

MODEL STUDY OF HOT AND DENSE BARYONIC MATTER

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Abstract

The properties of baryonic matter have been investigated at finite density and temperature using different models. The variation of baryon masses and fractional number densities with baryon density and temperature obtained from different models have been compared. The quark hadron phase transition have been studied using Chiral Colour Dielectric (CCD) model in the quark sector. No phase transition has been seen for the different variants of the Zimanyi-Moszkowski model. However, a phase transition is observed for the linear and non-linear Walecka model.

1 Introduction

The study of hot and dense hadronic matter is a very old sphere of activity [1]. It is quite well known now that QCD is the fundamental theory of strong interaction where the fundamental particles are quarks and gluons. In principle, one should be able to study the low energy hadronic phenomena using QCD. But due to the nonperturbative nature of the problem, it becomes difficult to use QCD to describe hadronic matter. This leads people to calculate hadronic matter properties from effective models.

The quark structure of hadrons lead people to believe that under extreme conditions, *i.e.* at very high temperature and/or density, the quarks, which are confined inside the hadrons, may have larger spatial extension [2, 3] compared to the typical size of a hadron which is about $1fm$. So hadron to quark matter phase transition may occur inside neutron stars because of the very high density. On the other hand, a soup of quarks and gluons may be produced in heavy ion collisions. Such a matter was termed as Quark-Gluon Plasma (QGP) [3]. Independent of the fact whether QGP will be produced, it can be said that a hot and dense matter will be produced under such extreme conditions. That can be either QGP or hot and dense hadronic matter. Furthermore, even if QGP is produced it will undergo a phase transition to hadronic matter. These phenomena make the study of hadronic matter an interesting area of research.

In this paper we have studied the properties of baryonic matter at high temperature and density from the five different hadronic models. These are the Walecka model (LW), the Zimanyi-Moszkowski (ZM) models and the non-linear Walecka (NW) model. The Walecka model [4] contains nucleons, scalar σ -meson and vector ω -meson and it reproduces the features of strong interaction *i.e.* short range repulsive and

long range attractive forces. Extensive studies have been carried out, after the invention of this model, towards the calculation of hadronic matter properties from this model [1, 5]. This revealed certain inconsistencies in the model. It was found that this model yields the incompressibility $K = 524MeV$ compared to the accepted value $K = 210 \pm 30MeV$. Furthermore the model gives a very low nucleon mass at the nuclear matter density. As a remedy of these problems other models of hadronic matter were proposed. In the NW [6] model, in addition to the interactions present in the usual Walecka model, cubic and quartic self interactions of the σ -meson are also included.

The ZM model [7] differs from the usual Walecka model in the form of coupling between the scalar meson and the nucleon, which is a derivative coupling in the new model. As a result of this derivative coupling the model reproduces some of the experimental results nicely [7, 8]. It yields the incompressibility $K = 224.49MeV$, which is much closer to the accepted value compared to the Walecka model, and a nucleon effective mass $M_N^* = 797.64MeV$ at the nuclear matter saturation density ($\rho = 0.15fm^{-3}$). But, the price one has to pay is that the model loses its renormalisability due to the derivative coupling. This is acceptable as far as the discussion of the effective models goes, since the description of hadronic matter need only be valid upto the temperatures $T \leq 200MeV$ provided the deconfinement phase transition to QGP is a reality.

In the present work the LW and ZM models have been extended to include hyperons. Some efforts have already been made to include hyperon degrees of freedom in the nuclear equation of state [6, 9, 10, 11] but, most of these studies were restricted to zero temperature scenario. Section 2 is devoted to the study of different models of baryonic matter. In section 3 the quark model, which is chosen to be the Chiral Colour Dielectric (CCD) model has been discussed. Section 4 contains the study

of the temperature and density dependence of baryon masses and number densities. The hadron-quark phase transition is discussed in section 5 and finally the results and conclusions are given in section 6.

