MRI-P-010801
hep-ph/0108150

Effects of lepton number violating interactions on $t\bar{t}$ production at NLC

Anindya Datta [1],
Harish-Chandra Research Institute,
Chhatnag Road, Jhusi, Allahabad - 211 019, India

Abstract

We discuss the effects of lepton number violating interactions namely, R-parity violation and leptoquarks on top-quark pair production at the upcoming e^+e^- linear colliders. Effects of $SU(2)$ singlet, doublet and triplet leptoquark interactions are investigated. R-parity violating minimal supersymmetric standard model also allows certain kinds of lepton number violating interactions which are same as singlet leptoquarks with left-handed interactions. We have calculated the cross-section of $e^+e^- \to t\bar{t}$ in presence of the above interactions. With conservative values of lepton number violating coupling strengths we got enhancement of top-pair production cross-section in all of the above cases.

1 Introduction

It is well known that lepton-number conservation in the Standard Model (SM) is an accidental symmetry. It is a mere outcome of particle content and gauge structure of the SM. In many extensions of the SM, lepton number violating interactions occur in a natural way. Minimal Supersymmetric Standard Model (MSSM) without R-parity [1] and non-SUSY theories with leptoquarks [2] are well cited examples of it. The key feature of these theories, relevant for the following analysis, is the presence, in their spectrum, of a scalar (leptoquark) which couples to a quark and a lepton at the same time. Leptoquarks arise in many models of extended gauge symmetry including the grand unified theories. In many of these models, vector leptoquarks can also arise. The gauged vector leptoquarks are superheavy. Their mass is related to the scale of spontaneous breaking of the lepton number. On the other hand, interactions involving the non-gauged vector leptoquarks are non-renormalizable. Several interesting phenomenological analyses have been done considering both of these interactions. In this article, we will focus on how these scalar leptoquark (a class

[1]E-mail: anindya@mri.ernet.in

that also includes the squarks in R-parity violating SUSY) interactions can modify the top-quark pair production cross-section significantly at the next generation e^+e^- colliders. The choice of this particular process has several advantages. The foremost is the copious production of top pairs at these machines. Also the cleaner environment of leptonic colliders helps one to make the precision studies like measuring such deviations more accurately, if exists, from the SM expectation. One of the major goals of these e^+e^- machines is to measure the top-quark interactions to a high level of precision [3]. Measurement of the lepton number violating couplings involving light quarks (mainly of first generation) and leptons can also be done at the hadron colliders by studying the processes like Drell-Yan pair production of the leptons. But at a hadronic machine, the couplings in which we will be interested in our analysis can only be probed in the decays of the heavy quarks. Though production cross-section of a heavy quark pair is huge at a hadron collider, presence of competing QCD backgrounds may intervene such precise measurements.

Baryon (B) and lepton number (L) violating processes involving the top-quark have been investigated by several authors. For example, effects of B- and L-violation in top-quark production at hadronic colliders have been analysed in ref. [4] in the context of R-parity violating SUSY. People have extensively studied the single top production [5] and decay of top-quark [6] mediated by R-parity violating interactions. Effects of R-parity violation on the top mass have been discussed in [7]. So a lot of attention has already been given to the top phenomenology [8] in the context of R-parity violation. Though leptoquark interactions has similarities with that of R-parity violating SUSY, in some of the cases the chiral structure of the relevant couplings differs from it. People also have payed a lot of attention to leptoquark phenomenology. Apart from direct leptoquark searches at future lepton and hadron colliders [9], effects of these interactions have been studied in the context of neutrinoless double beta decay [10], muon anomalous magnetic moment [11] and needless to mention, to explain the HERA anomaly [12]. Indirect effects of leptoquark interactions have also been investigated in the context of e^+e^-, $e\gamma$ and hadronic colliders [13]. In this paper we will try to discuss, in some detail, how these lepton number violating couplings can affect the pair production and decay of the heaviest quark. This has been studied previously in [14] in a slightly different manner. Using polarised electron beam in e^+e^- collision, constrains are derived, in the above reference, on leptoquark mass and couplings by comparing (and then doing a χ^2-analysis) the angular distribution of leptoquark mediated process with that of the pure SM. It was shown in this ref. that an 1 TeV, e^+e^- collider will be more efficient than a 500 GeV machine, in exploring/excluding the parameter space of leptoquark interactions. We will focus this point more on later. People have also considered the effects of vector leptoquarks on $t\bar{t}$ production from e^+e^- collision [15]. Authors in ref. [15] also used polarised e^- beams to differentiate the vector leptoquark interactions from the SM. They have presented the variation of total number of $t\bar{t}$ events with vector leptoquark mass assuming the leptoquark couplings to an e and t of the order of unity. Though the structures of the vector leptoquark interactions are different from those of the scalar leptoquarks, qualitatively the variation of production cross-section with leptoquark mass, agrees with us. In this article, we will concentrate on how the total cross-section would change in presence of such particles and how angular asymmetry in $t\bar{t}$ production and decay can be used to discriminate the different types of leptoquark interactions. Plan of the rest of the article is as follows. In the next section, we will discuss the models briefly with special emphasis on the relevant couplings and the similarities and differences in two models of our interest. The third section will contain the result of our analysis followed by a conclusion in the last section.

