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The neutrino mass scale and the mixing angle θ_{13} for quasi-degenerate Majorana neutrinos

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ABSTRACT

Considering a general mass matrix for quasi-degenerate neutrinos and treating the experimentally known oscillation parameters as inputs, we study the correlation between the degenerate mass scale (m) and the mixing angle θ_{13}. We find that, corresponding to different values of m, there exist upper bounds on θ_{13}, so that a precise determination of the latter in future may put upper limit on the former, and vice versa. One can also find a possible correlation between m and lower bound of θ_{13}, depending on the relative strength of the unperturbed degenerate mass matrix and the perturbation. The possible constraints on the parameters of few models of quasi-degenerate neutrinos are briefly discussed.

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The currently available experimental data on neutrino oscillation indicate that the mass squared differences of neutrinos are quite small. Although the mass-squared differences and mixing angles (except θ_{13}) are known from neutrino oscillation experiments, the absolute values of the neutrino masses still remain unknown. Depending on the scale, the pattern of neutrino masses may be hierarchical or quasi-degenerate in nature. In particular, if one assume that relic neutrinos constitute the hot dark matter of the universe, then the scale of neutrino masses are expected to be somewhat higher than their differences, and neutrinos are expected to be quasi-degenerate. Considering the recent results [1] from WMAP experiment, there might be some improvements in the upper bound on the neutrino mass. However, keeping in mind all possible uncertainties in the cosmological bounds, an upper limit of 2 eV on the sum of the masses of three neutrino flavours can be said to exist [2].

Quasi-degeneracy of neutrinos has also been suggested in a number of theoretical models proposed in the literature [3]. For example, considering neutrino masses as degenerate at some seesaw scale, various authors [4, 5, 6, 7, 8, 9] have shown that large mixing angles for solar as well as atmospheric neutrinos can be obtained after extrapolating masses and mixing to the weak scale using renormalization group equations.

Among the neutrino oscillation parameters, the values of two mass-squared differences and two mixing angles, viz. θ_{12} and θ_{23}, are already known to a reasonable degree from the solar [10] and atmospheric [11] neutrino data. However, quantities still unknown are the mixing angle θ_{13}, mass scale m of the neutrinos and the CP violating phase δ in the neutrino mixing matrix. It should be noted that the CHOOZ-Palo-Verde experiments [12, 13] have put an upper bound on $\theta_{13}(< 12^\circ)$. Furthermore, the absence of neutrino-less double beta decay implies an upper bound on m [14], which is compatible with the limit coming from the hot dark matter content of the universe [1].

In this note, we would like to illustrate the interplay among the unknown quantities, namely, m, elements of the perturbation matrix and θ_{13}, assuming that neutrinos are of Majorana nature and the masses are quasi-degenerate. We have taken into account the solar and the atmospheric data as well as neutrino less double beta decay constraint in our analysis, which has not been considered in earlier analyses. As we shall see, the maximum allowed value of θ_{13} and the cosmological upper limit on the overall neutrino mass scale actually restrict the parameter space of the perturbation matrix lifting the degeneracy. This can enable one to constrain various theoretical models of the neutrino mass matrix, which in turn determine the way degeneracy is lifted. We will also try to find any possible correlation between the degenerate mass scale m with minimum value of θ_{13}.

Without going to a specific model, we shall first consider the most general form of degenerate mass matrix in the weak interaction basis, which allows mixing among different flavours of neutrinos due to different intrinsic CP-properties. This degeneracy is lifted, for example, by the breaking of some symmetry at the seesaw scale [15]. Thus we add a small perturbation matrix to the original degenerate mass matrix in a model-independent way. We use as our inputs the known oscillation parameters, namely:

- $|\Delta m^2_{23}| \simeq 2.12^{+1.09}_{-0.81} \times 10^{-3}$ eV^2, $\theta_{23} \simeq 45.0^{\circ}{}^{+10.55^\circ}_{-9.33^\circ}$ (from the atmospheric ν_μ deficit [11]).
- $\Delta m^2_{12} \simeq 7.9^{+1.0}_{-0.8} \times 10^{-5}$ eV^2, $\theta_{12} \simeq 33.21^{\circ}{}^{+4.85^\circ}_{-4.55^\circ}$ (from the solar ν_e deficit [10]).

where $\Delta m^2_{ij} = m^2_j - m^2_i$.