2 Hadronic Models

We have five different hadronic models to describe the properties of baryonic matter. These models are the Linear Walecka model, the three variants of the Zimanyi-Moszkowski model and the Non-linear Walecka model. The different variants of the ZM model is denoted by ZM, ZM2 and ZM3 (according to the notations followed in ref.[8, 12]) models respectively. The ZM models discussed in refs. [8, 12] contain neutrons and protons only and they have been used to describe symmetric nuclear matter. Here the ZM models as well as the linear Walecka model are extended to include hyperons, ρ -meson and leptons (electrons and muons). The ρ -meson and leptons have been included to describe the asymmetric baryonic matter in β -equilibrium.

The Lagrangian density for the NW model [6] is written as,

$$\begin{aligned} \mathcal{L}_{NW} = & \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i + g_{\sigma i} \sigma + g_{\omega i} \omega_\mu \gamma^\mu - g_{\rho i} \rho_\mu^a \gamma^\mu T_a) \psi_i - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \rho_{\mu\nu}^a \rho_a^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_a^\mu \\ & - \frac{1}{3} b m_N (g_{\sigma N} \sigma)^3 - \frac{1}{4} c (g_{\sigma N} \sigma)^4 + \sum_l \bar{\psi}_l (i\gamma^\mu \partial_\mu - m_l) \psi_l \end{aligned} \quad (1)$$

The Lagrangian densities for LW model and ZM models can be written in a unified form as given below [4, 7, 8, 12]:

$$\begin{aligned} \mathcal{L}_R = & \sum_i \bar{\psi}_i (i\gamma_\mu \partial^\mu - M_i + m^{*\beta} g_{\sigma i} \sigma - m^{*\alpha} g_{\omega i} \gamma_\mu \omega^\mu + m^{*\alpha} g_{\rho i} \gamma_\mu \rho_a^\mu T_a) \psi_i \\ & + m^{*\alpha} \left(\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} - \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \right) + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m^2 \sigma^2) \end{aligned}$$

$$- m^{*\alpha} \left(\frac{1}{4} \rho_{\mu\nu}^a \rho_a^{\mu\nu} - \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_a^\mu \right) + \sum_l \bar{\psi}_l (i\gamma_\mu \partial^\mu - M_l) \psi_l \quad (2)$$

with $\alpha = 0, \beta = 0$ for the Walecka model, $\alpha = 0, \beta = 1$ for the ZM model, $\alpha = 1, \beta = 1$ for the ZM2 model, and $\alpha = 2, \beta = 1$ for the ZM3 model.

In the Lagrangians (1) and (2) mentioned above, ψ_i is the baryon field which can be neutron (n), proton (p), lambda (Λ) or sigma (Σ^-) baryons, ω , ρ and σ are the corresponding meson fields, $g_{\sigma i}$, $g_{\omega i}$ and $g_{\rho i}$ are the coupling strengths of σ , ω and ρ mesons, respectively, with the corresponding baryons. The electrons and muons are denoted by ψ_l . The Non-linear Walecka Lagrangian includes cubic and quartic self-interactions of the σ field with couplings b and c respectively [6]. The quantity m^* is given by

$$m^* = \left(1 + \frac{g_{\sigma n} \sigma_0}{M_n} \right)^{-1} \quad (3)$$

where $g_{\sigma n}$ is the coupling of scalar meson with nucleon, M_n is the nucleon mass and σ_0 is the mean field value of *sigma* field.

In the Mean Field Approximation (MFA) the meson fields are replaced by their ground state expectation values. Then the equation of motion for the baryon field can be written as

$$\left[i\gamma \cdot \partial - M_i^* - g_{\omega i} \gamma_0 \omega^0 + T_3 \gamma_0 \rho^{30} \right] = 0 \quad (4)$$

where T_3 is the z-component of the isospin and M_i^* is the effective mass of the baryon which is