2 Relevant interactions

In this section we will discuss briefly the phenomenology of lepton number violating interactions in the context of $t\bar{t}$ production in e^+e^- collision. As we emphasised earlier, two main kinds of models which allow these interactions are MSSM with R-parity violation and non-SUSY theories with leptoquarks. As it has been noted in the literature, unless a discrete symmetry [1] is introduced by hand, the MSSM superpotential contains the following terms [16]:

$$W_{\not R} = \lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}^c_k + \lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}^c_k + \lambda''_{ijk}\hat{U}^c_i\hat{D}^c_j\hat{D}^c_k + \epsilon_i\hat{L}_i\hat{H}_2 \qquad (1)$$

However, such a symmetry is *ad hoc*. So it is of interest to consider possible violation of this symmetry especially when it has some interesting experimental consequences in detecting the supersymmetric particles [17]. One can easily see that the first two and the last term in the superpotential violate the lepton number/flavour explicitly while the third term breaks the baryon number. As we are interested in the top pair production in electron positron annihilation, we will be interested in the second term. One can expand this piece in terms of the normal fields. This in turn, yields (with many other) the relevant interactions involving a lepton and a quark along with a squark. One can easily write the interaction of our interest.

$$\mathcal{L}_{\lambda'} = -\lambda'_{13k}\,(\tilde{d}^k_R)^*(\bar{e}_L)^c t_L + h.c. \qquad (2)$$

Now we will turn our attention to the leptoquark interactions. The interactions necessary for our purpose are listed in a tabular form in the following [18].

Here we have suppressed the $SU(2)$ indices. One can very easily write the interactions relevant for our purpose involving e, t and a particular leptoquark from the above table. Below we write the interaction Lagrangians separately for singlet, doublet and triplet leptoquarks [2]

$$\mathcal{L}_1 \;=\; -\left[\lambda^{(1)}_{13}\,(\bar{e})^c\,P_L\,t + \tilde{\lambda}^{(1)}_{13}\,(\bar{e})^c\,P_R\,t\right]\phi_1 + h.c.$$

[1]This symmetry is called R-symmetry. R is defined as: $(-1)^{3(B-L)-2S}$. All the SM fields have $R = 1$ and all the SUSY partners have $R = -1$. Apart from ruling out both B and L violating interactions this symmetry has an additional consequence of rendering the lightest super-particle absolutely stable.

[2]In eqn. 3 and Table. 2, one should not confuse the λ couplings with that in eqn.2. The λ couplings here, have more similarities with the λ'-coupling in eqn. 1.

leptoquark Type	Coupling	$SU(3)_c \times SU(2)_L \times U(1)_Y$
Φ_1	$\left[\lambda_{ij}^{(1)} \bar{Q}_{Lj}^c L_{Li} + \tilde{\lambda}_{ij}^{(1)} \bar{u}_{Rj}^c e_{Ri}\right] \Phi_1$	$(\bar{3},\, 1,\, \frac{2}{3})$
Φ_2	$\left[\lambda_{ij}^{(2)} \bar{Q}_{Lj} e_{Ri} + \tilde{\lambda}_{ij}^{(2)} \bar{u}_{Rj} L_{Li}\right] \Phi_2$	$(3,\, 2,\, \frac{7}{3})$
Φ_3	$\lambda_{ij}^{(3)} \bar{Q}_{Lj}^c L_{Li} \Phi_3$	$(\bar{3},\, 3,\, \frac{2}{3})$

Table 1: *Different kinds of leptoquark interactions relevant for our analysis. R-parity violating MSSM interaction in eqn.2 corresponds to the left-handed (proportional to $\lambda_{13}^{(1)}$) interaction of Φ_1.*

$$\begin{aligned} \mathcal{L}_2 &= \left[\lambda_{13}^{(2)}\, \bar{t}\, P_L\, e - \tilde{\lambda}_{13}^{(2)}\, \bar{t}\, P_R\, e\right] \phi_2 + h.c. \\ \mathcal{L}_3 &= \lambda_{13}^{(3)}\, (\bar{e})^c\, P_L\, t\, \phi_3 + h.c. \end{aligned} \tag{3}$$

There are some similarities and differences between the above interactions and that in eqn. 2. The triplet and the left-handed singlet (proportional to $\lambda_{13}^{(1)}$) have similar structures to the R-parity violating interaction. Charges of the leptoquarks in such cases are also the same with that of the squark involved in eqn.2. At the same time ϕ_1 has a coupling with e and t which is right-handed in nature. This type of interaction is not allowed in SUSY. $SU(2)$ doublet leptoquark ϕ_2 has a similar kind of interactions like ϕ_1. The only difference is it's electromagnetic charge which is equal to $\frac{5}{3}$.

The operators, those will contribute to the top-quark pair production via e^+e^- annihilation, follow very easily from the Lagrangian. They are given in Table 2.

Apart from SM s-channel diagram (mediated by γ or Z), one has to calculate an extra diagram mediated by the squark or leptoquarks (see fig.1) due to these lepton-number violating interactions. Looking at the Lagrangians, one can easily check that in R-parity violating contribution, one vertex is proportional to P_L and the other is proportional to P_R. While in the leptoquark mediated contributions P_L or P_R can arise in both the vertices.