Thereafter, the application of degenerate perturbation theory, in conjunction with the constraints arising on the perturbation matrix after using the above experimental values, allows us to obtain the correlation sought after. Specific choices of models for generating neutrino mass are likely to restrict further the general form of the perturbation matrix. We study the correlation in the context of a few models having family symmetries of abelian and non-abelian nature, and check whether these in turn can constrain some parameters of the models even further.

With the charged lepton matrix taken as diagonal, real and positive, the Majorana mass terms in the flavour basis can be expressed as

$$\mathcal{L}_{mass} = -(\nu_{L_\alpha})^T C^{-1} M_{\alpha\beta} \nu_{L_\beta} + h.c \tag{1}$$

where $M_{\alpha\beta}$ is a 3×3 complex symmetric mass matrix and $\nu_{L\alpha}$ is the weak eigenstate basis of neutrinos corresponding to three generations. The neutrino flavour states $|\nu_\alpha\rangle$, $\alpha = e, \mu, \tau$, in the weak basis are related to the neutrino mass eigenstates $|\nu_i\rangle$, $i = 1, 2, 3$, with masses m_i :

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \ . \tag{2}$$

where U is a 3×3 unitary matrix. In general, the mass matrix M can be diagonalised by a transformation of the form

$$U^T \ M \ U = M_{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{3}$$

where M_{diag} is the diagonal mass matrix. If one considers the three masses to be degenerate, then U can be rotated away for Dirac neutrinos. This cannot, however, be done in the case of Majorana neutrinos if the CP property of one of the fields is different from those of the other two. It was shown by Branco *et al* [16] that, in such a case, the fact that the CP-eigenvalue of a Majorana state can be $+i$ or $-i$ implies a still non-trivial form of the the diagonalising matrix U which contains two mixing angles and one phase. Implications of such (or similar) scenarios have also been explored, for example, in reference [4].

For degenerate Majorana neutrinos remembering that the mixing angles θ_{12} and θ_{23} are required to be large by observation, we shall consider the intrinsic CP-eigenvalue associated with ν_2 to be opposite to that of the other two neutrinos. Without losing any generality we can thus write

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & s'_{23} & -c'_{23} \end{pmatrix} \begin{pmatrix} c'_{12} & s'_{12} & 0 \\ s'_{12} & -c'_{12} & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

where $c'_{ij} = \cos\theta'_{ij}$, $s'_{ij} = \sin\theta'_{ij}$ and $\beta = \pi/2$ corresponds to different intrinsic CP property of ν_2 with respect to other two neutrinos and α corresponds to CP violating phase in general. We shall choose it to be zero for the CP conserving case. Following ref. [16] the degenerate mass matrix M in the flavour basis can be written as

$$M = m \ U^* U^\dagger \tag{5}$$

where degenerate mass scale $m = m_1 = m_2 = m_3$.

However, in spite of such a mixing, neutrinos cannot oscillate, since the quantity governing oscillations are the mass-squared differences. In order to lift the degeneracy, we assume that degenerate mass scale m is somewhat higher than the required mass-squared differences as indicated by the oscillation experiments. There are studies on the mechanism for lifting such degeneracy. One can assume degeneracy at a high scale and envision that it is lifted through running, as some symmetry which holds at high scale is broken. Alternatively, one can remain confined to the electroweak scale itself and consider mass splitting effects there. In either case, the mechanism consists in a perturbation to the mass matrix, in a basis where it is diagonal and degenerate. This perturbation matrix is real and symmetric in several studies in recent past. We follow the same practice here. Moreover, most of these investigations do not throw much light on the *numerical values* of the elements of the perturbation matrix. This is the point we wish to address, namely, how the limits on θ_{13} as well as the neutrino mass scale restrict the values of the perturbation matrix elements. We believe that , in spite of a six-fold multiplicity of the elements, such constraints can be useful in shortlisting viable models.