$$M_i^* = M_i - g_{\sigma i} m^{*\beta} \sigma_0 \quad (5)$$

for the linear Walecka and ZM models. For the NW model M_i^* can be written as,

$$M_i^* = M_i - g_{\sigma i} \sigma_0 \quad (6)$$

The mean field values of the meson fields are given by

$$\begin{aligned}\omega_0 &= \sum_i \frac{g_{\omega i}}{m_\omega^2} \langle \bar{\psi}_i \gamma_0 \psi_i \rangle \\ \rho_0^3 &= \sum_i \frac{g_\rho}{m_\rho^2} \langle \bar{\psi}_i I^3 \gamma_0 \psi_i \rangle\end{aligned}\quad (7)$$

for all the models. The value of σ_0 for the Lagrangian (2) is

$$\sigma_0 = \sum_i \frac{g_{\sigma i}}{m_\sigma^2} \langle \bar{\psi}_i \psi_i \rangle + \frac{\alpha}{2} \left[\frac{m_\omega}{m_\sigma} \right]^2 \frac{\alpha}{M} m^{*(\alpha+1)} \omega_0^2 \quad (8)$$

and for the non-linear Walecka model (1) is

$$\sigma_0 = \sum_i \frac{g_{\sigma i}}{m_\sigma^2} \langle \bar{\psi}_i \psi_i \rangle - b m_N g_{\sigma N}^3 \sigma_0^2 - c g_{\sigma N}^4 \sigma_0^3 \quad (9)$$

The Lagrangian (2) has six parameters. These are $g_{\sigma n}/m_\sigma$, $g_{\sigma \Lambda}/m_\sigma$, $g_{\rho n}/m_\rho$ and $g_{\rho \Lambda}/m_\rho$, $g_{\omega n}/m_\omega$ and $g_{\omega \Lambda}/m_\omega$. The nucleon couplings are determined from the nuclear matter saturation properties [8, 12]. The values are given as follows: $C_\sigma^2 = 357.4$ and $C_\omega^2 = 273.8$ for the Walecka model; 169.2 and 59.1 for the ZM model; 219.3 and 100.5 for the ZM2 model and 443.3 and 305.5 for the ZM3 model where $C_\sigma^2 = g_\sigma^2 (M_N/m_\sigma)^2$ and $C_\omega^2 = g_\omega^2 (M_N/m_\omega)^2$. The value of $g_{\rho n}$ is 8.55451. The other three parameters are coupling constants of the hyperon-meson interactions and are not well known. These cannot be determined from nuclear matter properties since the nuclear matter does not contain hyperons. Furthermore, properties of hypernuclei do not fix these parameters in a unique way. Here the hyperon couplings are taken to be $\frac{2}{3}$ of the nuclear coupling [13].

The non-linear Walecka model has eight parameters out of which five are determined by the saturation properties of nuclear matter. These are nucleon couplings to scalar g_σ/m_σ , isovector g_ρ/m_ρ and vector mesons g_ω/m_ω and the two coefficients in the scalar self interaction i.e. b and c . Here the ρ and ω meson masses are chosen to be their physical masses.

The values of the parameters obtained are

$$g_{\sigma n}/m_\sigma = 1.52505 \times 10^{-2}, g_{\omega n}/m_\omega = 8.63433/782, g_\rho/m_\rho = 8.55451/770,$$

$$b = 3.41799 \times 10^{-3} \text{ and } c = 1.46 \times 10^{-2}.$$

Once again the hyperon couplings are taken to be $\frac{2}{3}$ of the nuclear coupling.

3 Quark Model

The Colour Dielectric Model is based on the idea of Nielson and Patkos [14]. In this model, one generates the confinement of quarks and gluons dynamically through the interaction of these fields with a scalar field. In the present work, the chiral extension (CCD) of this model has been used to calculate the quark matter equation of state [13]. The Lagrangian density of CCD model is given by [13]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x)\{i\gamma^\mu\partial_\mu - (m_0 + m/\chi(x)U_5) + (1/2)g\gamma_\mu\lambda_a A_\mu^a(x)\}\psi \\ & + f_\pi^2/4Tr(\partial_\mu U\partial^\mu U^\dagger) - 1/2m_\phi^2\phi^2(x) - (1/4)\chi^4(x)(F_{\mu\nu}^a(x))^2 \\ & + (1/2)\sigma_v^2(\partial_\mu\chi(x))^2 - U(\chi) \end{aligned} \quad (10)$$