For the sake of completeness, we write down the expressions for the amplitudes, arising due to different types of interactions listed in Table. 2, along with the SM.

$$\begin{aligned} \mathcal{M}_{SM} &= -\frac{1}{s - m_V^2 + i\, m_V\, \Gamma_V}\; (\bar{v}(p_1)\, \gamma_\mu\, (a_e + b_e \gamma_5)\, u(p_2))\; (\bar{u}(p_3)\, \gamma^\mu\, (a_t + b_t \gamma_5)\, v(p_4)) \\ \mathcal{M}_{LQ}^{S/T} &= \frac{|\lambda|^2, |\tilde{\lambda}|^2, \lambda\tilde{\lambda}}{t - m_\phi^2}\; (\bar{u}(p_3)\; P_i\; u(p_1))\; (\bar{v}(p_2)\; P_j\; v(p_4)) \\ \mathcal{M}_{LQ}^{D} &= \frac{|\lambda|^2, |\tilde{\lambda}|^2, \lambda\tilde{\lambda}}{t - m_\phi^2}\; (\bar{u}(p_3)\; P_i\; u(p_2))\; (\bar{v}(p_1)\; P_j\; v(p_4)) \end{aligned} \tag{4}$$

h

1.	Squark, Singlet/Triplet-leptoquark (Left-handed)	$\lvert\lambda_{13}^{(i)}\rvert^2\,(\bar{t}P_R e^c\ \ \bar{e}^c P_L t)\,\widehat{\phi_i\phi_i^*}$ (**RL**)
2.	Singlet-leptoquark (Right-handed)	$\lvert\tilde{\lambda}_{13}^{(i)}\rvert^2\,(\bar{t}P_L e^c\ \bar{e}^c P_R t)\,\widehat{\phi_i\phi_i^*}$ (**LR**)
3.	Singlet-leptoquark (Right-Left)	$\lvert\lambda_{13}^{(1)}\tilde{\lambda}_{13}^{(1)}\rvert\,(\bar{e}^c P_\alpha t\ \bar{t}P_\alpha e^c)\,\widehat{\phi_1\phi_1^*}$ $\alpha = L, R$ (**LL, RR**)
4.	Doublet-leptoquark (Left)	$\lvert\lambda_{13}^{(2)}\rvert^2\,(\bar{t}P_L e\ \bar{e}P_R t)\,\widehat{\phi_2\phi_2^*}$ (**LR**)
5.	Doublet-leptoquark (Right)	$\lvert\tilde{\lambda}_{13}^{(2)}\rvert^2\,(\bar{t}P_R e\ \bar{e}P_L t)\,\widehat{\phi_2\phi_2^*}$ (**RL**)
6.	Doublet-leptoquark (Right-Left)	$\lvert\lambda_{13}^{(2)}\tilde{\lambda}_{13}^{(2)}\rvert\,(\bar{t}P_\alpha e\ \bar{e}P_\alpha t)\,\widehat{\phi_2\phi_2^*}$ $\alpha = L, R$ (**LL, RR**)

Table 2: *Different types of operators contributing to the process $e^+e^- \to t\bar{t}$, made out of interactions in eqns. 2,3. R-parity violating MSSM corresponds to case 1. For the first two cases i can be 1 or 3.*

The first ($\mathcal{M}_{SM}$) of the above equations stands for the two SM s-channel diagrams. For the photon- exchange diagram, $b_e = b_t = 0$, $a_e = -e, a_t = \frac{2}{3}e$ and $m_V = \Gamma_V = 0$. For the Z-exchange diagram, $a_e = \frac{g}{\cos\theta_W}\,(-\frac{1}{4} + \sin^2\theta_W)$, $b_e = \frac{g}{4\cos\theta_W}$, $a_t = \frac{g}{\cos\theta_W}\,(\frac{1}{4} - \frac{2}{3}\sin^2\theta_W)$, $b_t = -\frac{g}{4\cos\theta_W}$. Next two expressions, $\mathcal{M}_{LQ}^{S/T}$ and $\mathcal{M}_{LQ}^{D}$, are for singlet/triplet and doublet leptoquark mediated diagrams respectively. p_1, p_2, p_3 and p_4 are the momenta of e^+, e^-, t and $\bar{t}$. The Mandelstum variables are defined as: $s = (p_1 + p_2)^2$ and $t = (p_1 - p_3)^2$ for singlet/triplet and $t = (p_2 - p_3)^2$ for doublet leptoquarks. Amplitudes for leptoquark mediated diagrams, are proportional to $|\lambda|^2$ when $P_i = P_R$, $P_j = P_L$; to $|\tilde{\lambda}|^2$ when $P_i = P_L$ and $P_j = P_R$ and to $\tilde{\lambda}\lambda$ when both are P_L or P_R. Following the tables 1 and 2, triplet leptoquark contribution can only be proportional to $|\lambda|^2$. The other cases do not arise for the triplet leptoquark mediation.

Now let us discuss the experimental bounds on the relevant couplings. The R-parity violating contribution is proportional to the coupling λ'_{13k}, where k is the generation index. We will consider only one R-parity violating coupling to be non-zero at a time. Looking at the literature [19], one can check easily that the coupling λ'_{132} is the most loosely constrained [16][3]. So we will use this particular coupling in the following analysis. This implies the exchanged squark in fig.1 is the supersymmetric partner of s-quark. The same constrains would also exactly apply on the left-handed singlet ($\lambda_{13}^{(1)}$) and triplet ($\lambda_3^{(13)}$) leptoquark couplings to e and t. The product of the

[3]This particular coupling is constrained from the forward-backward asymmetry in e^+e^- collision.