We thus consider a small perturbation to the degenerate mass matrix M_{diag}, parameterized as

$$Q = \epsilon \begin{pmatrix} e & a & b \\ a & g & 1 \\ b & 1 & f \end{pmatrix} \tag{6}$$

where ϵ^2 is of the order of 10^{-3} eV^2 and other parameters are ≤ 1. In general, the smallness of θ_{13} present in the standard parametrisation of neutrino mixing matrix (as mentioned in equation (15) below) hints at the CP violating effect being small in the neutrino sector. So keeping CP violating phase only in M we have neglected it in the small perturbation matrix Q making it real. Q is written in a basis in which the degenerate mass matrix is diagonal. It is also obvious that in the flavour basis Q becomes

$$Q_0 = U^* \, Q \, U^\dagger \tag{7}$$

where U is of the form indicated in (4). Unlike Q in the flavor basis Q_0 is complex. We next employ the methodology of degenerate perturbation theory, with the diagonal and degenerate mass matrix determining the unperturbed basis. Besides, we absorb the phase associated with $\beta = \pi/2$ in the Majorana neutrino field, thus enabling it to disappear from the matrix U and we define this as U_1. Considering M in (5) and using U_1 as the diagonalising matrix from eq. (3) we get the diagonal elements of M_{diag} as ,

$$m_2 = -m, \; m_1 = m_3 = m \tag{8}$$

Now, only m_1 and m_3 are degenerate. If we wish to parameterize the lifting of degeneracy in such a manner that the mass m_1 remains unchanged, then one may set the following condition on the parameters in Q:

$$b^2 = ef \tag{9}$$

Using first order degenerate perturbation theory, we obtain the following mass eigenvalues for $M_{diag} + Q$, lifting the degeneracy to first order in ϵ:

$$m_1 = m, \; m_2 = -m + \epsilon g, \; m_3 = m + \epsilon(e + f) \tag{10}$$

and the diagonalising matrix as :

$$U' = \left(\begin{array}{ccc} \cos\psi & -ar & \sin\psi \\ r\,(a\cos\psi - \sin\psi) & 1 & r\,(a\sin\psi + \cos\psi) \\ -\sin\psi & -r & \cos\psi \end{array} \right) \mathcal{N} \tag{11}$$

where $\tan 2\psi = 2b/(f-e)$; $r = \frac{\epsilon}{2m}$. $\mathcal{N} \equiv diag(N_1^{-1}, N_2^{-1}, N_3^{-1})$ where N_1, N_2, N_3 are the proper normalisation constants for the eigenvectors. These can be easily expressed in terms of the parameters of the theory.

$$\begin{aligned} \mathcal{N}_1 &= \left[1 + r^2(a\cos\psi - \sin\psi)^2\right]^{\frac{1}{2}} \\ \mathcal{N}_2 &= \left[1 + r^2(1+a^2)\right]^{\frac{1}{2}} \\ \mathcal{N}_3 &= \left[1 + r^2(a\sin\psi + \cos\psi)^2\right]^{\frac{1}{2}} \end{aligned} \tag{12}$$

To go to the physical mass eigenstate basis, however, we keep $\beta = \pi/2$ in U and now the eigenvalues are same as in equation 8 except there will be overall change in sign in m_2 making it positive. So replacing U_1 by U we write

$$U_0 = UU' \tag{13}$$

which will diagonalise the mass matrix $M + Q_0$. Although there was no mixing of ν_e and ν_τ in U but U_0 has that mixing. One may note that, since the solar neutrino data indicates that m_2 is heavier than m_1 and that there is no sign ambiguity in Δm_{12}^2, the parameter g should be negative in our case. To relate the different parameters with the experimental data we shall write U_0 in the standard form of a 3×3 unitary mixing matrix:

$$U_0 = \left(\begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{array} \right), \tag{14}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ is the CP violating phase.

The parameters of Q in equation 6 can be constrained from the mass squared differences obtained from solar and atmospheric neutrino data. Using equation 10 up to the order of ϵ, one can write

$$2m\epsilon g = \Delta m_{12}^2, \;\; 2m\epsilon(e+f) = \Delta m_{23}^2 \tag{15}$$

From equation 9 it follows that e and f cannot be of opposite sign which implies

$$Q_{11} = \epsilon e \leq \Delta m_{23}^2/2m \tag{16}$$

It follows from the above that

$$x(\equiv em^2) \leq \Delta m_{23}^2|/r \tag{17}$$

We consider $\epsilon^2 a^2 < |\Delta m_{12}^2|$ so that solar neutrino data is satisfied.

