

# EVOLUTION OF INDUCED AXIAL MAGNETIZATION IN A TWO-COMPONENT MAGNETIZED PLASMA

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(November 1, 2018)

In this paper, the evolution of the induced axial magnetization due to the propagation of an EM-wave along the static background magnetic field in a two-component plasma has been investigated using the Block equation. The evolution process induces a strong magnetic anisotropy in the plasma medium, depending non-linearly on the incident wave amplitude. This induced magnetic anisotropy can modify the dispersion relation of the incident EM-wave, which has been obtained in this paper. In the low frequency Alfvén wave limit, this dispersion relation shows that the resulting phase velocity of the incident wave depends on the square of the incident wave amplitude and on the static background magnetic field of plasma. The analytical results are in well agreement with the numerically estimated values in solar corona and sunspots.

52.30.-q, 52.40.-w, 52.35.Nx, 52.35.-g

## I. INTRODUCTION

The investigation of the propagation of electromagnetic waves is a long studied subject of plasma physics ( see [1] and references therein). In the traditional approach one studies low amplitude waves propagating in an uncorrelated plasma. Important information on the properties of plasmas in the linear response regime can be obtained from the knowledge of the dielectric tensor. The dielectric function of a magnetized uncorrelated plasma has been extensively studied by Horing [2]. The dispersion of low amplitude waves or the interaction of low intensity particle beams with plasmas may be studied by employing the dielectric function. Recently, the stopping power of an uncorrelated plasma has been investigated [3]. There have been two basic lines beyond the traditional investigations of electromagnetic modes propagation. One line considers the influence of correlation effects on the plasma dispersion relations. Recent papers are devoted to the study of the dielectric tensor of correlated magnetized plasmas and to the investigation of the electromagnetic mode dispersion in coupled magnetized plasmas [4,5]. The other line is aimed at the investigation of nonlinear effects in uncorrelated plasmas. A growing number of papers is dedicated to the study of the propagation of intense radiation in plasmas (recent works are cited in [6]).

One of the important area in these investigations is the generation of magnetic fields under the influence of electromagnetic (EM) waves [7]. One of the sources of the generation of induced magnetization is the inverse Faraday effect (IFE). The induced magnetization from IFE due to propagation of several waves in plasma, has been previously investigated (see Ref. [8–12] and references therein). This phenomenon arises from magnetic moment per unit volume of the ordered motion of charges of both signs, in the presence of an electromagnetic wave propagating in plasma [7,13]. This induced field must have axial as well as lateral component depending on the nature of the wave-wave and wave-particle interactions. For an elliptically polarized Alfvén wave propagating along the static background magnetic field in a two component plasma, this induced magnetization were found to be inversely proportional to the cube of the ambient magnetic field and the square of the incident wave amplitude, and acts in the direction of the incident wave propagation. Such effects are expected to be significant in the study of various processes in the sun and other stars, including pulsars. This effect may be demonstrated in laboratory plasmas.

In this paper, it has been shown that the zero harmonic magnetic moment generated from an elliptically polarized EM-wave along the direction of its propagation induces strong dc magnetic permeability depending non-linearly on the incident wave amplitude and acts in the same direction as the induced magnetization. Moreover, a small perturbation of the self-generated zero harmonic magnetic moment starts to evolve. This evolution can be investigated by using Block equation model [14]. In this paper this evolution has been studied and it has been shown that it induces strong magnetic anisotropy in the plane perpendicular to the direction of the incident wave propagation. This induced magnetic anisotropy is evident from the existence of non-vanishing off-diagonal elements of the magnetic permeability tensor, which also depend non-linearly on the incident wave amplitude.

In general plasma medium is not a magnetic material. However, the propagation of an incident EM wave in a plasma generates a self generated uniform magnetization  $\vec{M}_a$  which induces a strong magnetic permeability in the plasma medium depending nonlinearly on the incident wave amplitude. As magnetic permeability of a ferromagnet is very large, we can assume our resulting plasma medium as a weakly ferromagnetic medium in which magnetic permeability is large but not as large as for a ferromagnetic material medium. The self generated uniform magnetization may be considered as the ground state magnetization. Since we have considered long wave length excitations, a continuum theory is appropriate to study the evolution of small perturbation in the ground state magnetization.