where $U = e^{i\lambda_a\phi^a/f_\pi}$ and $U_5 = e^{i\lambda_a\phi^a\gamma_5/f_\pi}$, $\psi(x)$, $A_\mu(x)$, $\chi(x)$ and $\phi(x)$ are quark, gluon, scalar (colour dielectric) and meson fields respectively, m_ϕ and m are the meson and quark masses, f_π is the pion decay constant, $F_{\mu\nu}(x)$ is the usual colour electromagnetic field tensor, g is the colour coupling constant and λ_a are the Gell-Mann matrices. The flavour symmetry breaking is incorporated in the Lagrangian through the quark mass term $(m_0 + m/\chi U_5)$, with $m_0 = 0$ for u and d quarks. So the

masses of u , d and s quarks are m , m and $m_0 + m$ respectively. The self interaction $U(\chi)$ of the scalar field is assumed to be of the form

$$U(\chi) = \alpha B \chi^2(x) [1 - 2(1 - 2/\alpha)\chi(x) + (1 - 3/\alpha)\chi^2(x)], \quad (11)$$

so that $U(\chi)$ has an absolute minimum at $\chi = 0$ and a secondary minimum at $\chi = 1$. Thus the parameter B is the analog of the bag pressure of the bag model. The interaction of the scalar field with quark and gluon fields is such that quarks and gluons can not exist in the region where $\chi = 0$. In the limit of vanishing meson masses, the Lagrangian of eqn.(10) is invariant under chiral transformations of quark and meson fields.

The parameters of the CCD model are the quark masses m_q ($q = u, d, s$), strong coupling constant α_s , the Bag pressure B and the constant α . These are obtained by fitting the baryon masses. In the present calculation we have used $m_{q(u,d)} = 92MeV$, $m_{q(s)} = 295MeV$, $B^{1/4} = 152MeV$, $\alpha = 36$ and $\alpha_s = 0.08$.

The quark matter calculation proceeds as follows. It is assumed that in the presence of non-zero quark/anti-quark densities, the square of the meson field ($\langle\phi^2\rangle$) may develop a non-zero vacuum expectation value. This assumption is analogous to the assumption that, in the linear-sigma model [15] the σ -field acquires non-zero vacuum expectation values. It has been found that this occurs when the quark density exceeds a certain critical value. As a result of this the effective quark masses decrease with the increase in $\langle\phi^2\rangle$ and hence with increase in baryon density. Thus at large quark densities one obtains from CCD model an equation of state similar to the equation of state of free quarks and gluons. Furthermore, the colour dielectric field χ and $\langle\phi^2\rangle$ in quark matter are evaluated using mean field approximation. Quark-gluon interaction is incorporated upto two loop in thermodynamic potential.

4 Hot and Dense Baryonic Matter

The thermodynamic equilibrium in the hot and dense matter can be imposed from the weak interaction conditions. This introduces the following relations among different bare chemical potentials as,

$$\begin{aligned}
 \mu_p &= \mu_n - \mu_e \\
 \mu_\Lambda &= \mu_n \\
 \mu_{\Sigma^-} &= \mu_n + \mu_e \\
 \mu_\mu &= \mu_e
 \end{aligned} \tag{12}$$

In the medium the baryon chemical potentials are modified as,

$$\begin{aligned}
 \bar{\mu}_n &= \mu_n - g_{\omega n}\omega_0 + \frac{1}{2}g_\rho\rho_0 \\
 \bar{\mu}_p &= \mu_p - g_{\omega p}\omega_0 - \frac{1}{2}g_\rho\rho_0 \\
 \bar{\mu}_\Lambda &= \mu_\Lambda - g_{\omega\Lambda}\omega_0 \\
 \bar{\mu}_{\Sigma^-} &= \mu_{\Sigma^-} - g_{\omega\Sigma^-}\omega_0 + g_\rho\rho_0
 \end{aligned} \tag{13}$$

where $\bar{\mu}_i$ is the chemical potential of the i 'th baryon in the medium and μ_i is the bare chemical potential for the same.

