

Instability in scalar channel of fermion-antifermion scattering amplitude in massless QED₃ and anomalous dimensions of composite operators

Indrajit Mitra^{a*}, Raghunath Ratabole^{b†} and H. S. Sharatchandra^{b‡}

^a Theory Group, Saha Institute of Nuclear Physics, 1/AF Bidhan-Nagar, Kolkata 700064, India

^b The Institute of Mathematical Sciences, C.I.T. Campus, Taramani P.O., Chennai 600113, India

Abstract

Instability in the scalar channel of the fermion-antifermion scattering amplitude in massless QED₃ for number of flavours less than the critical value $128/3\pi^2$ is demonstrated. The anomalous dimensions of gauge-invariant composite operators are determined to $O(1/N)$. Exponentiation of the $O(1/N)$ infrared logarithm is explicitly demonstrated by evaluating the contribution of the ladder diagrams.

Keywords: Anomalous dimension; Dynamical breaking of chiral symmetry; Massless QED₃
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*E-mail: indrajit.mitra@saha.ac.in

†E-mail: raghu@imsc.res.in; present address: Birla Institute of Technology and Science, Pilani, Goa Campus, Zuarinagar 403726, Goa, India

‡E-mail: sharat@imsc.res.in

Massless QED₃ provides a theoretical laboratory for studying infrared (IR) divergences, and for investigating dynamical breaking of chiral symmetry [1, 2, 3, 4, 5, 6, 7, 8]. It has also emerged as a leading paradigm for understanding the pseudogap phase of cuprate superconductors [9, 10]. Lattice simulations of this theory also form an active area of research [11, 12]. Recently, we have obtained a rather detailed understanding of the IR behaviour of this theory [13, 14, 15]. For a particular value of the gauge parameter in a non-local gauge [16, 17, 18, 19, 20], the IR behaviour is of a conformal field theory with canonical dimension for the fermion but a scale dimension one for the photon [13]. This value of the gauge parameter is fixed at each order in $1/N$ (N being the number of fermion flavours). For other values of the gauge parameter in this non-local gauge, the IR behaviour is again a conformal field theory, but with an anomalous dimension of the fermion which is linear in the gauge parameter, and can take arbitrary values. In the usual local gauge, the fermion correlation functions are exponentially damped in the IR, as if a mass gap is present. However, this is only a gauge artifact and illustrates the pitfalls involved in summing the severe IR divergences of the perturbation theory [14].

This emphasizes the need to study the IR behaviour of gauge-invariant correlation functions, as they correspond to physical observables. The simplest such objects are the Green functions involving only photons. They have a power law behaviour in the IR, as per a scale dimension two (in contrast to the canonical dimension $3/2$) for the field strength $F_{\mu\nu}(x)$. Another class of objects, which can be related to experiments which probe single fermions, are the correlations of gauge-invariant dressed fermions. We have argued in Ref. [15] that only the isotropic (in Euclidean space-time) dressing is relevant, and with this dressing, the fermion correlations have again a power law behaviour but with a negative anomalous dimension.

There is another class of correlations that are directly relevant to the experiments. These are the response functions of QED₃ [21] corresponding to correlations of gauge-invariant composite operators of the form $\bar{\psi}(x)\Gamma\psi(x)$, where the matrix Γ is any element of the algebra of gamma-matrices. We address their IR behaviour in this paper. We will first perform an $O(1/N)$ calculation which will give an IR logarithm, heralding an anomalous dimension for the composite operator. We will then explicitly demonstrate how the ladder diagrams lead to an exponentiation of this IR logarithm.¹

A crucial issue in QED₃ is dynamical breaking of chiral symmetry and the critical number of flavours below which this takes place [4, 5, 6, 7, 8]. It is always very difficult to calculate the condensate $\langle \bar{\psi}\psi \rangle$. There are different claims for the critical number of flavours N_c , ranging from $N_c = 1$ to $N_c = 4$.² In the course of our analysis, we calculate the fermion-antifermion scattering amplitude³ in the scalar channel for vanishing total energy-momentum. We demonstrate instability in the vacuum in which this scattering amplitude is calculated, for number of flavours less than the critical value $N_c = 128/3\pi^2$. Our calculation is important for the following reasons. Firstly, *we need only ladder diagrams* and not self-consistent solution of the gap equation. Secondly, *we relate the instability to a robust physical mechanism*: an attractive inverse-square potential (with momentum variables in

¹The anomalous dimensions for the cases $\Gamma = 1$ and $\Gamma = \gamma_5$ in the ladder approximation were also addressed in Ref. [22]. The result is discussed Ref. [23].