Thus a weak ferromagnetic behavior of plasma is expected which can change the orientation of the bulk magnetization of the plasma that can reduce the mobility of electrons and ions and as a result the displacement current dominates over the conduction current [14]. This effect modifies the dispersion characteristics of the incident EM-wave. The dispersion relation of the incident EM-wave in the resulting plasma medium has been obtained in this paper. In the low frequency Alfvén wave limit, it has been seen that the phase velocity of the incident Alfvén wave in the resulting plasma medium depends on the static back-ground magnetic field of the plasma as well as on the square of the incident wave amplitude. As the induced magnetization is directly proportional to the square of the incident wave amplitude, the increase in the wave amplitude causes to increase the induced IFE magnetization. This pronounces the induced magnetic anisotropy, and ultimately inhibits the Alfvén wave propagation in the resulting plasma medium. These results have been verified numerically both in the Solar corona and Sunspots.

On the basis of this mechanism many authors have already developed a new mechanism of stabilization of stimulated Brillouin scattering (SBS) in laser produced plasmas, which is a consequence of the self-generated magnetic field in the SBS process [11,15]. In that case, a temporally exponentially growing zero harmonic magnetic field was generated in both axial and lateral directions. The lateral magnetic field was found to be responsible for the initiation of magnetic anisotropy in the plasma medium, which can exponentially reduce the phase velocities of incident and scattered light waves. However, for an elliptically polarized EM-wave propagating parallel to the static back-ground magnetic field, a zero harmonic induced axial magnetization is only generated. This axial magnetization in the ground state cannot induce magnetic anisotropy. The evolution of its linear perturbation induces magnetic anisotropy in the plasma medium, which has been investigated in this paper by using Block equation model.

In section 2, the dc magnetic permeability induced by the self-generated axial magnetic moment has been obtained. Its evolution has been studied in section 3. The effect of this evolution on the incident EM-wave is investigated in section 4. Section 5 describes these results in the low frequency Alfvén wave limit. Calculation of the magnetic moment induced by the Alfvén wave is given in the Appendix. Numerical estimation has been followed by discussion cited in section 6.

## II. DC MAGNETIC PERMEABILITY INDUCED BY SELF-GENERATED AXIAL MAGNETIC MOMENT

In the classical approximation, the bulk magnetization present in a magnetic material should be due to orbital angular momentum of charges, because of the distortion of orbital motion under the inference of EM fields [8–12,7,13]. When an EM wave propagates along the static background magnetic field, in a two component plasma, the magnetic moment is generated from the Inverse Faraday effect (IFE) mechanism along the  $z$ -direction,

$$\vec{M}_0 = M_0 \vec{z}, \quad (1)$$

which has been presented in the Appendix . This magnetic moment can be expressed in the form

$$\vec{M}_0 = (M_{0x}, M_{0y}, M_{0z}), \quad (2)$$

where  $M_{0x} = 0$ ,  $M_{0y} = 0$ , and

$$M_{0z} = -\frac{n_0 c}{2\omega} \sum_{s=e,i} \frac{q_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{(1 - Y_s^2)^2}, \quad (3)$$

with

$$\alpha_s = \frac{q_s a}{m_s \omega c}, \quad \beta_s = \frac{q_s b}{m_s \omega c}; \quad Y_s = \frac{\Omega_s}{\omega}; \quad \Omega_s = \frac{q_s H_0}{m_s c}, \quad (4)$$

$q_s, m_s, \Omega_s, (s = e, i)$  are charge, mass and cyclotron frequencies of electrons and ions, respectively,  $a$  and  $b$  are the amplitudes of the incident elliptically polarized EM wave,  $H_0$  is the static background magnetic field. The unperturbed

plasma density is given by  $n_0 = n_{0e} = n_{0i}$  and  $c$  is the velocity of light in vacuum,  $\omega$  is the frequency of the incident EM wave,  $\vec{z}$  is the direction of incident wave propagation.

Hence the induced magnetization is,

$$H_z^{in} = 4\pi M_{0z} = -\frac{4\pi n_0 c}{2\omega} \sum_{s=e,i} \frac{q_s(\alpha_s + Y_s \beta_s)(\beta_s + Y_s \alpha_s)}{(1 - Y_s^2)^2}, \quad (5)$$

which also acts along the direction of wave propagation. Substituting Eqs.(1,2,3) in the constitutive relation

$$\vec{B} = \hat{\mu} \vec{H}, \quad (6)$$

with

$$\vec{B} = \vec{H} + 4\pi \vec{M}; \quad \vec{H} = \vec{H}_0; \quad \vec{M} = \vec{M}_0; \quad \vec{\mu} = \vec{\mu}_0, \quad (7)$$

we obtain

$$H_0 + 4\pi M_{0z} = \mu_{0z} H_0, \quad (8)$$

and hence

$$\mu_{0z} = 1 - \frac{1}{2} \sum_{s=e,i} \frac{X_s (\alpha_s + Y_s \beta_s)(\beta_s + Y_s \alpha_s)}{Y_s (1 - Y_s^2)^2}, \quad (9)$$

and  $\alpha_s, \beta_s$  are the dimensionless amplitude of the incident EM wave, where  $X_s = \omega_{ps}^2/\omega^2$  and  $\omega_{ps}^2 = 4\pi q_s^2 n_0/m_s$  is the plasma frequency of s-th species of charges.

