a pre-QCD theory of nucleons that interact via an effective two body interaction. This today is reinterpreted as a theory with quark degrees of freedom. The Lagrangian density of this model incorporates the essential symmetries of QCD, the most important being the chiral symmetry. The actual mechanism via which the chiral symmetry breaking occurs in the NJL model follows closely the microscopic theory of superconductivity.

The NJL model had its own shortcomings as well. The interaction between quarks is assumed to be point like and the model is non-renormalizable. Furthermore, confinement is not incorporated in the NJL model; see ref. [15]. Nevertheless, NJL model is very useful for our present purpose, to understand the behaviour of pions and kaons (or more accurately, pion or kaon like excitations built out of quarks) in a hot quark medium.

The formulation of NJL model in flavour SU(3) was first introduced by Hatsuda *et al.* [14] and Bernard *et al.* [16]. The three flavour NJL model Lagrangian is written in terms of u, d and s quarks, the interaction between them being constrained by the $SU(3)_L \otimes SU(3)_R$ chiral symmetry, explicit symmetry breaking due to the current quark masses and the $U(1)_A$ breaking due to the axial anamoly [15]. The full Lagrangian with KMT (Kobayashi- Maskawa -'t-Hooft) term is given below [15].

$$\begin{aligned}\mathcal{L} &= \bar{q}(i\gamma\cdot\partial - \mathbf{m})q + \frac{1}{2}g_s\sum_{a=0}^{8}[(\bar{q}\lambda_a q)^2(\bar{q}i\lambda_a\gamma_5 q)^2] \\ &+ g_D[det\bar{q}_i(1-\gamma_5)q_j + h.c.] \qquad (1)\end{aligned}$$

where the quark fields q_i has three colours ($N_c = 3$) and three flavours ($N_f = 3$) and λ_a ($a = 1, 8$) are the Gell-Mann matrices. The quark mass matrix is given by $\mathbf{m} = diag(m_u, m_d, m_s)$.

In the mean field approximation, the quark condensates at finite temperature are given by [15],

$$<< \bar{q}_i q_i >>= -2N_c \int \frac{d^3p}{2\pi^3}\frac{M_i}{E_{ip}} f(E_{ip}) \tag{2}$$

where E_{ip} is the quark single particle energy for the i-th specie and $f(E_{ip}) = 1 - n_{ip} - \bar{n}_{ip}$, n_{ip} and $\bar{n}_{ip}$ being the Fermi-Dirac distributions for quarks and anti-quarks. If quark chemical potential is zero, then $n_{ip} = \bar{n}_{ip} = [exp(E_{ip}/T) \quad + \quad 1]^{-1}$.

The temperature dependent constituent quark masses M_i are obtained from the expressions below,

$$\begin{aligned} M_u &= m_u - 2g_s\alpha - 2g_D\beta\gamma \\ M_d &= m_d - 2g_s\beta - 2g_D\alpha\gamma \\ M_s &= m_s - 2g_s\gamma - 2g_D\alpha\beta \end{aligned} \tag{3}$$

where

$$<< \bar{u}u >>\equiv \alpha, \qquad << \bar{d}d >>\equiv \beta, \qquad << \bar{s}s >>\equiv \gamma \tag{4}$$

The actual formation of pions from quarks and gluons would require an involved analysis through the Bethe-Salpeter equation. Such a study is very much on our agenda but we do not address this issue here. In the present work, we concern ourselves with the decay of pionic and kaonic excitations, the properties of which we assume to be given by the NJL model. It should be reiterated that at temperatures above the critical temperature, these mesonic excitations are more like resonances with large effective masses [9, 10]. In the following, we study the decay width of such pionic excitations in the hot quark medium as a function of temperature, starting with the Lagrangian given above in equation (1).

The quark mass M_i appearing in eq. (2) and in eq. (3) is a very important ingredient in our calculation. In the absence of any medium and/or dynamic effect,

M_i should assume the value of the current quark mass. On the other hand, we know that due to the spontaneous breakdown of the chiral symmetry, quarks attain the value of the constituent quark mass [13].