²For a review, see, for example, Ref. [24].

³The fermion-antifermion scattering amplitude was also considered in Ref. [25].

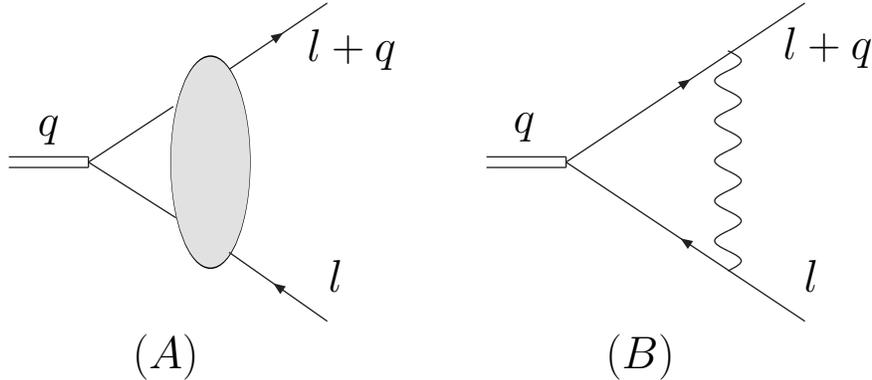


Figure 1: (A) The correlation function $\langle \bar{\psi}(x)\Gamma\psi(x)\psi_\alpha(y)\bar{\psi}_\beta(z) \rangle$ in momentum space (the figure is for $\Gamma = 1$). (B) The $O(1/N)$ contribution.

the place of coordinate variables) arising from the masslessness of the fermions and the feature that the small-momentum behaviour of the photon propagator is inversely linear.

Let us consider (the Fourier transform of) the correlation function $\langle \bar{\psi}(x)\Gamma\psi(x)\psi_\alpha(y)\bar{\psi}_\beta(z) \rangle$ (see Fig. 1A). This correlation is gauge-dependent, but from it, one can extract the (gauge-invariant) anomalous dimension of the composite operator $\bar{\psi}\Gamma\psi$, as follows.

The $O(1/N)$ contribution to this correlation function, as shown in Fig. 1B, is given by

$$\int \frac{d^3k}{(2\pi)^3} e^{\gamma_\sigma} \frac{1}{\not{l} + \not{q} + \not{k}} \Gamma \frac{1}{\not{l} + \not{k}} e^{\gamma_\rho} \frac{\delta_{\sigma\rho} - \xi k_\sigma k_\rho / k^2}{k^2 + \mu k} \quad (1)$$

with $\mu = Ne^2/8$ (for N four-component spinors). Here we have used the photon propagator [13] in the $1/N$ expansion (in which there is a resummation of chains of one-loop vacuum polarization diagrams on every photon propagator) with a non-local gauge-fixing term.⁴ This non-local gauge ensures that the IR behaviour of the propagator is inversely linear in momentum for arbitrary value of the gauge parameter ξ . The value of the gauge parameter can then be fixed to each order in $1/N$ such that the IR logarithms are absent in the fermion self-energy and in other Green functions of elementary fields [13]. This choice of ξ will simplify the ladder diagrams, to be considered later in this paper.

We will now follow the calculation of Appendix A of Ref. [13]. Choose $l_\mu = \rho L_\mu$ and $q_\mu = \rho Q_\mu$. Also let $k_\mu = \rho K_\mu$. Then the integral equals

$$e^2 \int \frac{d^3K}{(2\pi)^3} \gamma_\sigma \frac{1}{\not{L} + \not{Q} + \not{K}} \Gamma \frac{1}{\not{L} + \not{K}} \gamma_\rho \frac{\delta_{\sigma\rho} - \xi K_\sigma K_\rho / K^2}{\rho K^2 + \mu K}. \quad (2)$$