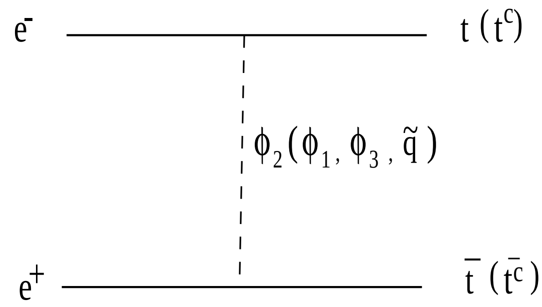

Figure 1: *Feynman diagram for the process* $e^+e^- \to t\bar{t}$ *in* R_p *violating SUSY or leptoquarks.*

couplings $\lambda_{13}^{(i)}\tilde{\lambda}_{13}^{(i)}$ in these two cases are unconstrained. The $SU(2)$ doublet leptoquark couplings $\lambda_{13}^{(2)}$, $\tilde{\lambda}_{13}^{(2)}$ (left and right) are individually constrained from the e^+e^- partial decay width of Z-boson [20]. It is interesting to observe that the left-handed couplings are more stringently constrained than their right-handed counterparts. Numerical values of the upper bounds on the left-handed couplings of the $SU(2)$ doublet leptoquarks are comparable with the upper bounds obtained for R-parity violating coupling strengths. There is no upper bound on the product of the left- and right-handed leptoquark couplings. So we may take their values as free parameter, keeping in mind that the value should be perturbatively viable.

3 Discussion of the results

We will discuss, in this section, the numerical results from our analysis. We have only estimated the Born level diagrams corresponding to the operators in Table 2. All the coupling constants scale with the scalar mass. In the case of R-parity violation, λ'_{132} scales linearly with $\tilde{t}_L$ mass. This particular coupling is constrained to be less than 0.28 for a 100 GeV $\tilde{t}_L$ mass [19]. As we discussed earlier, this bound equally applies to the left-handed singlet and triplet leptoquark couplings. We will also use the same values for $\tilde{\lambda}_{13}^{(1)}$ $(= 0.3)$ and the product $\tilde{\lambda}_{13}^{(1)}\lambda_{13}^{(1)}$ $(= 0.09)$ as there are no phenomenological bounds available for those. Again for numerical values of the couplings involving doublet leptoquarks we follow the ref. [20]. For a 100 GeV scalar, upper bound for left- $(\lambda_{13}^{(2)})$ or right-type $(\tilde{\lambda}_{13}^{(2)})$ coupling is almost the same (and is nearly equal to 0.4). While the upper bound on the $\lambda_{13}^{(2)}$ coupling is not very sensitive to leptoquark mass, upper bound on the other one rises pretty fast with the scalar mass. We will use the same values as before (like singlet and triplet leptoquark) for these couplings which makes our estimate conservative.

In figure. 2 and 3 we present the numerical estimates of the cross-sections. We do not consider any higher order corrections to the process of our interest. Higher order corrections are important [21]. In the case of SM, inclusion of higher order effects increase the cross-section significantly. The aim of this paper is to show the enhancement of the total cross-section (of $t\bar{t}$ production) over its SM value, when one includes the lepton number violating interactions arising from leptoquarks or

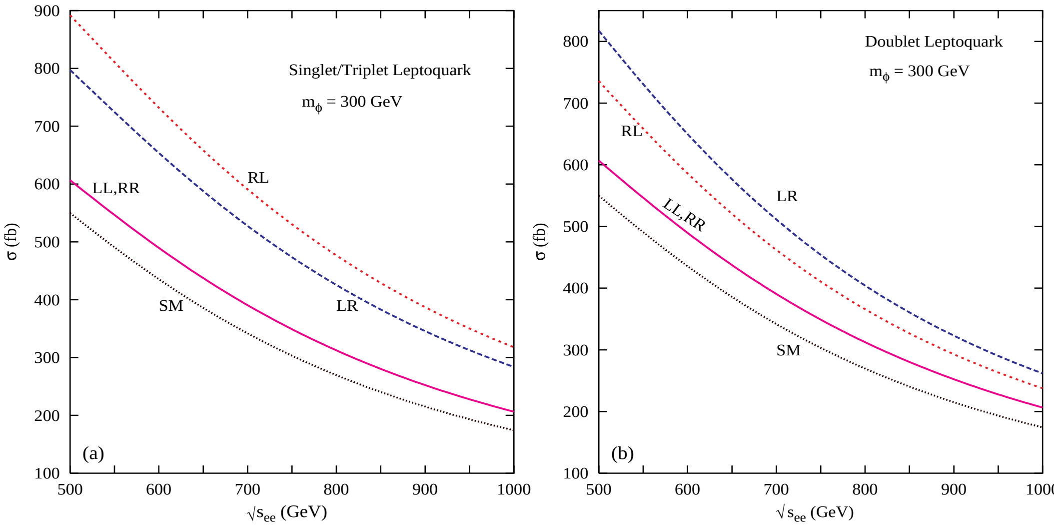

Figure 2: *Variation of top-pair production cross-section (in presence of (a) singlet, triplet leptoquarks, R-parity violating interactions and (b) doublet leptoquarks) with* e^+e^- *centre-of-mass energy,* $\sqrt{s_{ee}}$*. Leptoquark mass* (m_ϕ) *is fixed at 300 GeV. For comparison we have also plotted the pure SM contribution. Different lines are for different kinds of interactions. Legends follow from the Table 2. The curve marked by RL in (a) corresponds to R-parity violating SUSY.*

R-parity violation. We have calculated the cross-section at centre-of-mass energies away from the $t\bar{t}$ threshold. Around the centre-of-mass energy of 350 GeV ($\sim 2\,m_t$), threshold effects are very important [22]. And we wanted to avoid this extra complicacy. But this does not reduce the very essence of our analysis.