This shows that the zero harmonic magnetic moment  $\vec{M}_0 = M_{0z} \vec{z}$  induces a strong dc magnetic permeability  $\mu_{0z}$  depending nonlinearly on the incident wave amplitude and in the  $z$ -direction. Thus the resulting plasma medium behaves as a ferromagnetic medium with the IFE magnetization as the ground state magnetization. In the next section, the dynamics of this self-generated axial IFE magnetization will be studied.

### III. EVOLUTION OF SELF-GENERATED AXIAL MAGNETIC MOMENT IN A WEAKLY FERRO-MAGNETIC MEDIUM

From a macroscopic point of view, we may consider the ferromagnetic media as continua characterized by a magnetic moment density called magnetization. The ground state of a ferromagnet is of uniform magnetization at absolute zero temperature. A small disturbance in this magnetization will propagate in such a medium and this propagation can be studied by Bloch equation model [10],

$$\frac{d\vec{M}}{dt} = \frac{\gamma}{c} \left( \vec{M} \times \vec{H}_{eff} \right), \quad (10)$$

where  $\vec{M}$  is the bulk magnetization and  $\vec{H}_{eff}$  is the effective magnetic field in the medium,  $\gamma$  is the charge to mass ratio and  $c$  is the velocity of light in vacuum.

The propagation of an elliptically polarized EM wave in a two component magnetized plasma induces a zero harmonic axial magnetic moment from IFE, which generates a nonlinear magnetic permeability in the same direction. The plasma behaves as a weakly ferromagnetic medium and the induced magnetic moment  $\vec{M}_0$  acts as its bulk magnetization. This bulk magnetization is immediately perturbed and the resulting magnetization follows the equation

$$\frac{d\vec{M}_s}{dt} = \frac{\gamma_s}{c} \left( \vec{M} \times \vec{H}_{eff} \right), \quad (11)$$

where  $s=e$ (electron)/ $i$ (ion) and the effective magnetic field  $\vec{H}_{eff}$  is the sum of the background magnetic field  $\vec{H}_0$  and the 1<sup>st</sup> harmonic magnetic field  $\vec{H}_1$  of the incident em wave. Thus, we have

$$\vec{H}_{eff} = \vec{H}_0 + \vec{H}_1. \quad (12)$$

Moreover, the resulting magnetization,

$$\vec{M}_s = \vec{M}_{0s} + \vec{M}_{1s}, \quad (13)$$

where  $\vec{M}_{1s}$  is the linearized perturbation of the bulk magnetization  $\vec{M}_{0s}$ . Both  $\vec{M}_{1s}$  and  $\vec{H}_1$  satisfy the condition

$$|\vec{M}_{1s}| \ll |\vec{M}_{0s}|; \quad |\vec{H}_1| \ll |\vec{H}_0|. \quad (14)$$

$\vec{M}_{0s}$  being the zero harmonic magnetic moment from the orbital motion of sth species of charges given by

$$\vec{M}_{0s} = (0, 0, M_{0s_z}), \quad (15)$$

where

$$M_{0s_z} = -\frac{n_0 c q_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{2\omega (1 - Y_s^2)^2} \quad (16)$$

is independent of both space and time. Here,

$$\vec{M}_{1s} = (M_{1s_x}, M_{1s_y}, 0) \text{ and } \vec{H}_1 = (H_{1x}, H_{1y}, 0) \quad (17)$$

are the first order perturbations in  $\vec{M}_s$  and  $\vec{H}_{eff}$ , respectively. Using Eqs.(12,13) in (11) and linearizing we obtain

$$\frac{d\vec{M}_{1s}}{dt} = \frac{\gamma_s}{c} \left\{ \left( \vec{M}_{0s} \times \vec{H}_1 \right) + \left( \vec{M}_{1s} \times \vec{H}_0 \right) \right\}. \quad (18)$$

Substitution of (15) and (17) in the RHS of (18) gives

$$\dot{M}_{1s_x} = -\omega_{Ms} H_{1y} + \Omega_s M_{1s_y}, \quad (19)$$

$$\dot{M}_{1s_y} = \omega_{Ms} H_{1x} - \Omega_s M_{1s_x}, \quad (20)$$

where  $\omega_{Ms} = q_s M_{0s_z} / m_s c$  is the magnetization frequency of the sth species of charge particles, which depends on the induced magnetization  $M_{0s_z}$ . It is actually the frequency of gyration of charge particles about the lines of forces of the induced magnetic field  $\vec{M}_{0s}$ . Substituting  $M_{0s_z}$  from (16) in  $\omega_{Ms}$ , we obtain

$$\omega_{Ms} = -\frac{\omega X_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{8\pi (1 - Y_s^2)^2}. \quad (21)$$