For $\rho \rightarrow 0$, with L_μ and Q_μ of $O(1)$, this is IR finite but logarithmically ultraviolet (UV) divergent. Letting the divergent part be $C \ln \rho$, the coefficient C can be obtained by the action of $[\rho(d/d\rho)]_{\rho=0}$ on the last expression (it is convenient to replace K by k/ρ before setting $\rho = 0$):

$$C = -e^2 \int \frac{d^3k}{(2\pi)^3} \gamma_\sigma \frac{\not{k}}{k^2} \Gamma \frac{\not{k}}{k^2} \gamma_\rho \frac{\delta_{\sigma\rho} - \xi k_\sigma k_\rho / k^2}{(k + \mu)^2}. \quad (3)$$

⁴The gauge-fixing term is given by Eq. (25) of Ref. [14]. The gauge parameter α of Ref. [14] is related to the gauge parameter ξ of the present paper and of Ref. [13] by $\alpha = 1 - \xi$.

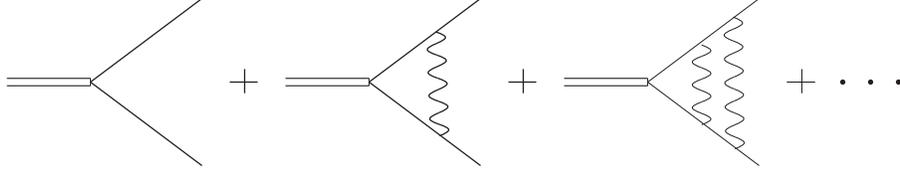


Figure 2: Ladder diagrams contributing to the correlation function.

In the numerator, the part proportional to ξ equals $-\xi k^2 \Gamma$. Writing the other part as $k_\mu k_\nu \gamma_\sigma \gamma_\mu \Gamma \gamma_\nu \gamma_\sigma$, we replace $k_\mu k_\nu$ by $\frac{1}{3} k^2 \delta_{\mu\nu}$. Now consider two types of vertices [21]:

$$\gamma_\sigma \gamma_\mu \Gamma \gamma_\mu \gamma_\sigma = \kappa \Gamma \quad (4)$$

with $\kappa = 9$ if Γ commutes or anticommutes with all the three gamma-matrices, and $\kappa = 1$ if Γ anticommutes with one or two of the gamma-matrices and commutes with the rest. Thus the numerator equals $k^2 (\frac{1}{3} \kappa - \xi) \Gamma$, and on doing the angular integration,

$$C = -\frac{4}{\pi^2 N} \left(\frac{1}{3} \kappa - \xi \right) \Gamma. \quad (5)$$

Now, if we choose the value $\xi = 1/3$ for which the $O(1/N)$ logarithmic divergence in the fermion self-energy is absent [13], the coefficient of Γ in C gives the anomalous dimension of the composite operator $\bar{\psi} \Gamma \psi$.⁵ Thus for the vertices with $\kappa = 9$, the composite operator (for example, $\bar{\psi} \psi$) has an anomalous dimension

$$\eta = -\frac{32}{3\pi^2 N}, \quad (6)$$

while for the vertices with $\kappa = 1$, the composite operator does not have an anomalous dimension. Our result agrees with that of Ref. [21]. The apparent discrepancy of a factor of -2 between Eq. (6) and the result of Ref. [21] is merely due to difference in the definition of anomalous dimension. In particular, the authors of Ref. [21] consider the dependence of $\langle \bar{\psi}(x) \Gamma \psi(x) \bar{\psi}(y) \Gamma \psi(y) \rangle$ on the external momentum, and this correlation has *two* factors of the wave-function renormalization $Z_{\bar{\psi} \Gamma \psi}$ instead of *one* factor as in our correlation. It is to be noted that we have obtained the anomalous dimension by performing a one-loop calculation, instead of the two-loop calculation of Ref. [21].

The anomalous dimension was heralded by the appearance of an IR ($\rho \rightarrow 0$) logarithm in external momenta. In order to address the exponentiation of this logarithm, we consider the contribution of the ladder diagrams (see Fig. 2). We choose to work with the special gauge parameter ξ_0 of our non-local gauge for which the logarithmic terms are absent at all orders in the fermion self-energy and in other Green functions of the elementary fields ψ , $\bar{\psi}$ and A_μ . Then the full fermion propagator has the behaviour of the free propagator $1/\not{k}$, and the

⁵This follows from the application of the Callan-Symanzik equation to the correlation function under consideration; see Ref. [26] for a similar situation.