In figure 2a., we plotted variation of the total cross-section of top pair production with centre-of-mass energy for singlet and triplet leptoquarks. For the purpose of illustration, we present the cross-section with one value of scalar mass (say 300 GeV, which is well above the bounds quoted by CDF and D0 [23] from Tevatron search limits for squarks and leptoquarks.) and setting the value of all ($\lambda_i, \tilde{\lambda}_i, i = 1, 2, 3$) the couplings at say 0.3. There are several cases of interest, following Table 2. The LL and RR types of interactions do not interfere with the SM contribution. It is also worth mentioning here that LL, RR and LR lines in fig.2a come from the singlet leptoquarks only. The others, namely LR and RL, interfere constructively with the SM. For comparison, we plotted the pure SM contribution as well. It is clear from the figures that presence of any one kind of lepton number violating interactions increase the $t\bar{t}$ cross-section over its SM value. It is worth mentioning that the R-parity violating MSSM contribution corresponds to the RL case of figure 2a. Incidentally, this case shows the maximum enhancement. MSSM with or without R-parity conservation is one of the strongest contender of physics beyond the SM which we expect to see at the next generation of colliding machines. So any enhancement of top cross-section at e^+e^-

linear colliders may be a positive signal of this kind of scenario. The LR case is also interesting to observe. Here also the enhancement is pretty prominent. Finally the LL or RR, which can only arise from leptoquark interactions (this is also true for LR case), enhances the total cross-section by 10% or so over the entire range of centre-of-mass energy we have considered.

Plots in Fig. 2(b) are for doublet leptoquarks. Structure of the interactions, here, are little different from that of the singlet case. Otherwise one easily see, comparing the figures 2(a) and (b) that contributions are nearly the same for both the cases. Here again the LL or RR type of interactions do not interfere with the SM. Enhancement of $t\bar{t}$ cross-section is also exactly the same in magnitude as the singlet case with LL or RR interaction.

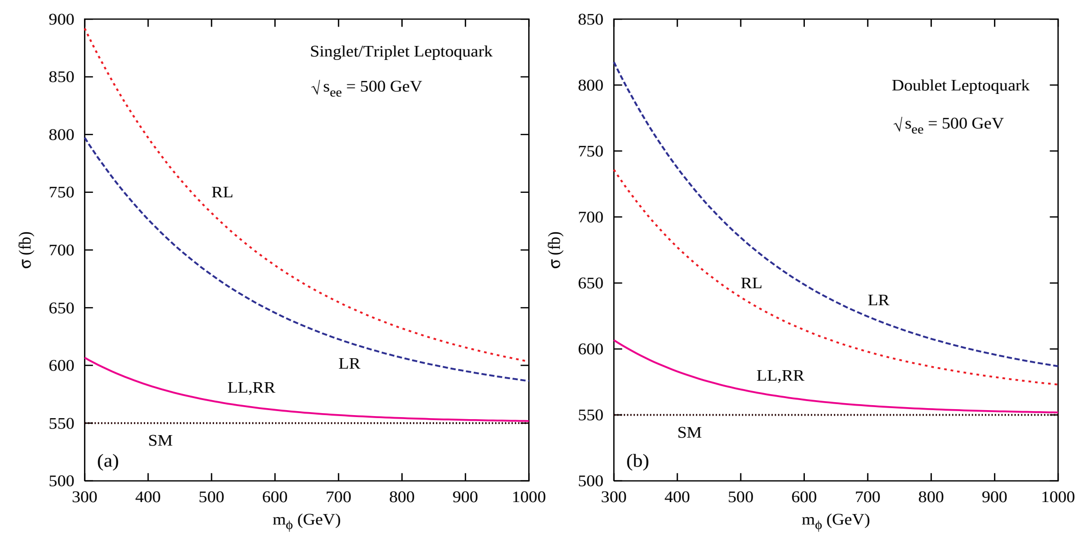

Figure 3: *Variation of top-pair production cross-section (in presence of (a) singlet, triplet leptoquarks, R-parity violating interactions and (b) doublet leptoquarks) with leptoquark mass (m_ϕ). For comparison we have also plotted the pure SM contribution which is independent of m_ϕ . e^+e^- centre-of-mass energy, $\sqrt{s_{ee}}$, is fixed at 500 GeV. Different lines are for different kinds of interactions. Legends follow from the Table 2. The curve marked by RL in (a) corresponds to R-parity violating SUSY.*

Now let us consider the variation of the cross-section with leptoquark mass. For this purpose, we fixed the centre-of-mass energy of the e^+e^- system at 500 GeV. One can easily see that the leptoquark (or squark) mass acts as the scale of the new physics we are interested in. This particular feature is reflected in Fig. 3(a) and (b) where we plotted the total cross-section with m_ϕ. As m_ϕ increases all the cross-sections are converging to the SM value, indicating the decoupling nature of the leptoquark interactions at higher energies.

From the above discussions, it is evident that the presence of lepton number violating couplings may enhance the total rate of top-quark pair production in electron positron annihilation. Absence

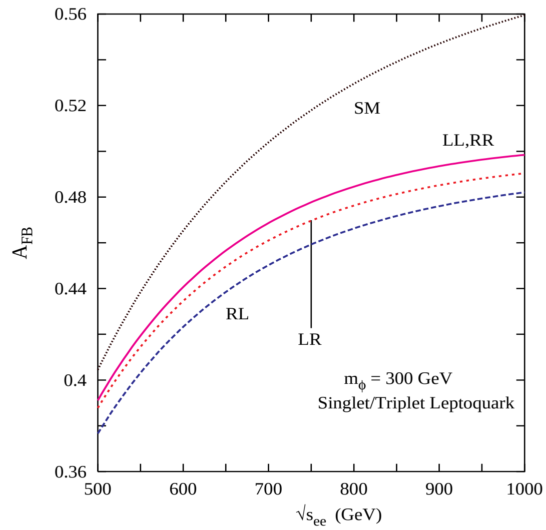

Figure 4: *Variation of forward-backward asymmetry, A_{FB}, (in presence of singlet, triplet leptoquarks, R-parity violating interactions) with* e^+e^- *centre-of-mass energy . For comparison we have also plotted the pure SM contribution. Leptoquark mass is fixed at 300 GeV. Different lines are for different kinds of interactions. Legends follow from the Table 2. The curve marked by RL corresponds to R-parity violating SUSY.*