From (19) and (20) we obtain the coupled differential equations,

$$(D^2 + \Omega_s^2) M_{1s_x} = -\omega_{Ms} \dot{H}_{1y} + \Omega_s \omega_{Ms} H_{1x}, \quad (22)$$

$$(D^2 + \Omega_s^2) M_{1s_y} = \omega_{Ms} \dot{H}_{1x} - \Omega_s \omega_{Ms} H_{1y}. \quad (23)$$

Since  $H_{1x}$ ,  $H_{1y}$  are the  $x$  and  $y$  components of the magnetic field  $\vec{H}_1$  of the incident EM wave, propagating along the  $z$ -direction, and hence  $M_{1s_x}$ ,  $M_{1s_y}$  are all proportional to  $\exp[i(kz - \omega t)]$ . Hence (22) and (23) can be written in the form

$$M_{1s_x} = \frac{\Omega_s \omega_{Ms}}{\Omega_s^2 - \omega^2} H_{1x} + \frac{i\omega \omega_{Ms}}{\Omega_s^2 - \omega^2} H_{1y}, \quad (24)$$

$$M_{1s_y} = \frac{-i\omega \omega_{Ms}}{\Omega_s^2 - \omega^2} H_{1x} + \frac{\Omega_s \omega_{Ms}}{\Omega_s^2 - \omega^2} H_{1y}, \quad (25)$$

or equivalently,

$$\vec{M}_{1s} = \hat{\chi}_s \vec{H}_1, \quad (26)$$

where

$$\hat{\chi}_s = \begin{pmatrix} \frac{\Omega_s \omega M_s}{\Omega_s^2 - \omega^2} & \frac{i\omega \omega M_s}{\Omega_s^2 - \omega^2} & 0 \\ \frac{-i\omega \omega M_s}{\Omega_s^2 - \omega^2} & \frac{\Omega_s \omega M_s}{\Omega_s^2 - \omega^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (27)$$

is the magnetic susceptibility tensor, whose non vanishing component are

$$\begin{aligned} \chi_{s_{xx}} = \chi_{s_{yy}} &= \frac{\Omega_s \omega M_s}{\Omega_s^2 - \omega^2}, \\ \chi_{s_{xy}} = \chi_{s_{yx}} &= \frac{\omega \omega M_s}{\Omega_s^2 - \omega^2}. \end{aligned} \quad (28)$$

Hence the net induced magnetic susceptibility of the resulting plasma medium is

$$\hat{\chi} = \hat{\chi}_e + \hat{\chi}_i = \sum_{s=e,i} \hat{\chi}_s. \quad (29)$$

Consequently the induced magnetic permeability of the medium becomes

$$\hat{\mu} = \hat{I} + 4\pi \sum_{s=e,i} \hat{\chi}_s, \quad (30)$$

where  $\hat{I}$  is the unit matrix of order 3. Substitution of (28) in (30) gives

$$\mu_{xx} = 1 + \sum_{s=e,i} \frac{4\pi \Omega_s \omega M_s}{\Omega_s^2 - \omega^2}, \quad (31)$$

$$\mu_{xy} = - \sum_{s=e,i} \frac{4\pi \omega \omega M_s}{\Omega_s^2 - \omega^2}, \quad (32)$$

and hence

$$\mu_{xx} = 1 + \frac{1}{2} \sum_{s=e,i} \frac{X_s Y_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{(1 - Y_s^2)^3}, \quad (33)$$

$$\mu_{xy} = -\frac{1}{2} \sum_{s=e,i} \frac{X_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{(1 - Y_s^2)^3}. \quad (34)$$

It is seen from the expression (33) and (34) that  $\mu_{xx}$  and  $\mu_{xy}$  depend on the square of the incident wave amplitude. The nonvanishing off-diagonal elements  $\mu_{xy}$  and  $\mu_{yx}$  indicate that a strong magnetic anisotropy is developed in the  $xy$ -plane, perpendicular to the direction of incident wave propagation (along the z-direction). This anisotropy is exclusively due to the evolution of the perturbation  $\vec{M}_1$  in a plasma medium, having weakly ferromagnetic properties.