In the present calculation we have used the parametrisation of ref. [16] ($\Lambda =$ 631.4 , $g_s\Lambda^2 = 3.67$, $g_D\Lambda^5 =$ -9.29 and current mass $m_{u,d}(m_s) = 5.5$ (135.7) MeV) to calculate the quark and meson masses. The constituent quark masses are calculated using the gap equations(eq. 3). These quark masses are then put into the dispersion equation [15] for mesons to get dynamical masses of mesons (π and K, here).

$$1 \quad + \quad 2G_\phi \Pi_{ij}(\omega, \vec{q} \rightarrow 0) \quad = \quad 0 \tag{5}$$

where Π_{ij} is the one loop polarization due to u and d quark for pions and u or d and s quark for kaons. G_ϕ is the coupling constant with ϕ coerresponding to π or K. The general expression for polarization function is

$$\begin{aligned}\Pi(q_0, \vec{q}) = \frac{N_c}{(2\pi)^3}\int_0^\Lambda \frac{d^3p}{E_pE_k}\Bigg[(n_k - n_p)\left\{\frac{1}{E_p - E_k + q_0 + i\epsilon} + \frac{1}{E_p - E_k - q_0 - i\epsilon}\right\} \\ \times(-E_pE_k + \vec{p}.\vec{k} + M_1M_2) \\ +(n_k + n_p - 1)\left\{\frac{1}{E_p + E_k + q_0 + i\epsilon} + \frac{1}{E_p + E_k - q_0 - i\epsilon}\right\} \\ \times(E_pE_k + \vec{p}.\vec{k} + M_1M_2)\Bigg]\end{aligned} \tag{6}$$

where N_c is the number of colours and $\vec{k} = \vec{p} + \vec{q}$. The energies $E_p = \sqrt{p^2 + {M_1}^2}$ and $E_k = \sqrt{(\vec{p} + \vec{q})^2 + {M_2}^2}$. For pion, $M_1 = M_2 = M_u$. For kaon, $M_1 = M_{u(d)}$ and $M_2 = M_s$. n_k and n_p are the Fermi-Dirac distribution functions defined earlier. The pseudoscalar couplings are,

$$\begin{aligned} G_\pi = g_s \quad &+ \quad g_D\gamma \\ G_{K^\pm} = g_s \quad &+ \quad g_D\beta \\ G_{K^0} = g_s \quad &+ \quad g_D\alpha \end{aligned} \tag{7}$$

where α, β and γ are defined in eq. (4).

The decay width is evaluated using the imaginary part of the eq.(6) as given below,

$$\Gamma_\phi = -\frac{{G_{\phi q}}^2 Im\Pi(\omega, \vec{q} \to 0)}{\omega} \tag{8}$$

where $G_{\phi q}$ is the empirical meson-quark coupling as obtained in NJL. Here we have used $G_{\pi q} = 3.5$ and $G_{Kq} = 3.6$[16].

The variation of quark and meson masses is shown in figure 1. The u or d quark masses starting from 135 MeV drops to the current quark mass value just after a temperature of 200 MeV. On the other hand, the drop in the strange quark mass is much smaller around that temperature, showing the effect of explicitly broken chiral symmetry, by a larger amount, in the SU(3) sector. Pion and kaon both show a similar qualitative behaviour. The masses of pion and kaon remain constant at their free masses upto a temperature 200 MeV but increases sharply after that with a pion mass of 900 MeV and Kaon mass of 100 MeV around 450 MeV temperature, thus giving a slower increment for Kaons compared to pions. The difference in the behaviour of Kaon and pion can be attributed to the difference in the behaviour of u and s quark masses.

The difference in the behaviour of pion and kaon is more prominent in the decay width, as shown in figure 2. Figure 2 shows that the decay width is very high at high temperature and decreases with decreasing temperature, going to zero at around $T = 0.2$ GeV. It is worth noticing that at around the same temperature, the effective pion mass attains the value of the free pion mass (figure 1). The decay width of Kaon is around 3 GeV where as that of pion is around 1.4 GeV at 500 MeV temperature. This is a very significant result due to two reasons. Firstly, our results show that though there will be pions and kaons along with the quarks at high temperature phase, the numbers of mesons will be very small due to their large decay width. Moreover, the number of kaons will be much less compared to pions at high temperature phase, though both the mesons will become stable around the same temperature (below 200 MeV temperature).