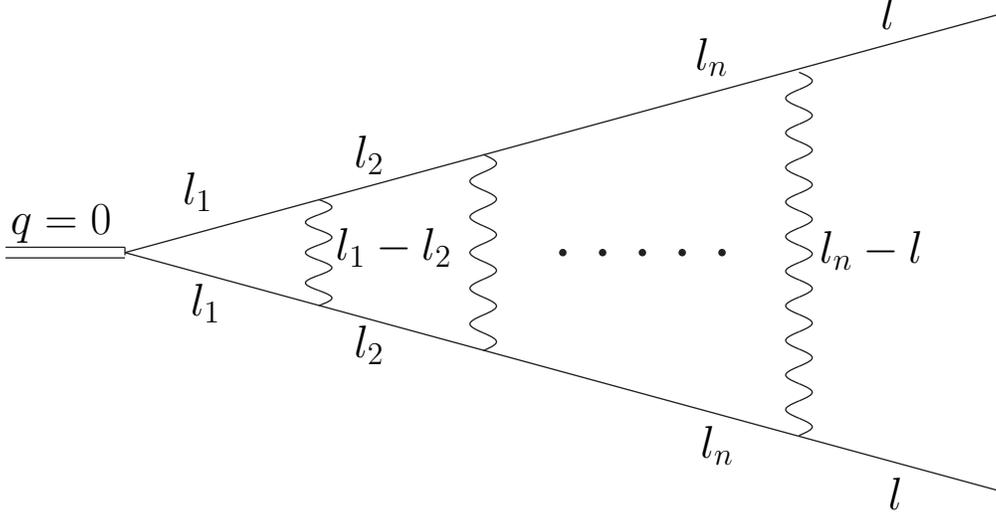


Figure 3: The n -loop ladder diagram: the spinor algebra is drastically simplified for $q = 0$.

full vertex $e\gamma_\mu$, when all the momenta are small. Therefore, *keeping only the bare fermion propagators and the bare vertices in Fig. 2 is justified*. Moreover, we will demonstrate the exponentiation of only the lowest order contribution to the anomalous dimension. So we will not consider the diagrams with crossed photons, which will contribute at higher orders in $1/N$.

Power counting of the subdiagrams shows that there are no UV divergences. We are interested in the IR behaviour. We will also consider the case $q = 0$. To extract the anomalous dimension of the composite operator, we need the correlation function at non-exceptional and Euclidean external momenta. There is a possibility of additional IR divergences at exceptional values. However, we consider this case as there is an enormous simplification in the spinor algebra, so that the power law in the IR comes out in a simple and straightforward way.

We take Γ to commute or anticommute with all the gamma-matrices, which is the case for which we obtained a log divergence in our one-loop calculation. Now from the l_1 -loop in Fig. 3, we get the spinor factors

$$e^2 \gamma_\mu \not{l}_1 \not{l}_1 \gamma_\nu (\delta_{\mu\nu} - \xi_0 (l_1 - l_2)_\mu (l_1 - l_2)_\nu / (l_1 - l_2)^2) = e^2 (3 - \xi_0) l_1^2. \quad (7)$$

As four gamma-matrices are involved, this contribution is insensitive to our sign convention and the choice of the action. Similar simplification occurs for each of the loops of Fig. 3. Thus, all spinor dependence is absent, as if the fermion is replaced with a scalar. We get a contribution

$$(e^2 (3 - \xi_0))^n \int \frac{d^3 l_1}{(2\pi)^3 l_1^2} \int \frac{d^3 l_2}{(2\pi)^3 l_2^2} \cdots \int \frac{d^3 l_n}{(2\pi)^3 l_n^2} \frac{1}{\mu |\vec{l}_1 - \vec{l}_2|} \frac{1}{\mu |\vec{l}_2 - \vec{l}_3|} \cdots \frac{1}{\mu |\vec{l}_n - \vec{l}|}, \quad (8)$$

from n ladders, where we have kept the leading IR behaviour of the photon propagator, and presumed a cut-off in the UV for each integration. Integration over l_1 leads to $\sim \ln l_2$ for small l_2 (this is because l_2 serves as the IR cutoff for an otherwise log divergent l_1