of any such increase in $t\bar{t}$ cross-section at the future e^+e^- machines would help us to constrain the parameter space of the theories which allow such interactions. As we emphasised, there can be several types of such interactions. Now it is important to consider how one can differentiate those if at any experiment such an enhancement is detected. Different chiral structures of the interactions point to the fact that angular distribution may be helpful. The most useful signal of top-quark pair production comes from when one top decays semi-leptonically and the other decays hadronically. The cleaner environment of an electron positron collider enables us to reconstruct the scattering angle from the hadronically decaying top. So we have tried to compare the angular distributions of pure SM case with the leptoquark case. At lower $\sqrt{s}_{ee}$, there is a very little difference between these cases. At higher centre-of-mass energies ($\sim 1\ TeV$), angular distribution in leptoquark cases become less (than the SM) asymmetric in $\cos\theta$ (θ is the scattering angle). To quantify this we calculate the forward backward asymmetry, A_{FB}, defined as,

$$A_{FB} = \frac{\sigma_B - \sigma_F}{\sigma_B + \sigma_F} \tag{5}$$

where $\sigma_B = \int_{-1}^{0} \frac{d\sigma}{d(\cos\theta)}\, d(\cos\theta)$ and $\sigma_F = \int_{0}^{1} \frac{d\sigma}{d(\cos\theta)}\, d(\cos\theta)$.

We have plotted this asymmetry with e^+e^- centre-of-mass energy in fig. 4 for the singlet/triplet leptoquark interactions, along with the SM. As expected, for the SM, A_{FB} grows with centre-of-mass energy. From the figure it is evident, though at lower energies, A_{FB} for all the four cases remain very close to the SM value, at higher energies angular distributions for leptoquark mediated cases become less asymmetric. This in turn reduces A_{FB} in all these cases from the SM value. Forward-backward asymmetries for LL and RR cases come out to be equal. At higher energies also, values of A_{FB}, for different kinds of leptoquark interactions remain very close to each other. So one needs a large number of clean background free events (which looks possible in the next generation e^+e^- machines) to differentiate these scenarios. Once again we will try to compare our results with that obtained in ref. [14] in a qualitative manner. According to this work, an 1 TeV electron positron collider will explore a larger area in leptoquark parameter space than a 500 GeV machine. When one looks at the total cross-sections (see fig. 2a and fig. 2b), one can see that at higher centre-of-mass energies, differences between the SM cross-section and that of different leptoquark (+ SM) mediated processes are less than the differences at lower energies. But when we look at the forward backward asymmetries at different energies, it is evident at higher energies the differences between the SM case and leptoquarks are higher than those evaluated at smaller centre-of-mass energies. So comparison of the forward backward asymmetry (which is also the reflection of the angular distribution of the processes) will be more efficient at higher energies to discriminate the leptoquark models from the SM which is in consonance with the results in ref. [14].

For the doublet leptoquarks, there are no qualitative differences in A_{FB} from the singlet case. Numerically, for different types of doublet leptoquark interactions (LL, RL, LR *etc.*) A_{FB} differ very little from the corresponding singlet/triplet cases. We do not present them here.

Finally we want to make some comments about the top-quark decay mediated via these new interactions. As we assume this particular coupling (involving e, t and a scalar leptoquark, *i.e.* λ'_{132}, $\lambda_{13}^{(i)}$ or $\tilde{\lambda}_{13}^{(i)}$) to be non-zero, top-quark decay width to $b\, e\, \nu_e$ could also be modified. We have not written the relevant interactions involving a b-quark, a neutrino and a leptoquark. Looking at the interactions in ref. [18], one can easily check that this particular decay cannot be mediated via the $SU(2)$ doublet leptoquarks. Operators (apart from SM contribution mediated by W-boson) contributing to this process can be written as;

$$\begin{aligned} Singlet/Triplet &: \ |\lambda_{13}^{(i)}|^2 \ (\bar{e}^c\, P_L\, t) \ (\bar{\nu}_e\, P_R\, b^c)\, \widehat{\phi_i \phi_i^*} \\ Singlet &: \ \lambda_{13}^{(1)} \tilde{\lambda}_{13}^{(1)} \ (\bar{e}^c\, P_R\, t) \ (\bar{\nu}_e\, P_R\, b^c)\, \widehat{\phi_1 \phi_1^*} \end{aligned} \quad (6)$$

R-parity violating SUSY corresponds to the first one of eqn. 6. There can be other decay modes, but as long as we confine to the specific coupling (which we have used so far) this is the only one. We have calculated the decay widths corresponding to the cases in eqn. 6. With the values of couplings and leptoquark masses we have used before, the width comes out to be very nearly equal to the SM value. This looks surprising because with the same values of the parameters we get pretty good enhancement in $t\bar{t}$ production. The smallness of new-physics contribution can be attributed to the fact that, dominant contribution to the amplitudes, corresponds to eqns. 6,