#### IV. EFFECT OF MAGNETIC ANISOTROPY ON THE INCIDENT EM WAVE PROPAGATION

As we are studying magnetic moment dynamics in the plasma medium under the action of long range coulomb forces between the charge particles, classical theory is more appropriate than quantum mechanical theory because any disturbance propagates through a plasma medium with a wave length much greater than the atomic distances.

In this section we shall investigate how this induced magnetic anisotropy changes the dispersion characteristics of the incident elliptically polarized EM wave.

The induced magnetic anisotropy reduces the mobility of the charge particles. Hence the conduction current becomes negligible and the displacement current dominates over conduction current. Thus in such an insulated ferromagnet, the propagation of an EM wave obeys the following Maxwell equations

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad (35)$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t}, \quad (36)$$

with constitutive relations

$$\begin{aligned} \vec{D} &= \hat{\epsilon} \vec{E}, \\ \vec{B} &= \hat{\mu} \vec{H}, \end{aligned} \quad (37)$$

where  $\hat{\epsilon}$ ,  $\hat{\mu}$  are respectively the dielectric tensor and magnetic permeability tensor of the resulting plasma medium,  $\hat{\mu}$  has been already obtained in section 3. Since the plasma under consideration is initially magnetized, it has a dielectric anisotropy of the form

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_{xx} & -i\epsilon_{xy} & 0 \\ i\epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (38)$$

where

$$\begin{aligned} \epsilon_{xx} &= 1 - \sum_{s=e,i} \frac{X_s}{1 - Y_s^2}, \\ \epsilon_{xy} &= \sum_{s=e,i} \frac{X_s Y_s}{1 - Y_s^2}. \end{aligned} \quad (39)$$

Since the electric field  $E_{1x}$ ,  $E_{1y}$  and magnetic field  $H_{1x}$ ,  $H_{1y}$  of the incident electromagnetic wave contain the phase factor  $\exp[i(kz - \omega t)]$ , where  $\omega$  and  $k$  are its frequency and wave number, the Maxwell equations (35) and (36) together with the state relations (37) reduce to

$$n \begin{pmatrix} E_{1x} \\ E_{1y} \end{pmatrix} = \begin{pmatrix} i\mu_{xy} & \mu_{yy} \\ -\mu_{xx} & i\mu_{xy} \end{pmatrix} \begin{pmatrix} H_{1x} \\ H_{1y} \end{pmatrix}, \quad (40)$$

$$n \begin{pmatrix} H_{1x} \\ H_{1y} \end{pmatrix} = \begin{pmatrix} i\epsilon_{xy} & \epsilon_{yy} \\ -\epsilon_{xx} & i\epsilon_{xy} \end{pmatrix} \begin{pmatrix} E_{1x} \\ E_{1y} \end{pmatrix}. \quad (41)$$

Substituting  $E_{1x}$ ,  $E_{1y}$ ,  $H_{1x}$ ,  $H_{1y}$  from (A15) and (A16), we obtain

$$n^2 = (\epsilon_{xx} \pm \epsilon_{xy}) (\mu_{xx} \pm \mu_{xy}), \quad (42)$$

which is the modified dispersion relation of the incident EM wave in the anisotropic plasma medium. Substitution of  $\mu_{xx}$ ,  $\mu_{xy}$ ,  $\epsilon_{xx}$ ,  $\epsilon_{xy}$  from (33,34) and (39) in (42) gives

$$n^2 = \left( 1 - \sum_{s=e,i} \frac{X_s}{1 \pm Y_s} \right) \left( 1 \mp \frac{1}{2} \sum_{s=e,i} \frac{X_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{(1 \pm Y_s) (1 - Y_s^2)^2} \right), \quad (43)$$

where  $n = kc/\omega$  is the refractive index of the resulting plasma medium. This dispersion relation shows that the dispersion characteristics of the incident EM wave depend on the product of the incident EM wave amplitude. In the next section we shall investigate such characteristics for the case of Alfvén waves.

## V. ALFVEN WAVE APPROXIMATION

If the incident EM wave propagating in a two component plasma along the static background magnetic field is an Alfvén wave, the wave frequency satisfies the condition

$$\omega \ll \Omega_e, \Omega_i. \quad (44)$$

Under the approximation (44) the dispersion relation (43) reduces to

$$n_{\pm}^2 = \left( \frac{k^2 c^2}{\omega^2} \right)_{\pm} = \left( 1 + \frac{c^2}{c_A^2} \right) \left( 1 - \frac{c^2}{c_A^2} \frac{a^2 \mp ab + b^2}{H_0^2} \right), \quad (45)$$