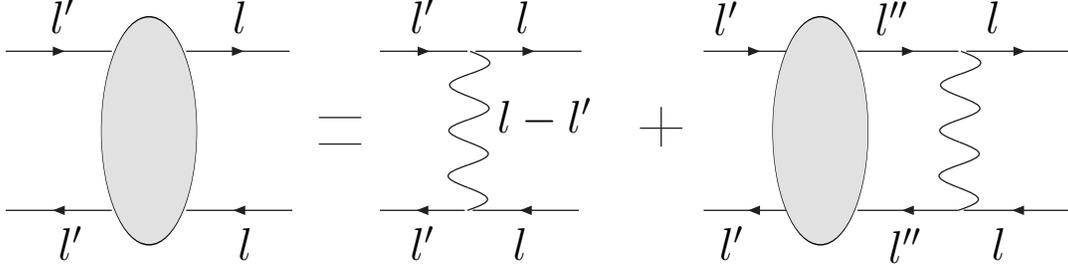


Figure 4: Integral equation for the fermion-antifermion scattering amplitude in the ladder approximation (and at vanishing total energy-momentum).

integration). This when fed into the l_2 integration, gives $\sim d(\ln l_2) \ln l_2$ with an IR cutoff l_3 , which is $\sim \frac{1}{2!}(\ln l_3)^2$. Continuing in this way, the n -loop integration of Fig. 3 gives a contribution $\sim \frac{1}{n!}(\ln l)^n$. A sum over the number of ladders n , as in Fig. 2, then gives a power in the fermion momentum l , namely, $\exp[c(e^2/\mu)(3 - \xi_0) \ln l] = l^{c(e^2/\mu)(3 - \xi_0)}$, where c is a numerical factor. This argument thus suggests an anomalous dimension for composite operators like $\bar{\psi}\psi$.

It should be noted that it is the logarithm from the l_1 -loop which eventually gets exponentiated. The vertex in the l_1 -loop is not a $\bar{\psi}A\psi$ vertex, and so our proof [13] of absence of anomalous dimension for Green functions involving *elementary* fields in the gauge $\xi = \xi_0$ does not apply for this correlation function.

We now analyse this situation using an integral equation. Let us consider the kernel $I(\vec{l}, \vec{l}')$ depicted in Fig. 4. By joining the l' lines, we can recover the correlation function considered earlier. We have

$$I(\vec{l}, \vec{l}') = \frac{1}{|\vec{l} - \vec{l}'|} + \lambda \int d^3 l'' \frac{1}{l''^2} \frac{I(\vec{l}'', \vec{l}')}{|\vec{l} - \vec{l}''|} \quad (9)$$

where

$$\lambda = \frac{e^2}{(2\pi)^3 \mu} (3 - \xi_0). \quad (10)$$

Eq. (9) clearly holds for the case when the l' lines are joined to give a $\bar{\psi}\psi$ vertex (that is, $\Gamma = 1$), which carries zero spin. Thus, Eq. (9) is for the scalar channel of the fermion-antifermion scattering amplitude. We may convert this integral equation into a differential equation by operating on it with ∇_l^2 :

$$\left(\nabla_l^2 + \frac{4\pi\lambda}{l^2} \right) I(\vec{l}, \vec{l}') = -4\pi\delta^{(3)}(\vec{l} - \vec{l}'). \quad (11)$$

This means that $I(\vec{l}, \vec{l}')$ is a propagator (Green function) for the three-dimensional Schrödinger equation (with momentum variables in the place of coordinate variables) with the potential $V(\vec{l}) = -4\pi\lambda/l^2$. As $\lambda > 0$ (ξ_0 being $1/3$) for us, this is an attractive inverse square potential. This is a well-studied problem in quantum mechanics [27], and has unusual properties due to exact scale invariance. We will return to this connection a little later.