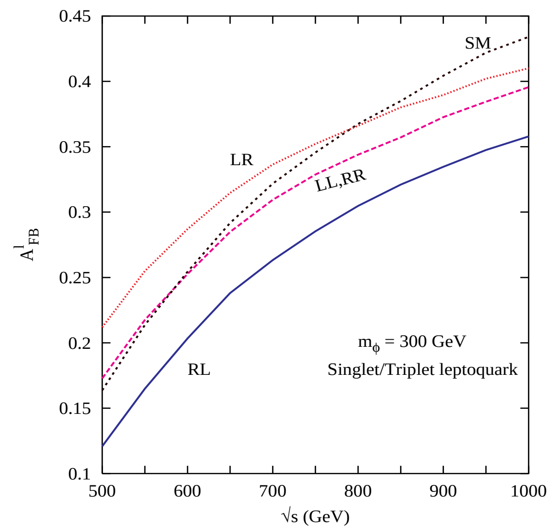

Figure 5: *Variation of* e^+ *(coming from t-quark decay) forward-backward asymmetry,* A^l_{FB}*, (in presence of singlet, triplet leptoquarks, R-parity violating interactions) with* e^+e^- *centre-of-mass energy . For comparison we have also plotted the pure SM contribution. Leptoquark mass is fixed at 300 GeV. Different lines are for different kinds of interactions. Legends follow from the Table 2. For top decay we have used the first of eqn. 6. The curve marked by RL corresponds to R-parity violating SUSY.*

are proportional to $m_t\, m_b$ when in the case of the top-pair production these are proportional to m_t^2. So the top semi-leptonic branching ratio (to electron) is barely changed in the presence of these new interactions, unless the couplings are big enough.

Operators responsible for top-quark decay (eqn. 6) have a distinctly different structure form the SM case. Though the total width shows a little enhancement over the SM value, it would be interesting to see how the angular distribution of the decay products differ from the later. As we pointed out, the cleanest signal for top pair production comes from when one top decays semileptonically and the other decays hadronically. We have calculated the angular distribution of the e^+ coming from the *top* decay keeping the full spin correlation between the top production and decay, in presence of leptoquark interactions as well as the SM. From the angular distribution one can easily calculate forward backward asymmetry of the e^+ (A^l_{FB}). For the purpose of illustration, we have presented the result of our analysis for singlet/triplet leptoquarks in fig 5. We have chosen the first one of eqn. 6 to calculate the top-quark decay matrix element. Fig 5 clearly shows the difference in A^l_{FB} between the SM and leptoquarks interaction over the energy range we have considered. Despite of the fact that these new interactions (with the coupling strength we have considered) could not change the top semileptonic branching ratio to a significant extent, angular asymmetries still play a crucial role in discriminating these effects from the SM. With the ballpark values of the $t\bar{t}$ cross-sections at these energies (see fig. 2a) and with the projected e^+e^- luminosities, one can easily detect these asymmetries. A comparison of fig. 5 with fig. 4, reveals that the A_{FB} in $t\bar{t}$ production differs from that of A^l_{FB} over the whole range of centre-of-mass energy. This can be accounted by the chiral structure of decay matrix element which plays a crucial role in determining the angular distribution of the top decay products.

R-parity violating MSSM allows the t-quark to decay to left-handed selectron ($\tilde{e}_L$) and a b-quark via the same λ'_{132} coupling. $\tilde{e}_L$ will in turn decay to a electron and to the lightest neutralino ($\tilde{\chi}^0_1$). $\tilde{\chi}^0_1$ is no longer stable and would decay to s-quark, ν_e and b-quark. This has been discussed in detail in ref. [24]. This decay will lead to 3 jets (including one b-quark), an electron and missing energy originating from a neutrino. So R-parity violation can be separated out from non-SUSY leptoquarks by this kind of top-decay signals.

4 Conclusion

To summarise, we show that presence of lepton number violating interactions can enhance the top-quark pair production cross-section in electron positron annihilation at next generation linear collider machines. We have considered different kinds of leptoquark interactions. R-parity violating interactions, involving one lepton, and two quark superfields, belong to one of these above cases. Non-SUSY theories with leptoquarks allow both left and right handed couplings involving a scalar leptoquark, a top-quark and an electron. We have estimated the cross-sections in all the cases separately. With moderate values of these lepton number violating Yukawa couplings one gets pretty good enhancement of total cross-section over the SM value. Depending on the e^+e^- centre-of-mass energy and leptoquark mass, enhancement varies from a few percent to 60%. With higher values of leptoquark mass cross-section converges to the SM value. This clearly points to the fact that these interactions are decoupling in nature at higher energies. We have also considered the effects of this coupling on the top semi leptonic decay. Top-decay width changes very little after inclusion of these new interactions. Forward-backward asymmetry in top pair production and top-decay may be used to differentiate these lepton number violating interactions from the SM and among themselves at higher centre-of-mass energies. But this will need a large sample of $t\bar{t}$ events which looks feasible at the next generation e^+e^- linear colliders.

Acknowledgement

The author thanks A. Raychaudhuri for carefully reading the manuscript and P. Konar for computational help. Thanks are also due for D. Choudhury for pointing me out some earlier important works in this direction and for many useful suggestions regarding this work.

References

[1] C.S. Aulakh, R.N. Mohapatra, Phys. Lett. **B119** 136 (1982); L.J. Hall, M. Suzuki, Nucl. Phys. **B231** 419 (1984); G. Ross, J.W. F. Valle Phys. Lett. **B151** 375 (1985); S. Dawson Nucl. Phys. **B261** 297 (1985); H. Dreiner in *Perspective in Supersymmetry* ed., G. Kane, World Scientific, hep–ph/9707435.