Our results will have a strong bearing on the study of hadronisation. As already mentioned, the lack of thermodynamic equilibrium in a QGP system implies that one may not get a clear-cut phase transition from QGP to hadrons. Thus, to understand the process of hadronisation, one should really start from a very high temperature ($>>$ expected T_c) and then let the system evolve dynamically towards lower temperatures. Here what one would find, as indicated from our present calculation, is that initially a very small number of pions and kaons would be present in the system along with quarks and gluons. Then, even if additional mesons are formed through $q\bar{q}$ fusion and/or bound state formation, the total number of mesons would not increase very fast, as most of them must decay immediately due to the large decay width at such high temperatures. Only in the vicinity of $T \sim 200MeV$, where the decay width is small, the number of mesons would start increasing significantly and gradually become dominant compared to the number of quarks at some lower temperature. However, the exact value of the temperature, at which the decay width goes to zero, will depend on the value of the quark mass considered.

To summarise, we have calculated, for the first time, the decay of pions and kaons, in a hot quark medium. The most interesting and noteworthy feature is that, even without any consideration of the detailed evolution and dynamics of the system, the pionic modes are found to dominate around a temperature of 200 MeV. Though the question whether this is a signature of a phase transition cannot be addressed within the framework of the present work, the fact that most of the pions and kaons decay into quarks, owing to a large decay width at temperatures higher than $T = 200$

MeV, is a remarkable finding. Moreover, both the pionic as well as kaonic modes start becoming important at about the same temperature, thus providing a hint of some kind of a transition temperature.

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References

[1] E.V.Shuryak, *Phys. Rep.* **61**, 71 (1980).

[2] See, for example, *Quark-Gluon Plasma*, Ed. R.C.Hwa, World Scientific, Singapore (1990)

[3] F.Karsch, *Z. Phys.* **C38**, 147 (1988).

[4] J.Alam, S.Raha and B.Sinha, *Phys. Rep.* **273**, 243 (1996).

[5] P.Hasenfratz and J.Kuti, *Phys. Rep.* **40**, 73 (1978).

[6] It should be noted that, in the context of QGP or hot hadronic matter, one comes across two kinds of phase transitions, namely, the chiral phase transition and the deconfinement phase transition. The current consensus is that although these two phase transitions are certainly distinct from each other, the critical temperatures may not be much different. We have taken them to be same for the present purpose.

[7] E.V.Shuryak, *Phys. Rev. Lett.* **68**, 3270 (1992); S.Chakraborty, S.Raha and B.Sinha, *Mod. Phys. Lett.* **A7**, 927 (1992); J.Alam, S.Raha and B.Sinha, *Phys.*

Rev. Lett. **73**, 1895 (1994); T.S.Biro, B.Müller, M.H.Thoma and X.N.Wang, *Nucl. Phys.* **A566**, 543c (1994).

[8] M.Plümer, S.Raha and R.M.Weiner, *Phys. Lett.* **B139**, 198 (1984); *Nucl. Phys.* **A418**, 549c (1984).

[9] K.Born *et al.*, *Phys. Rev. Lett* **67**, 302 (1991).

[10] A. Gocksh, *Phys. Rev. Lett.* **67**, 1701 (1991).

[11] G. Boyd, S. Gupta, F. Karsch and E. Laermann, *Z. Phys.* **C64**, 331 (1994).

