Effective Theory Approach to SUSY Hadron Spectroscopy

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Abstract

Supersymmetric hadrons of the type $\tilde{g}\tilde{g}$, $\tilde{g}g$ and $\tilde{g}q\bar{q}$ could exist depending on the masses of the gluino and the squarks being in an appropriate range of values. We find the energy levels of $\tilde{g}g$ and $\tilde{g}q\bar{q}$ bound states (where q denotes a light quark, u, d or s), as well as the strong interaction transition rates among them, using a Heavy Gluino Effective Theory, much in the same spirit as the well-known Heavy Quark Effective Theory. The results are obtained with greater ease and elegance in comparison to other approaches.

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1. Introduction

The trend of the ever-accumulating data from precision experiments indicates that new physics beyond the Standard Model, if any, must be of a weak and decoupling nature. The prime candidate for such a theory is the Supersymmetric Standard Model, either minimal or nonminimal. A huge amount of literature [1] exists on the predicted properties and signatures of the superpartners. However, none of them have been experimentally detected so far, and one can only hope for signals from the hadronic colliders (like LHC) or e^+e^- colliders (such as LEP-200) coming into operation in the not-so-distant future.

In LHC, the most copiously produced superparticle is expected to be the gluino, due to its colour-octet nature and its dominant production mechanism via strong interaction. The gluino is also regarded as the most promising candidate to form a supersymmetric hadron. Assuming the right- and the left-handed squarks to be degenerate in mass, the most dominant decay mode of the gluino is a C-conserving 3-body one $(\tilde{g} \rightarrow q\bar{q}\tilde{\gamma})$. For a heavy gluino $(m_{\tilde{g}} > 100 \text{ GeV})$, if the squark is also heavy (such that the factor $m_{\tilde{g}}^5/m_{\tilde{q}}^4 \leq 100 \text{ GeV})$, the decay will be suppressed by the squark propagator and, as a result, bound states involving the gluino $(\tilde{g}\tilde{g},\ \tilde{g}g,\ \tilde{g}q\bar{q})$ can appear in the spectra, *viz.*, the lifetime will be long enough for a bound state to form. For a light gluino $(1 \text{ GeV} < m_{\tilde{g}} < 4 \text{ GeV})$, the formation of SUSY hadrons would be a near certainty.

In this paper, we will analyse the nature of glueballino $(\tilde{g}g)$ and meiktino $(\tilde{g}q\bar{q})$ spectra using the framework of Heavy Gluino Effective Theory

(HGET) [2], which, apart from some conceptual differences outlined in the next section, is analogous to the well-studied Heavy Quark Effective Theory (HQET) [3]. The said spectra have also been analysed using the bag model [4] and the Bethe-Salpeter formalism [5]; however, we will show that one can arrive at similar, and even more predictive, results in a simpler and more elegant way using an effective theory approach.

2. SUSY hadrons

Let us assume that the gluino undergoes only 3-body tree-level decay $\tilde{g} \to q\bar{q}\tilde{\gamma}$. The 2-body decay mode $\tilde{g} \to g\tilde{\gamma}$ will be suppressed if the right- and left-handed squarks are degenerate in mass ($\tilde{m}_R = \tilde{m}_L$). (For a light gluino ($m_{\tilde{g}}/m_{\tilde{q}} \ll 1$) this mode will practically be forbidden.) The total decay rate is given by [1]

$$\Gamma(\tilde{g} \to q\bar{q}\tilde{\gamma}) = \frac{\alpha \alpha_s e_q^2}{48\pi} \frac{m_{\tilde{g}}^5}{m_{\tilde{q}}^4} \times \mathcal{P} \tag{1}$$

where $\mathcal{P}$ is the phase-space factor

$$\mathcal{P} = (1 - y^2)(1 + 2y - 7y^2 + 20y^3 - 7y^4 + 2y^5 + y^6) + 24y^3(1 - y + y^2)\ln y \tag{2}$$

with $y = m_{\tilde{\gamma}}/m_{\tilde{g}}$. We use the preferred value of $y = 1/7$. The results do not significantly change if we vary this value, or, even if we relax the earlier assumption that the 2-body mode is suppressed. Summing over all possible final-state channels involving various quark-antiquark pair, the gluino lifetime τ comes out to be

$$\tau = 1.2 \times 10^{-13}\text{s} \quad (m_{\tilde{g}} = 2 \text{ GeV}, m_{\tilde{q}} = 70 \text{ GeV})$$

$$\tau = 5.5 \times 10^{-22}\text{s} \quad (m_{\tilde{g}} = 100 \text{ GeV}, m_{\tilde{q}} = 70 \text{ GeV}). \tag{3}$$