[2] See, for example, W. Buchmüller, D. Wyler, Phys. Lett. **B177** 377 (1986); W. Buchmüller, R. Rückl, D. Wyler, Phys. Lett. **B191** 442 (1987); Erratum: *ibid.***B448** (1999) 320; S. Davidson, D. Bailey, B. Campbell Z. Phys. C**61** 613 (1994).

[3] H. Murayama, M. Peskin, Ann. Rev. Nucl. Part. Sci. **46** 533 (1996); E. Accomando et al., Phys. Rep. **299** 1 (1998).

[4] D. K. Ghosh, S. Raychaudhuri, K. Sridhar, Phys. Lett. **B396** 177 (1997); E. L. Berger, B.W. Harris, Z. Sullivan, Phys. Rev.**D63** 115001 (2001).

[5] A. Datta et al., Phys. Rev.**D56** 3107 (1997); U. Mahanta, A. Ghosal, Phys. Rev.**D57** 1735 (1998); R. J. Oakes et al., Phys. Rev.**D61** 534 (1998); P. Chiappetta et al., Phys. Rev.**D61** 115008 (2000); M. Chemtob, G. Moreau, Phys. Rev.**D61** 116004 (2000).

[6] K. Agashe, M. Graesser, Phys. Rev.**D54** 4445 (1996); J. M. Yang, B. -L. Young, X. Zhang, Phys. Rev.**D58** 055001 (1998); F. de Campos et al., hep–ph/9903245; S. bar-Shalom, G. Eilam, A. Soni, Phys. Rev.**D60** 035007 (1999); T. Han, M. Magro, Phys. Lett. **B476** 79 (2000); G. Eilam et al.; Phys. Lett. **B510** 227 (2001); K. J. Abraham, Phys. Rev.**D63** 034011 (2001).

[7] M. Carena, C. Wagner, Phys. Lett. **B186** 361 (1987).

[8] H. Dreiner, R.J.N. Phillips, Nucl. Phys. **B367** 591 (1999); L. Navarro, W. Porod, J. W. F. Valle, Phys. Lett. **B459** 615 (1999).

[9] M. Doncheski, S. Godfrey, Phys. Rev.**D49** 6220 (1994); M. Doncheski, S. Godfrey, Phys. Rev.**D51** 1040 (1995); F. Cuypers, Int. J. Mod. Phys. **A11** 1627 (1996); F. Cuypers, hep–ph/9602355; F. Cuypers, P. Frampton, Phys. Lett. **B390** 221 (1997); R. Ruckl, R.Settles, H. Spiesberger,hep–ph/9709315; T. Rizzo, Int. J. Mod. Phys. **A13** 2351 (1998).

[10] H.V. Klapdor-Kleingrothaus, hep–ex/9901021.

[11] I. Bigi, G. Kopp, P. M. Zerwas, Phys. Lett. **B166** 238 (1986); G. Couture, H. K önig, Phys. Rev.**D53** 555 (1996); E. Gabrielli, Phys. Rev.**D63** 055009 (1996); D. Chakraverty, D. Choudhury, A. Datta, Phys. Lett. **B508** 103 (2001); K. Cheung, Phys. Rev.**D64** 033001 (2001).

[12] J. Kalinowski et al., Z. Phys. C**74** 595 (1996); S. Lola, K. Sridhar, J. Ellis, Phys. Lett. **B408** 252 (1997). The second ref. in fact deals with R-parity violation.

[13] G. Bhattacharyya, D. Choudhury, K. Sridhar, Phys. Lett. **B349** 118 (1995); M. Hirsch, H.V. Klapdor-Kleingrothaus, S.G. Kovalenko, Phys. Lett. **B387** 17 (1996); M. S. Berger, hep–ph/9611386; J. Hewett, T. Rizzo, Phys. Rev.**D56** 5709 (1997).

[14] D. Choudhury, Phys. Lett. **B346** 291 (1995).

[15] T. Aliev, D. Demir, E. Iltan, N. Pak, J. Phys. **G**: Nucl. Part. Phys. **22** 611, (1996).

[16] V. Barger, G. Giudice, T. Han, Phys. Rev.**D40** 2987 (1989).

[17] D.P. Roy, Phys. Lett. **B283** 270 (1992); H. Dreiner, S. Lola, in DESY Report No. 96 -123D (1995); M. Guchait and D.P. Roy, Phys. Rev.**D54** 3276 (1996); J. Kalinowski et al., Phys. Lett. **B406** 314 (1997); D.K. Ghosh, S. Raychaudhuri, Phys. Lett. **B422** 187 (1998), D.K. Ghosh, R.M. Godbole, S. Raychaudhuri, hep–ph/9904233; R. Barate et al., (ALEPH Collab.), Eur.Phys.J **C13** 29 (2000).

[18] See the last of ref. [2].

[19] B. Allanach, A. Dedes, H. Dreiner Phys. Rev.**D60** 075014 (1999).

[20] G. Bhattacharyya, J. Ellis, K. Sridhar, Phys. Lett. **B336** 100 (1994); J. K. Mizukoshi, O.J.P Eboli, M. C. Gonzalez-Garcia, Nucl. Phys. **B443** 20 (1995).

[21] For example see, V. Ravindran, W.L. van Neerven, Nucl. Phys. **B589** 507 (2000); C. Macesanu, L. H. Orr, hep–ph/0012200.

[22] For example see, O. Yakovlev, S. Groote, hep–ph/0009014 and references therein.

[23] B. Abbott et al., (D0 Collab.), Phys. Rev. Lett. **83** 4476 (1999); B. Abbott et al., (D0 Collab.), Phys. Rev. Lett. **84** 2088 (2000).

[24] See the first ref. of [6].