The Green function can be determined using standard techniques [28]. The “radial” part $f_L(l, l')$ for the channel with angular momentum L satisfies

$$\frac{1}{l} \frac{d^2}{dl^2} (lf_L) - \frac{L(L+1) - 4\pi\lambda}{l^2} f_L = -\frac{4\pi}{l^2} \delta(l - l'). \quad (12)$$

Note that the λ -containing term (with $\lambda > 0$) provides a centripetal attraction in contrast to the centrifugal repulsion provided by the angular momentum. The radial equation (for $l \neq l'$) has the two power law solutions:

$$l^{-\frac{1}{2} \pm \alpha}, \quad \alpha = \sqrt{\left(L + \frac{1}{2}\right)^2 - 4\pi\lambda}. \quad (13)$$

If $4\pi\lambda < 1/4$ (which is true when the number of flavours $N > N_c = (16/\pi^2)(3 - \xi_0)$), α is real for each of $L = 0, 1, 2, \dots$. Now the boundary conditions on $f_L(l, l')$ follows from the IR and UV finiteness of the integral on the right-hand side of Eq. (9): $\lim_{l \rightarrow 0} lf_L(l, l') = 0$ and $\lim_{l \rightarrow \infty} f_L(l, l') = 0$. Also, $f_L(l, l') = f_L(l', l)$. Therefore, we have $f_L(l, l') = Cl_{<}^{-1/2+\alpha} l_{>}^{-1/2-\alpha}$, where $l_{<}$ ($l_{>}$) is the smaller (larger) of l and l' , and C is determined from the effect of the delta function in Eq. (12). Thus, the solution for the Green function is obtained as

$$I(\vec{l}, \vec{l}') = \sum_{L=0}^{\infty} \sum_{M=-L}^L \frac{2\pi}{\sqrt{(L + \frac{1}{2})^2 - 4\pi\lambda}} \frac{1}{\sqrt{l_{<} l_{>}}} \left(\frac{l_{<}}{l_{>}}\right)^{\sqrt{(L + \frac{1}{2})^2 - 4\pi\lambda}} Y_{LM}^*(\theta', \phi') Y_{LM}(\theta, \phi), \quad (14)$$

where (θ, ϕ) and (θ', ϕ') are the angular variables for \vec{l} and \vec{l}' respectively.

For the quantum mechanical problem of a particle in an attractive inverse-square potential, a coupling constant $4\pi\lambda$ exceeding the critical value $4\pi\lambda_c = 1/4$ leads to a singular situation: the particle falls to the centre [27]. In our problem, this corresponds to $N < N_c = 128/3\pi^2 \approx 4.3$ for $\xi_0 = 1/3$. This is same as the value for the critical number of flavours for dynamical chiral symmetry breaking, as obtained from an analysis of gap equation [16, 20].

Let the eigenfunctions and eigenvalues of the Schrödinger operator $-\nabla_{\vec{l}}^2 - 4\pi\lambda/l^2$ be denoted by $\chi_n(\vec{l})$ and ϵ_n respectively. Then, from Eq. (11),

$$I(\vec{l}, \vec{l}') = 4\pi \sum_n \frac{\chi_n(\vec{l}) \chi_n^*(\vec{l}')}{\epsilon_n}. \quad (15)$$

There are no bound states (that is, $\epsilon_n < 0$) for $\lambda < \lambda_c$ [27, 29], but bound states appear at the critical coupling. $I(\vec{l}, \vec{l}')$ is the Bethe-Salpeter amplitude in the channel with total spin zero and total energy-momentum zero. *This is possibly the simplest object which brings out instability in the vacuum.* For total energy-momentum zero, only the mass-squared terms in the poles in the amplitude survive. So having some $\epsilon_n < 0$ corresponds to a tachyon. This means an instability in the vacuum in which we have computed $I(\vec{l}, \vec{l}')$. The system cures this by forming a condensate.

Let us now join the l' lines in Fig. 4 to obtain the desired correlation function. Thus, we are to evaluate

$$\mathcal{I}(l) \equiv \int \frac{d^3 l'}{(2\pi)^3} \frac{1}{l'^2} e^2 (3 - \xi_0) \frac{1}{\mu} I(\vec{l}, \vec{l}') \quad (16)$$

in the limit $l \rightarrow 0$. Because of the integral over the angular variables for \vec{l}' , only the $L = 0$ term is picked out from $I(\vec{l}, \vec{l}')$. Then

$$\mathcal{I}(l) = \frac{e^2 (3 - \xi_0)}{4\pi^2 \mu \sqrt{1/4 - 4\pi\lambda}} \int_0^\mu dl' \frac{1}{\sqrt{l_{<} l_{>}}} \left(\frac{l_{<}}{l_{>}} \right)^{\sqrt{1/4 - 4\pi\lambda}}. \quad (17)$$