We further simplify the analysis by setting $\delta = 0$ in equation 14 and $\alpha = 0$ in equation 4 which correspond to CP conserving case. Comparing 11, 12 and 13 elements of U_0 in eqs. 13 and 14, we obtain

$$\sin\theta_{13} = N_3^{-1}\left[\cos\theta'_{12}\sin\psi + \sin\theta'_{12}\left(a\sin\psi + \cos\psi\right)\frac{\epsilon}{2m}\right] \tag{18}$$

where

$$\tan\theta'_{12} = \left[\frac{ac_{12}\epsilon + 2ms_{12}\cos\psi}{2mc_{12} - \epsilon a s_{12}\cos\psi + \epsilon s_{12}\sin\psi}\right] \tag{19}$$

Equations 18 and 19 can be re-expressed as in term of x ($\equiv em^2$):

$$\sin\theta_{13} = \mathcal{N}_3^{-1}\left[\cos\theta'_{12}\sqrt{\frac{2rx}{\Delta m_{23}^2}} + r\sin\theta'_{12}\,(\sqrt{1-\frac{2rx}{\Delta m_{23}^2}} + a\sqrt{\frac{2rx}{\Delta m_{23}^2}})\right] \tag{20}$$

and

$$\tan\theta'_{12} = \frac{\frac{\mathcal{N}_2}{\mathcal{N}_1}\tan\theta_{12}\sqrt{1-\frac{2rx}{\Delta m_{23}^2}} + ar}{1 - r\left(a\sqrt{1-\frac{2rx}{\Delta m_{23}^2}} - \sqrt{\frac{2rx}{\Delta m_{23}^2}}\right)} \tag{21}$$

Considering θ_{12} from the solar neutrino experimental data as discussed in the introduction, one can find the relation between the two unknowns m and θ_{13} in terms of the elements of Q. It may be noted here that for both normal and inverted hierarchy equations 20 and 21 remain unchanged.

Now one can see how θ_{13} and m are related. It is evident that for a given value of r, θ_{13} is a function of x only. A careful inspection of equation 18, tells the monotonically increasing

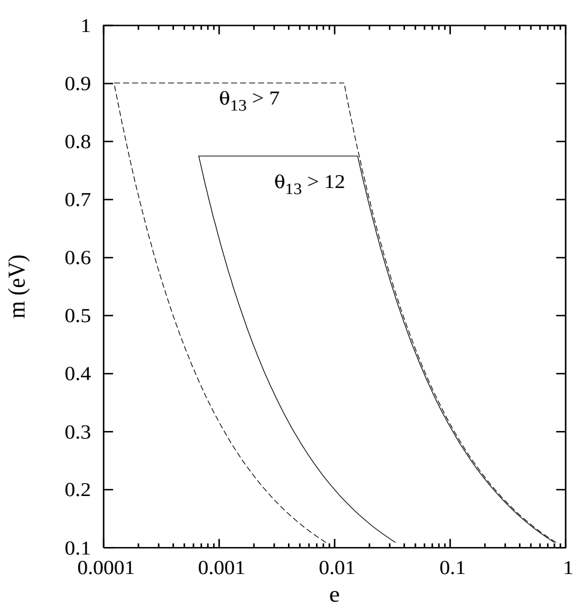

Figure 1: Allowed region for m and e depending on θ_{13}. $r \equiv \frac{\epsilon}{2m} = 0.1$ has been used in this plot.

dependence of θ_{13} on x. Thus maximum allowed value of θ_{13} corresponds to the limiting value of x as defined in equation 17. The contours of $\sin\theta_{13}$ thus correspond to specific values of x. In other words, they corresponds to contours of specific values of x in the $e-m$ plane.

In figure 1, we vary e in the range from 0 to its upper bound defined by equation 17. Corresponding to different values of θ_{13} we have shown the variation of m as obtained numerically. The region marked with $\theta_{13} > 12$ is thus disallowed from the CHOOZ result. We have argued earlier how the normal and inverted hierarchical ordering of neutrino masses would produce the same constant θ_{13} contours in the $m-e$ plane. Depending on the possible values of e, one can find an upper bound on m in both normal and inverted hierarchical case from this plot. If, in future, better bound on θ_{13} is obtained then it is possible to improve upper bound on the degenerate mass scale, m, compared to its present cosmological bound [1].