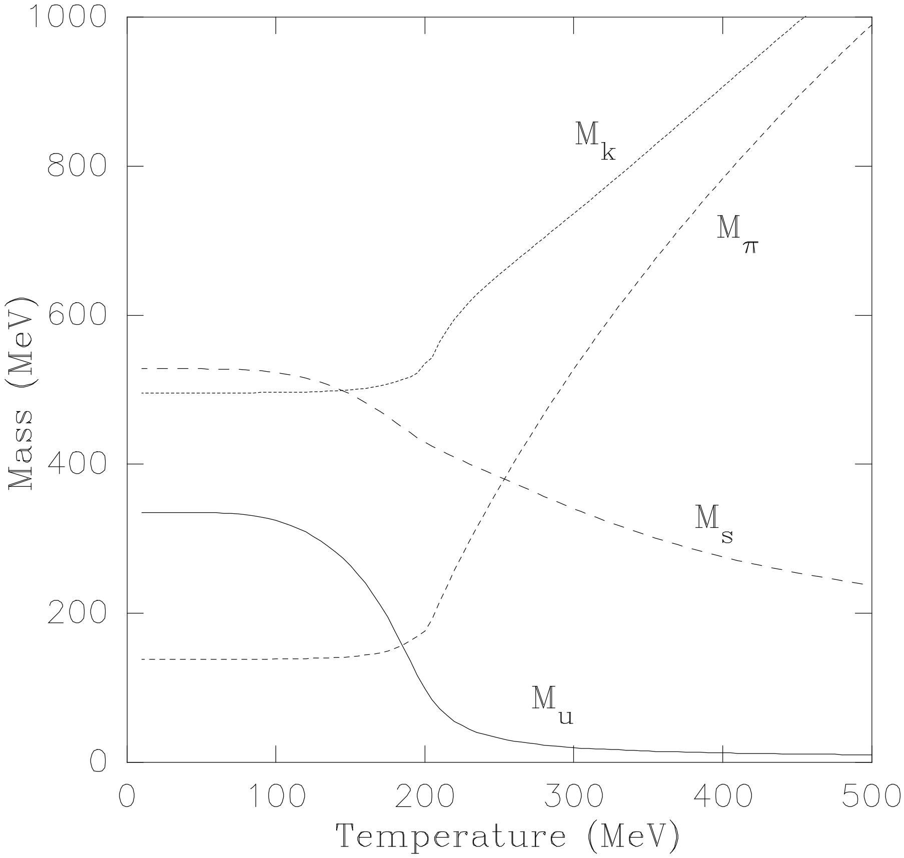
[12] E. Shuryak and T. Schaefer, *Nucl. Phys.* **B53**, 472 (1997).

[13] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961).

[14] T. Kunihiro and T. Hatsuda, *Phys. Lett* **B206**, 385 (1988).

[15] T. Hatsuda and T Kunihiro, Phys. Rep. **247**, 221 (1994).

[16] V. Bernard, R. L. Jaffe and U. -G Meissner, *Nucl. Phys.* **B308**, 753 (1988).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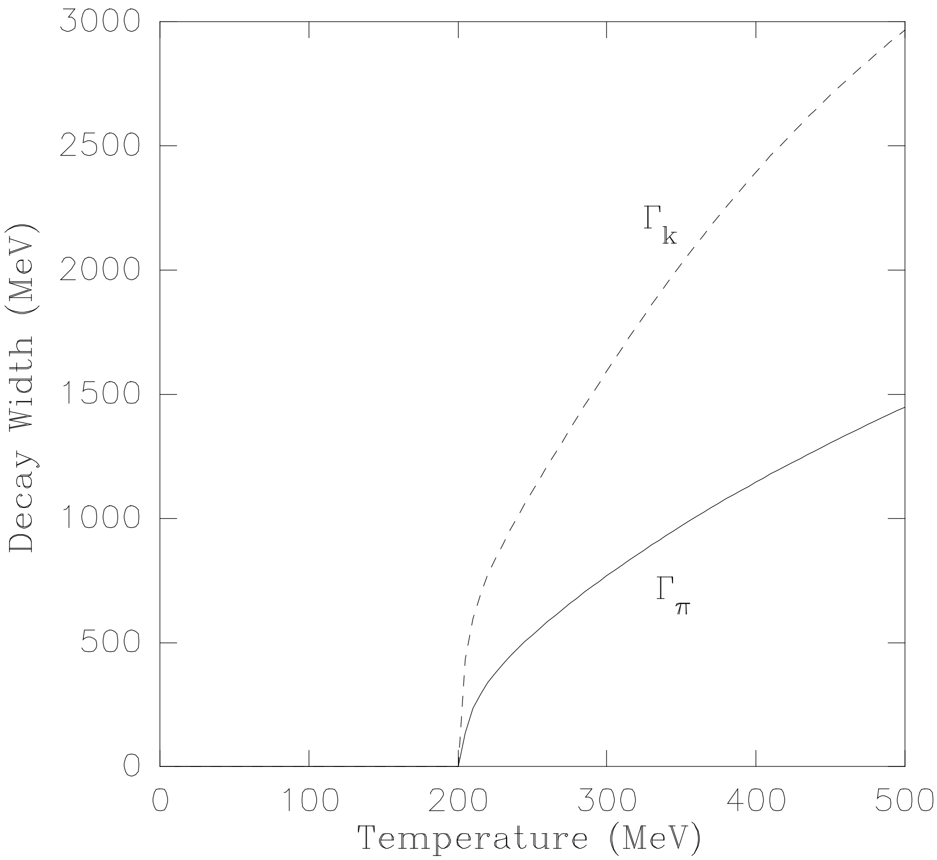
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