where  $c_A^2 = H_0^2/4\pi n_0 m_i$  is the Alfvén velocity in the plasma. Thus we get two branches of mode propagation. In the case of a low amplitude wave,  $a, b \rightarrow 0$ , we obtain from Eq. (45) the dispersion relation of the ordinary Alfvén wave with only one branch. Consider the modifications of wave propagation caused by the nonlinearity. First, the nonlinearity produces a splitting of the Alfvén branch into two branches corresponding to the left elliptically polarized ( $a$  and  $b$  of equal sign) or to the right elliptically polarized Alfvén wave ( $a$  and  $b$  of different sign), respectively. Second, the phase velocity of the incident Alfvén wave in the resulting anisotropic plasma medium becomes

$$\left( \frac{\omega}{k} \right)_{\pm} = \pm \frac{c_a c}{\sqrt{c_A^2 + c^2}} / \sqrt{1 - \frac{c^2}{c_A^2} \frac{a^2 \mp ab + b^2}{H_0^2}}. \quad (46)$$

This shows that Alfvén wave can propagate with very large but finite phase velocity (46), if

$$\frac{c_A^2}{c^2} \frac{H_0^2}{a^2 \mp ab + b^2} > 1. \quad (47)$$

Only very long wave length Alfvén waves can propagate, if

$$\frac{c_A^2}{c^2} \frac{H_0^2}{a^2 \mp ab + b^2} \cong 1, \quad (48)$$

The first branch of the nonlinear Alfvén wave has a cutoff, if

$$\frac{c_A^2}{c^2} \frac{H_0^2}{a^2 + ab + b^2} < 1, \quad (49)$$

the second branch has its cutoff, if

$$\frac{c_A^2}{c^2} \frac{H_0^2}{a^2 - ab + b^2} < 1. \quad (50)$$

We see that for a given amplitude of the Alfvén wave the magnetic field strength  $H_0$  should exceed a certain threshold  $H_0 > (4\pi n_0 m_i c^2 (a^2 \pm ab + b^2))^{1/4}$  to make propagation of the Alfvén wave possible.

## VI. NUMERICAL ESTIMATION

In the solar corona, the ambient magnetic field is,  $H_0 = 10^{-2}$ G and the plasma mass density is,  $\rho = 10^{-16}$ g/cm<sup>3</sup>. Hence the Alfvén speed in a solar corona is  $c_A = 2.75 \times 10^5$ cm/sec. If an Alfvén wave of amplitude  $a = 10^{-8}$ esu propagates in the solar corona, the induced magnetic field would be of the order of  $10^{-4}$ G and the phase velocity of the incident wave in the resulting anisotropic plasma medium is very large but finite. This implies that only very long wave length waves can propagate in such a medium. For an incident wave amplitude  $a = 10^{-7}$ esu, the induced magnetization is  $10^{-2}$ G and the wave phase velocity  $\omega/k$  becomes infinite. Moreover, if the incident wave amplitude increases to a value  $10^{-6}$ esu, the induced magnetization becomes 1.13G and the phase velocity of the incident wave becomes imaginary and no further wave propagation is possible. The refractive index of nonlinear circular polarized Alfvén wave with amplitude  $a = b$  and propagating in the solar corona is shown in Fig. 1. With increasing wave amplitude the refractive indices of both nonlinear Alfvén branches decrease. The first branch ( $n_{\pm}^2$ ) has its cutoff at an amplitude  $a = 5.3 \cdot 10^{-8}$ esu, whereas the second branch ranges up to an amplitude of  $a = 9.1 \cdot 10^{-8}$ esu.

Similar results have also been obtained in sunspots, where  $H_0 = 3000$ G, mass density  $\rho = 10^{-5}$ g/cm<sup>3</sup> and Alfvén speed  $c_A = 2.7 \times 10^5$ cm/sec. In that case for  $a = 10^{-2}$ esu,  $H^{in} = 420$ G and  $\omega/k$  is very large but finite. For  $a = 10^{-1.5}$ esu,  $H^{in}$  becomes 4200G and  $\omega/k$  is infinite, and for  $a = 10^{-1}$ esu,  $H^{in} = 4.2 \times 10^4$ G,  $\omega/k$  is imaginary.

These numerical results confirm that the increase in the incident wave amplitude produces a magnetic anisotropy via the increasing induced magnetization in the plasma medium and consequently inhibits the Alfvén wave propagation.

## VII. DISCUSSION

From the results so far obtained in this paper, it is evident that for a strong EM wave propagating along the static back ground magnetic field in a two component plasma, the self generated zero harmonic axial magnetic moment starts to evolve. This evolution induces a strong magnetic anisotropy in the plasma medium and the medium consequently behaves as a weakly ferromagnetic medium with the zero harmonic magnetization as the ground state magnetization. This anisotropy inhibits the incident wave propagation in the resulting plasma medium. Moreover, as the wave amplitude increases, the anisotropy becomes strong and absorption of the wave by the medium is pronounced.