(The full $1/(k^2 + \mu k)$ photon propagator provides an effective UV cutoff of $O(\mu)$.) On splitting the range of integration into $(0, l)$ and (l, μ) , and putting $l' = lx$, we get the two integrals:

$$\int_0^1 dx x^{-1/2 + \sqrt{1/4 - 4\pi\lambda}} + \int_1^{\mu/l} dx x^{-1/2 - \sqrt{1/4 - 4\pi\lambda}}. \quad (18)$$

While there is no divergence from the $x = 0$ end, there is a divergence at $x = \mu/l$ for $l \rightarrow 0$. On expanding $\sqrt{1/4 - 4\pi\lambda}$ in powers of $1/N$, it is found to equal $1/2 + \eta$ at $O(1/N)$, where η is given in Eq. (6). Also, the prefactor in the integral in Eq. (17) equals $-\eta$, to the lowest order in $1/N$. Thus,

$$\mathcal{I}(l) = -\eta \int^{\mu/l} dx x^{-1-\eta} \sim \left(\frac{l}{\mu} \right)^\eta \quad (19)$$

for $l \rightarrow 0$. (Since we have a divergence for $l \rightarrow 0$, the contribution of unity from the first diagram (the free-theory diagram) in Fig. 2 can be neglected.) We have thus demonstrated that the correlation function has a power law behaviour with the exponent η .⁶

Let us now briefly consider the case $q \neq 0$. Let the incoming fermion-antifermion pair in the scattering amplitude have the momenta $\vec{l} \pm \vec{q}/2$, and the outgoing pair $\vec{l}' \pm \vec{q}/2$. Let us define $\vec{L} = \vec{l}/q$ and $\vec{L}' = \vec{l}'/q$, and scale the loop momentum: $\vec{L}'' = \vec{l}''/q$. Also define $J(\vec{l}, \vec{l}', \vec{q}) = qI(\vec{l}, \vec{l}', \vec{q})$. Then $J(\vec{l}, \vec{l}', \vec{q})$ satisfies almost the same integral equation as Eq. (9) for $I(\vec{l}, \vec{l}')$ with $\vec{l} \rightarrow \vec{L}$, $\vec{l}' \rightarrow \vec{L}'$, $\vec{l}'' \rightarrow \vec{L}''$; only the factor $1/l''^2$ in the last term gets replaced with $1/(|\vec{L}'' + \hat{q}/2||\vec{L}'' - \hat{q}/2|)$, where \hat{q} is the unit vector along \vec{q} . (To simplify our analysis, we have used $1/k$ as the fermion propagator.) This then gives the differential equation

$$\left(\nabla_L^2 + \frac{4\pi\lambda}{|\vec{L} + \hat{q}/2||\vec{L} - \hat{q}/2|} \right) J(\vec{L}, \vec{L}', \vec{q}) = -4\pi\delta^{(3)}(\vec{L} - \vec{L}'). \quad (20)$$

So, for large \vec{L} , we still have an inverse square potential, but near $\vec{L} = \pm\hat{q}/2$ the potential is Coulomb-like. A potential of this form has also been obtained⁷ following other routes.

⁶This agrees with the result of Ref. [22] when the appropriate limits are taken.

⁷The authors of Ref. [30] find the same value of N_c in QED₃ as ours. The two-centre potential appears in Section 4 of Ref. [31].

In this paper, we demonstrated instability in the scalar channel of the fermion-antifermion scattering amplitude in massless QED₃, for number of flavours less than the critical value $128/3\pi^2$ for which spontaneous chiral symmetry breaking takes place. This was done using only the ladder diagrams, and the instability was linked to the robust physical mechanism of an attractive inverse-square potential. We also determined the anomalous dimensions of the gauge-invariant composite operators to $O(1/N)$, first as coefficient of IR logarithm, and then as exponent of power law.