The effective mass of a baryon in matter is as given in equations (11) and (12).

The charge neutrality gives,

$$n_p = n_e + n_\mu + n_{\Sigma^-} \tag{14}$$

In addition the baryon number conservation gives,

$$n_B = n_n + n_p + n_\Lambda + n_{\Sigma^-} \tag{15}$$

In the last two equations n_i is the number density of the i 'th particle and is given by

$$n_i = 2 \int \frac{d^3p}{(2\pi)^3} [f_i^+ - f_i^-] \quad (16)$$

According to the usual convention the baryon chemical potential is defined as $\mu_B = \mu_n$ [6].

All the above equations need to be solved self consistently to obtain the density dependence of baryon masses and number densities at different temperatures. In figures 1-3 the temperature dependence of n , Λ and Σ^- masses have been plotted for five different models which have been considered here and at three different densities - $\rho = 0, 0.15$ and $0.3fm^{-3}$. The density dependence of baryon masses at three temperatures - $T = 0, 100$ and $200MeV$ have been plotted in figures 4-6. The number densities for different models at $T = 0$ and $200MeV$ have been plotted in figures 7-11.

5 Quark Hadron Phase Transition

The quark-hadron phase transition can now be calculated by using the effective models discussed in section 2 and section 3. The phase transition point is determined by adopting Gibb's criterion. This says that the point at which the free energies (or pressure) of the two phases, for a given chemical potential, are equal corresponds to the phase transition point. In the present context, the baryon chemical potential (μ_B) is the only independent chemical potential. Then the crossing of the pressure curves for two phases in $P - \mu_B$ plane gives the phase transition point. Glendenning, on the other hand, considered a case where, in the mixed phase, neutron and quark matter are not charge neutral but the mixture as a whole is [16]. This aspect has been studied further by Heiselberg et.al. in ref [17]. This situation is not considered here.

For a first order transition, the derivatives of the P - μ_B curve for the two phases at the phase transition point are not equal and the difference in the two derivatives gives the discontinuity in the density of the two phases at the transition point. The two phases coexist in this range of density. The latent heat of transition is given by the difference in the energy densities of the two phases at the critical point. As mentioned earlier, the lattice QCD calculations indicate that hadron - quark phase transition may be weakly first order or second order. In a calculation, such as presented here, one would necessarily get a first order transition since two different models are employed to calculate the properties of quark and hadron phases.

The pressure for Walecka and ZM models can be written as

$$P = m^{*\alpha} \frac{1}{2} m_\omega^2 \bar{\omega}_0^2 + m^{*\alpha} \frac{1}{2} m_\rho^2 (\bar{\rho}_0^3)^2 - \frac{1}{2} m_{\sigma_0}^2 \bar{\sigma}^2 + \sum_i P_{FG}(\bar{m}_i, \bar{\mu}_i) + \sum_l P_{FG}(m_l, \mu_l) \quad (17)$$

For the Non-Linear Walecka model pressure P is given by

$$P = \frac{1}{2} m_\omega^2 \bar{\omega}_0^2 + \frac{1}{2} m_\rho^2 (\bar{\rho}_0^3)^2 - \frac{1}{2} m_{\sigma_0}^2 \bar{\sigma}_0^2 - \frac{1}{3} b m_N (g_{\sigma N} \bar{\sigma}_0)^3 - \frac{1}{4} c (g_{\sigma N} \bar{\sigma})^4 + \sum_i P_{FG}(\bar{m}_i, \bar{\mu}_i) + \sum_l P_{FG}(m_l, \mu_l) \quad (18)$$

In the above two equations P_{FG} is the pressure for the free fermi gas and is given by

$$P_{FG} = \frac{1}{3\pi^2} \int p^4 dp \sum_i \frac{1}{E_i} [f_i^+ + f_i^-] \quad (19)$$