### APPENDIX A: CALCULATION OF THE MAGNETIC MOMENT FOR AN ELLIPTICALLY POLARIZED EM-WAVE IN A TWO-COMPONENT MAGNETIZED PLASMA

We consider the propagation of a transverse EM-wave in a two component cold magnetized plasma in which electrons and ions are both mobile. Collisions have been neglected. The basic equations describing such a plasma model, in the cold plasma limit are

$$\frac{\partial \vec{u}_s}{\partial t} + (\vec{u}_s \cdot \vec{\nabla}) \vec{u}_s = \frac{q_s}{m_s} \vec{E} + \frac{q_s}{m_s c} (\vec{u}_s \times \vec{H}), \quad (\text{A1})$$

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot (n_s \vec{u}_s) = 0, \quad (\text{A2})$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}, \quad (\text{A3})$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j}, \quad (\text{A4})$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi \rho, \quad (\text{A5})$$

$$\vec{\nabla} \cdot \vec{H} = 0, \quad (\text{A6})$$

where  $\rho = \sum_{s=e,i} n_s q_s$  and  $\vec{j} = \sum_{s=e,i} n_s q_s \vec{u}_s$  are the charge and current densities in the plasma. Assuming the plasma is initially quasi-static and quasi-neutral, such that

$$\vec{u}_{s0} = 0; \quad n_0 = n_{0e} = n_{0i}. \quad (\text{A7})$$

We linearize the field variables,

$$\vec{u}_s = \vec{u}_{s0} + \vec{u}_{s1}; \quad n_s = n_{s0} + n_{s1}; \quad \vec{E} = \vec{E}_0 + \vec{E}_1; \quad \vec{H} = \vec{H}_0 + \vec{H}_1, \quad (\text{A8})$$

where  $\vec{H}_0 = (0, 0, H_0)$  is the ambient magnetic field acting along the  $z$ -direction and  $\vec{u}_{s1}$ ,  $n_{s1}$ ,  $\vec{E}_1$ ,  $\vec{H}_1$  are the first order perturbations in the field variables about their equilibrium value.

Linearizing the basic equations (A1,A2,A3,A4,A5,A6) with the help of (A8), we obtain

$$\frac{\partial \vec{u}_{s1}}{\partial t} = \frac{q_s}{m_s} \vec{E}_1 + \frac{q_s}{m_s c} (\vec{u}_{s1} \times \vec{H}_0), \quad (\text{A9})$$

$$\frac{\partial n_{s1}}{\partial t} + \vec{\nabla} \cdot (n_{0s} \vec{u}_{s1}) = 0, \quad (\text{A10})$$

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{c} \frac{\partial \vec{H}_1}{\partial t}, \quad (\text{A11})$$



$$\vec{\nabla} \times \vec{H}_1 = \frac{1}{c} \frac{\partial \vec{E}_1}{\partial t} + \frac{4\pi n_0}{c} \sum_{s=e,i} q_s \vec{u}_{s1}, \quad (\text{A12})$$

$$\vec{\nabla} \cdot \vec{E}_1 = 4\pi \sum_{s=e,i} q_s n_{s1}, \quad (\text{A13})$$

$$\vec{\nabla} \cdot \vec{H}_1 = 0. \quad (\text{A14})$$

Let the first order electric field of the em wave that induces the perturbation in the plasma model be

$$\vec{E}_1 = (a \cos \theta, b \sin \theta, 0); \quad \theta = kz - \omega t, \quad (\text{A15})$$

where  $\vec{z}$  being the direction of propagation of the em wave. Substitution of (A5) in (A11) gives

$$\vec{H}_1 = n(-b \sin \theta, a \cos \theta, 0), \quad (\text{A16})$$

where  $n = kc/\omega$  is the refractive index of plasma,  $\omega$  and  $k$  are the frequency and wave number of the incident em wave. Solution of the linearized set of equations (A9,A10,A11,A12,A13,A14) with the help of (A15) and (A16) gives the first order perturbed velocity of the charged particles  $\vec{u}_{s1}$ , induced by the first order transverse em wave

$$\vec{u}_{s1} = (u_{s1x}, u_{s1y}, 0), \quad (\text{A17})$$

$$\vec{u}_{s1x} = -\frac{q_s}{m_s} \frac{\omega a + \Omega_s b}{\omega^2 - \Omega_s^2} \sin \theta, \quad (\text{A18})$$

$$\vec{u}_{s1y} = \frac{q_s}{m_s} \frac{\omega b + \Omega_s a}{\omega^2 - \Omega_s^2} \cos \theta. \quad (\text{A19})$$