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References

- [1] R. Jackiw and S. Templeton, Phys. Rev. **D23** (1981) 2291.
- [2] T. Appelquist and R. D. Pisarski, Phys. Rev. **D23** (1981) 2305.
- [3] E.I. Guendelman and Z.M. Radulovic, Phys. Rev. **D27** (1983) 357; *ibid.* **D30** (1984) 1338.
- [4] T. Appelquist, M. J. Bowick, E. Cohler and L. C. R. Wijewardhana, Phys. Rev. Lett. **55** (1985) 1715.
- [5] N. E. Mavromatos and J. Papavassiliou, Phys. Rev. **D60** (1999) 125008.
- [6] A. Bashir and A. Raya, Nucl. Phys. **B709** (2005) 307 [arXiv:hep-ph/0405142].
- [7] C. S. Fischer, R. Alkofer, T. Dahm and P. Maris, Phys. Rev. **D70** (2004) 073007 [arXiv:hep-ph/0407104].
- [8] A. Bashir and A. Raya, arXiv:hep-ph/0511291.
- [9] W. Rantner and X.-G. Wen, Phys. Rev. Lett. **86** (2001) 3871 [arXiv:cond-mat/0010378].
- [10] M. Franz and Z. Tesanovic, Phys. Rev. Lett. **87** (2001) 257003 [arXiv:cond-mat/0012445].
- [11] S. J. Hands, J. B. Kogut, L. Scorzato and C. G. Strouthos, Phys.Rev. **B70** (2004) 104501 [arXiv:hep-lat/0404013].
- [12] R. Fiore *et al.*, Phys. Rev. **D72** (2005) 094508 [arXiv:hep-lat/0506020].

- [13] I. Mitra, R. Ratabole and H. S. Sharatchandra, Phys. Lett. **B611** (2005) 289 [arXiv:hep-th/0410120].
- [14] I. Mitra, R. Ratabole and H. S. Sharatchandra, Phys. Lett. **B634** (2006) 557 [arXiv:hep-th/0510055].
- [15] I. Mitra, R. Ratabole and H. S. Sharatchandra, Phys. Lett. **B636** (2006) 68 [arXiv:hep-th/0510057].
- [16] D. Nash, Phys. Rev. Lett. **62** (1989) 3024.
- [17] H. Georgi, E. H. Simmons and A. G. Cohen, Phys. Lett. **B236** (1990) 183.
- [18] E. H. Simmons, Phys. Rev. **D42** (1990) 2933.
- [19] T. Kugo and M. G. Mitchard, Phys. Lett. **B282** (1992) 162.
- [20] K. -I. Kondo, T. Ebihara, T. Iizuka and E. Tanaka, Nucl. Phys. **B434** (1995) 85 [arXiv:hep-ph/9404361].
- [21] M. Franz, T. Pereg-Barnea, D. E. Sheehy and Z. Tesanovic, Phys. Rev. **B68** (2003) 024508 [arXiv:cond-mat/0211119].
- [22] V. Gusynin, A. Hams and M. Reenders, Phys. Rev. **D63** (2001) 045025 [arXiv:hep-ph/0005241].
- [23] V. P. Gusynin, D. V. Khveshchenko and M. Reenders, Phys. Rev. **B67** (2003) 115201 [arXiv:cond-mat/0207372].
- [24] T. Appelquist and L. C. R. Wijewardhana, Cincinnati 2003, Quantum theory and symmetries, pp. 177-191 [arXiv:hep-ph/0403250].
- [25] T. Appelquist, J. Terning and L. C. R. Wijewardhana, Phys. Rev. Lett. **75** (1995) 2081 [arXiv:hep-ph/9402320].
- [26] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory, Addison-Wesley, Reading, MA, 1995, p. 431.
- [27] L. D. Landau and E. M. Lifshitz, Quantum Mechanics, third ed., Pergamon Press, Oxford, 1977, pp. 114-117.
- [28] See, for example, J. D. Jackson, Classical Electrodynamics, second ed., John Wiley, New York, 1975, pp. 111-113.
- [29] H. E. Camblong and C. R. Ordonez, Mod. Phys. Lett. **A17** (2002) 817 [arXiv:hep-th/0110278].
- [30] V.P. Gusynin, A.H. Hams and M. Reenders, Phys. Rev. **D53** (1996) 2227 [arXiv:hep-ph/9509380].
- [31] A. P. Isaev, Nucl. Phys. **B662** (2003) 461 [arXiv:hep-th/0303056].
- [32] D. Binosi and L. Theussl, Comp. Phys. Comm. **161** (2004) 76.