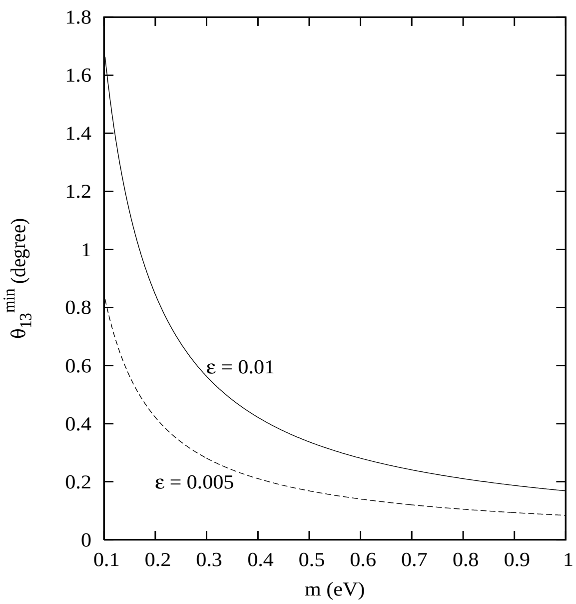

Figure 2: Minimum value for θ_{13} vs. the degenerate mass scale m for two different values of perturbation parameter ϵ.

In figure 2, we have plotted the minimum values for θ_{13} as a function of the degenerate mass scale m for two different values for the perturbation parameter ϵ. The range over which we vary m is fixed by two considerations. The upper limit should be less than 1 eV from cosmological considerations [1]. Furthermore, we would like to generate the tiny mass square differences by perturbing the degenerate matrix (with eigenvalues m). Thus the value of m should be bigger than the differences (which is generated by perturbation) in the masses. Thus, keeping in mind the atmospheric mass (square) difference, we take the lower limit for m to be 0.1 eV.

It is interesting to note that that there is lower bound on θ_{13} depending on the value of m. One can see that the minimum of θ_{13} decreases with increasing value of m. Increasing m and keeping the value of ϵ fixed implies, the relative strength of the perturbation with respect to the unperturbed degenerate mass matrix, keeps on decreasing. This in turn is taming down the value of θ_{13}. In the limit when perturbation vanishes, masses are exactly the same and

θ_{13} is exactly zero. However, to create qusi-degenerate masses there is perturbation and as such there is some lower bound on θ_{13} which will be nearer to 1^0 if the cosmological bound on neutrino masses improves further.

The general degenerate mass matrix M in the weak basis and Q_0 contain too many parameters. However, on the basis of some symmetry principle one can reduce the number of these parameters. One may consider a simple model of neutrino mass [8] using a leptonic Higgs doublet and three right-handed singlet fermions at TeV energy scale or below. Considering discrete A_4 symmetry one may obtain the degenerate mass matrix of the form

$$M = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{22}$$

as discussed in [8]. Such pattern correspond to $\theta'_{12} = 0$ and $\theta'_{23} = 3\pi/4$ in the degenerate mass matrix M. Then our perturbation matrix Q_0 in the flavor basis is

$$Q_0 = \epsilon \begin{pmatrix} e & (-ia+b)/\sqrt{2} & (ia+b)/\sqrt{2} \\ (-ia+b)/\sqrt{2} & (-2i+f-g)/2 & (f+g)/2 \\ (ia+b)/\sqrt{2} & (f+g)/2 & (2i+f-g)/2 \end{pmatrix} \tag{23}$$

This can be obtained by considering soft symmetry-breaking terms of the form $m_{ij}N_{iR}N_{jR}$ in the Lagrangian where N are right-handed neutrino fields. Thus parameters m_{ij} of the theoretical model of neutrinos will be related to the parameters of Q_0 and as such Q. From equation. 20 it is evident that θ_{13} is very close to $\psi \simeq \sqrt{\frac{2me\epsilon}{\Delta m^2_{23}}}$. As $\theta_{13} < 12^0$ using (15) one gets the relative estimate of the parameters e and f. So $f \sim 2 \times 10^{-3}/(m\epsilon)$ and e is much smaller than that as required by CHOOZ constraint.

Degenerate neutrino masses from an abelian family symmetry has also been considered in [4]. The neutrino mass matrix as considered in eq. (38) of that paper is similar in form as shown above in (22) & (23) provided that we consider $e = 0$ and $f = -g$ in our general form of the perturbation matrix Q so as to reproduce the Q_0 appropriate for their mass matrix.