After integrating (A18) and (A19), we obtain the first order wave induced displacement of the charge particles

$$\gamma_{s1x} = -\frac{q_s}{m_s \omega} \frac{\omega a + \Omega_s b}{\omega^2 - \Omega_s^2} \cos \theta, \quad (\text{A20})$$

$$\gamma_{s1y} = \frac{q_s}{m_s \omega} \frac{\omega b + \Omega_s a}{\omega^2 - \Omega_s^2} \sin \theta. \quad (\text{A21})$$

Hence the magnetic moment induced by the electrons and ion motion under the influence of the incident EM wave is

$$\vec{M} = \sum_{s=e,i} \vec{M}_s, \quad (\text{A22})$$

where

$$\vec{M}_s = \frac{1}{2c} \left( \vec{\gamma}_{s1} \times \vec{j}_{s1} \right), \quad (\text{A23})$$

with  $\vec{j}_{s1} = n_0 q_s \vec{u}_{s1}$ , is the first order perturbed current density due to the wave induced motion of charge particles. From (A18,A19,A20,A21,A22) we obtain

$$\vec{M} = (0, 0, M_z), \quad (\text{A24})$$

where

$$M_z = -\frac{n_0 c}{2\omega} \sum_{s=e,i} \frac{q_s (\alpha_s + Y_s \beta_s) (\beta_s + Y_s \alpha_s)}{(1 - Y_s^2)^2}, \quad (\text{A25})$$

where  $(\alpha_s, \beta_s) = q_s(a, b)/m_s\omega c$ ,  $Y_s = \Omega_s/\omega$ ,  $\Omega_s = q_s H_0/m_s c$ . Here  $q_s$  and  $m_s$  are charge and mass of the sth species of charge particles.

This is the induced magnetization from IFE generated from the distortion of the ordered motion of charge particles under the influence of incident EM wave. Eq. (A24) shows that this induced magnetic moment acts along the  $z$ -direction which is the direction of the incident wave propagation. This is the ground state magnetization  $\vec{M} = \vec{M}_0$  of the weakly ferromagnetic medium as discussed in this paper. Hence the induced magnetization  $\vec{H}^{in}$  is

$$\vec{H}^{in} = 4\pi\vec{M}. \quad (\text{A26})$$

Substituting (A24) and (A25) in (A26), we obtain

$$\vec{H}^{in} = (0, 0, H_z^{in}), \quad (\text{A27})$$

where,

$$H_z^{in} = -\frac{2\pi n_0 c}{\omega} \sum_{s=e,i} \frac{q_s(\alpha_s + Y_s\beta_s)(\beta_s + Y_s\alpha_s)}{(1 - Y_s^2)^2}. \quad (\text{A28})$$

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## Figure Captions

Fig. 1 Squares of refraction indices  $n_{\pm}^2$  of nonlinear left circular polarized Alfvén wave propagating in the solar corona vs. wave amplitude  $a$  (in esu): solid line -  $n_-^2$ , dashed line -  $n_+^2$ . Magnetic field strength is  $H_0 = 10^{-2}\text{G}$  and plasma mass density is  $\rho = 10^{-16}\text{g/cm}^3$ .

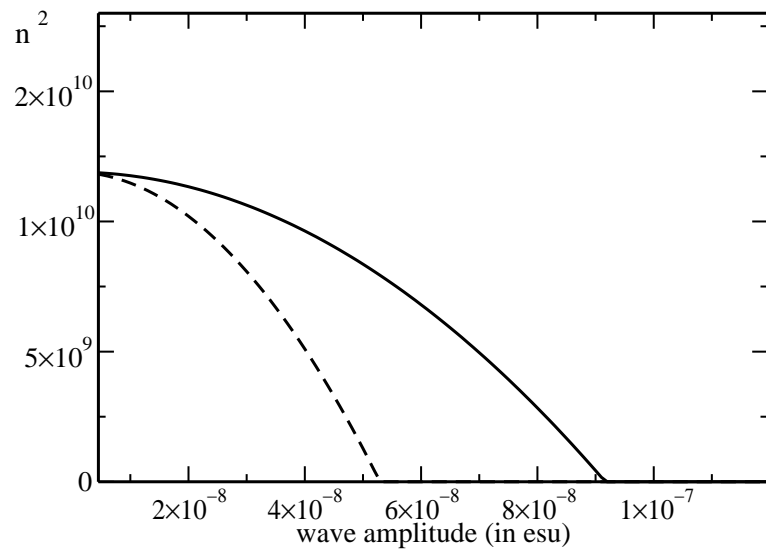


FIG. 1.