For the quark sector the pressure is calculated upto two loop in the quark-gluon interaction and is given by

$$\begin{aligned} P &= T \gamma_q \int \frac{d^3 p}{(2\pi)^3} \left[\ln(1 + e^{(\mu - \epsilon_q(p))/T}) + \ln(1 + e^{-(\mu + \epsilon_q(p))/T}) \right] \\ &- T \gamma_g \chi^4 \int \frac{d^3 p}{(2\pi)^3} \ln(1 + e^{-\epsilon_g(p)/T}) - T \sum_\phi \gamma_\phi \int \frac{d^3 p}{(2\pi)^3} \ln(1 - e^{-\epsilon_\phi(p)/T}) \\ &- \frac{16}{3} \pi \alpha_s T^2 \int \frac{d^3 p n(p)}{(2\pi)^3 \epsilon_q(p)} - 8 \pi \alpha_s \int \frac{d^3 p d^3 q}{(2\pi)^6 \epsilon_q(p) \epsilon_q(q)} \end{aligned}$$

$$\begin{aligned}
& \times \left\{ \left[\frac{2m^{*2}}{(\epsilon_q(p) - \epsilon_q(q))^2 - \epsilon_g^2} - 1 \right] \times [n^-(p)n^-(q) + n^+(p)n^+(q)] \right. \\
& + \left. \left[\frac{2m^{*2}}{(\epsilon_q(p) + \epsilon_q(q))^2 - \epsilon_g^2} - 1 \right] \times [n^-(p)n^-(q) + n^+(p)n^+(q)] \right\} \\
& - \frac{2}{3}\alpha_s\pi T^4
\end{aligned} \tag{20}$$

In the above expression ϵ_q , ϵ_g and ϵ_ϕ are the quark gluon and meson kinetic energies, γ_q , γ_g and γ_ϕ are the corresponding degeneracies, $n^-(p) = 1/(1 + \exp((\epsilon_q(p) - \mu)/T))$ and $n^+(p) = 1/(1 + \exp((\epsilon_q(p) + \mu)/T))$ are the quark and antiquark distribution functions respectively, $N(p) = 1/(\exp(\epsilon_\phi(p)/T) - 1)$ is the distribution function for mesons and $n(p) = n^+(p) + n^-(p)$. In the quark sector the chemical equilibrium under weak decay and the charge neutrality give

$$\mu_d = \mu_u + \mu_e; \quad \mu_s = \mu_u + \mu_e \tag{21}$$

and

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0 \tag{22}$$

The baryon density $n_B = \frac{1}{3}\sum_i n_i$ where $i = u, d, s$ and the baryon chemical potential is defined as $\mu_B = \mu_u + \mu_d + \mu_s$.

The $P - \mu_B$ curves for different models have been plotted in figures 12-16.

6 Results and Discussions

In the present work we have studied the properties of hot and dense asymmetric baryonic matter in β -equilibrium. The baryonic matter has been discussed within the framework of five effective hadronic models. They are the LW model [4], three variants of the ZM model [7] - ZM, ZM2 and ZM3 and the NW model [6]. We have extended the LW, ZM, ZM2 and ZM3 models to include hyperons and also discussed

the NW model. The models include four baryons - n, p, Λ and Σ^- , the scalar σ -meson, the vector mesons ω and ρ and the leptons (electron and muon). The baryonic matter properties *i.e.* the masses and fractional number densities have been calculated from these models in the mean field level. The pressure of the baryonic matter has been calculated in the usual canonical fashion. The pressure for quark matter has been calculated from CCD model [13].

The temperature dependence of baryon masses at different densities have been plotted in figures 1-3 for different models. Fig.1 is the temperature dependence of nucleon mass at different densities. This shows that the nucleon mass decreases with temperature and the maximum change in the nucleon mass is obtained in the LW model. Also, as the density is increased the baryon mass first increases slightly with temperature and then starts decreasing. The same qualitative behaviour is observed in the case of Λ and Σ^- masses (fig.2 and 3). With density, the variation of nucleon mass is even faster. In the LW model the nucleon mass goes to zero at about $0.8fm^{-3}$ whereas in the other models the nucleon mass saturates in the range $400 - 600MeV$. The higher value is for ZM model and lower is for ZM3 model.