So far we have considered the case of different intrinsic CP parities of Majorana neutrinos for which there is mixing even in the exact degenerate mass limit. However there may be a case for which intrinsic CP parities of all the Majorana neutrinos are the same [5]. In that case the U matrix can be rotated away and the mixing will be controlled by the perturbation matrix only. So it is not possible to perform the same analysis for the perturbed part in the quasi-degenerate mass matrix as done in this paper. Neutrino-less double beta decay in such a case may yield a direct constraint on the approximate degenerate mass m, although θ_{13} may not receive any additional constraint. However, same CP parity of all neutrinos is disfavoured from the viewpoint of the requirement of stability at the weak scale of quasi-degenerate neutrino masses and mixing pattern which emerge at the high scale [6] as the CHOOZ constraint on θ_{13} cannot be satisfied. In ref [9] although the same CP parity of quasi-degenerate neutrinos has been considered at the see-saw scale but at the weak scale it has been found that the appropriate mass-squared difference for oscillation of solar neutrinos cannot be obtained.

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References

[1] D. N. Spergel et al., astro-ph/0603449.

[2] M. Fukugita et al., Phys. Rev. D **74**, 027302 (2006).

[3] S. T. Petcov and A. Yu Smirnov, Phys. Lett **B 322**, 109 (1994); D. O. Caldwel and R. N. Mohapatra, Phys. Rev. **D 48**, 3259 (1993), Phys. Rev. **D 50**, 3477 (1994); M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. D **57**, 4429 (1998).

[4] P. Binetruy, S. Lavignac, S. T. Petcov and P. Ramond, Nucl. Phys. B **496**, 3 (1997)

[5] C. D. Carone and M. Sher, Phys. Lett. B **420**, 83 (1998)

[6] K. R. S. Balaji, A. S. Dighe, R. N. Mohapatra and M. K. Parida, Phys. Rev. Lett. **84**, 5034 (2000).

[7] R. Adhikari, E. Ma and G. Rajasekaran, Phys. Lett. B **486**, 134 (2000).

[8] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001); E. Ma, Phys. Rev. Lett. **86**, 2502 (2001).

[9] R. N. Mohapatra, M. K. Parida and G. Rajasekaran, Phys. Rev. D **69**, 053007 (2004); Phys. Rev. D bf 71, 057301 (2005).

[10] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1158 (1998); Phys. Rev. Lett. **82**, 1810 (1999); Phys. Rev. Lett. **82**, 2430 (1999); S. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **86**, 5651 (2001); Phys. Rev. Lett. **86**, 5656 (2001); Phys. Lett. **B 539**, 179 (2002); Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **87**, 071301 (2001); Phys. Rev. Lett. **89**, 011301 (2002); Phys. Rev. Lett. **89**, 011302 (2002); S. N. Ahmed *et al.* (SNO Collaboration), Phys. Rev. Lett. **92**, (2004) 181301; G. Giacomelli *et al* (MACRO Collaboration), Nucl. Phys. Proc. Suppl. **145**, 116 (2005).

[11] Y. Ashie *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **93** 101801 (2004); M. Ishitsuka (Super-Kamiokande Collaboration), NOON2004, The 5th Workshop on “Neutrino Oscillations and their origin”, 11-15 February, 2004, Tokyo, Japan; Y. Ashie *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **71**, 112005 (2005).

[12] K. Eguchi *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **90**, 021802 (2003); M. Apollonio *et al.* (CHOOZ Collaboration), Eur. Phys. J. C **27**, 331 (2003).

[13] A. Bandyopadhyay *et al.,* Phys. Lett. B **608**, 115 (2005); S. Choubey, Nucl. Phys. Proc. Suppl. **138**, 326 (2005).

[14] H. V. Klapdor-kleingrothaus, hep-ph/0512263.

[15] M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, ed. P. Van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdam, 1979), p315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, edited by O.Sawada and A. Sugamoto (KEK-Report No. 79-18, Tsukuba, Japan (1979)) p95; S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); E. Ma, Phys. Rev. Lett. **86**, 2502 (2001).

[16] G. C. Branco *et al.*, Phys. Rev. Lett. **82**, 683 (1999).