In figures 7-11 the variation of fractional number densities, (n_i/n_B) , of different particles with baryon density for different models have been plotted, each at two temperatures $T = 0MeV$ and $T = 200MeV$. Fig. 7 shows that the neutron number density in the LW model decreases and goes to zero at about $0.8fm^{-3}$. The proton number density increases initially from 0 to $0.25fm^{-3}$ then it decreases slightly upto $1.0fm^{-3}$ after which it saturates. The Λ baryon appears at about $n_B = 0.25fm^{-3}$, increases till $1.0fm^{-3}$ to about 0.95 and then saturates. The Σ^- also appears at $n_B = 0.25fm^{-3}$, it increases first and then starts decreasing going to zero at $n_B = 1.1fm^{-3}$. The matter becomes hyperonic at $n_B = 0.4fm^{-3}$. (We will use this terminology of "hyperonic matter", in the discussion of number densities, for the situation when the

Λ or Σ^- number density takes over n or p number density). In the same model, at $T = 200MeV$ the matter is hyperonic right from the beginning. The Λ number density varies from 0.39 to 0.85 as the density is varied from 0 to $1.5fm^{-3}$.

The nature of variation of number densities is somewhat different for the ZM model (fig. 8). At $T = 0MeV$, the neutron number density decreases with baryon density but does not go to zero. The proton number density saturates at a higher value compared to that in the LW model. The transition from nuclear to hyperonic matter takes place at $n_B = 1.1fm^{-3}$ which is considerably higher than that in the LW model. In the ZM model, at $T = 200MeV$, unlike the LW model, the matter is not hyperonic at the very beginning. The transition takes place at about $n_B = 1.6fm^{-3}$. For ZM2 model (fig. 9), at $T = 0MeV$, the transition takes place at $n_B = 1.6fm^{-3}$ whereas at $T = 200MeV$ it is at $n_B = 1.9fm^{-3}$. Figure 10 gives an interesting result. This is for the ZM3 model. The matter does not become hyperonic ever. It stays as nuclear matter at all temperatures and densities. In the NW model at $T = 0MeV$ the transition occurs at $n_B = 0.85fm^{-3}$. Unlike LW model the neutron and sigma number densities do not go to zero. The proton number density also saturates at a higher value compared to that in the LW model.

The hadron-quark phase transition has been studied using the Gibbs' criterion. The intersection of the two $P - \mu_B$ curves, for two different models for the two sectors, gives the transition density. We have a first order phase transition due to the consideration of two different models in two phases. In figures 12-16 we have plotted the $P - \mu_B$ curves for five different hadronic models along with the quark model at five different temperatures. The quark-hadron phase transition, for this set of models, is observed only for LW and NW models and that too at $T = 0MeV$ and $T = 50MeV$. The transition, for the LW model at $T = 0MeV$, takes place at $\mu_B = 1100MeV$ which corresponds to $\rho_c = 0.27fm^{-3}$. At $T = 50MeV$ the critical

density is $\rho_c = 0.26 fm^{-3}$. For the NW model these two values are $\rho_c = 0.47 fm^{-3}$ and $\rho_c = 0.37 fm^{-3}$ respectively.

To conclude, we have compared different hadronic models at finite temperature and density. These models give varying predictions for masses and number densities. This will have a strong bearing both for neutron star properties as well as heavy ion collisions in the laboratory. Also the neutrino emissivities will be different for these models.

The hadron-quark phase transition has been investigated using CCD model. The phase transition is seen for LW and NW whereas none of the variants of ZM model gives the phase transition for the parameter set considered here, even at $T = 0 MeV$. This will imply the absence of quark-hadron phase transition inside neutron star.

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