As it appears that the light gluino window cannot definitively be ruled out on the basis of presently available data, we have shown one result in that regime too, where the CDF limit on $m_{\tilde{q}}$ [6] slackens to 70 GeV. To form a bound state with lighter constituents, the components have to revolve round each other at least once, and the revolution time τ_{rev} is given by

$$\tau_{rev} \sim 1/\alpha_s^2 \Lambda_{QCD} = 1.6 \times 10^{-22} \text{s} \tag{4}$$

where we have taken $\Lambda_{QCD} = 300$ MeV as a measure of the 'mass' of the light constituents. For $\tilde{g}\tilde{g}$ bound states, Λ_{QCD} is to be replaced by $m_{\tilde{g}}/2$, which reduces τ_{rev} by one or more orders of magnitude, enhancing the probability of bound state formation. Comparing (1) with (4), one gets the condition for the formation of a bound state as

$$m_{\tilde{g}}^5/m_{\tilde{q}}^4 \leq 500 \text{ GeV}. \tag{5}$$

Taking into account the uncertainty in Λ_{QCD}, one can put a much more conservative bound of 100 GeV on the RHS of (5). This enables one to see that a sizable parameter space is allowed for the existence of SUSY hadrons.

To study the spectra of the SUSY hadrons, we use the HGET framework, developed in a way similar to the usual HQET. Gluinos being Majorana particles, one cannot distinguish between the particle and the antiparticle spinor. In conventional HQET, the four-component Dirac spinor is reduced to an effective two-component one (in the zeroth order) by projecting out the positive energy part only. However, for Majorana fermions, only two independent components exist, and hence one cannot dispense with the negative energy part in the zeroth order of the effective Lagrangian. This in

turn allows the fermion number violating Green's function in the theory, both for the propagators and the vertices. In other words, whereas in HQET the negative-energy pole disappears in the limit of infinite fermion mass (in the leading order) and there is no interaction involving antiparticles, this is not so in HGET, because here one cannot differentiate between particles and antiparticles. Due to this extra complication, some of the characteristic features of the HQET are either lost or modified in HGET. For example, the spin SU(2) symmetry is lost as the relevant vertices do explicitly contain γ-matrices coming from the charge-conjugation operator. (We recall that the spin symmetry of HQET originates from the fact that the $q\bar{q}g$ vertex involves the four-velocity v of the heavy particle and does not contain explicit γ-matrices.) This is reflected in the calculation, for example, of the elements of the anomalous dimension matrix; the extra diagrams as well as the extra vertex factors contribute to the said elements. However, the spin symmetry, is partially restored in the static limit $v = (1, \vec{0})$, and the elements of the anomalous dimension matrix simplify. This formulation will be developed in a subsequent paper [2], since one does not need to be concerned about these intricacies to study the spectra of SUSY hadrons. The spectra, as we will show in the next section, can be computed from a judicious comparison with the spectra of ordinary heavy hadrons. In this paper, we consider only those hadrons which consist of only one heavy gluino; thus gluinonium ($\tilde{g}\tilde{g}$) spectroscopy will not be discussed .

3. Sum rules

HQET (or HGET) can be successfully applied to find the energy levels of different hadronic states consisting of one heavy quark (gluino), modulo the uncertainty in the off-shellness scale Λ, which is of the order of Λ_{QCD}. To overcome this uncertainty, one compares the system under investigation with an already known one, and constructs sum rules for the masses [8].

Λ depends on the *brown muck*, i.e., lighter constituents of the bound state, including sea partons. Evidently, Λ is different not only for 2-body glueballino and 3-body meiktino states, but also for $1S$ and $1P$ levels of the bound states. When comparing with the known hadrons, one should take this feature into account; thus, $1S$ $\tilde{g}g$ states are to be compared with $1S$ $c\bar{q}$ or $b\bar{q}$ states, while $1S$ $\tilde{g}q\bar{q}$ states are to be compared with $1S$ cq_1q_2 (or bq_1q_2) states. (The same holds for $1P$ states.)

Being a Majorana particle, the gluino has imaginary parity, and so do meiktino and glueballino states. However, for convenience, we will denote the parity of a state as $+(-)$ when the actual parity is $+i(-i)$. This does not lead to any complication, since the final decay products of $\tilde{g}$ contain a LSP (lightest supersymmetric particle) which has imaginary intrinsic parity.

The nomenclature of states, which will be generally followed [5] is: $1S$ $\tilde{g}u\bar{d}$ states labeled as $\pi_{\tilde{g}}(\rho_{\tilde{g}}, \rho_{\tilde{g}}^*)$, where u and $\bar{d}$ combine to give $J_{light} = 0$ ($J_{light} = 1$); $\rho_{\tilde{g}}$ and $\rho_{\tilde{g}}^*$ denote the final $J = 1/2$ and $J = 3/2$ states (fig. 2). A similar scheme defines the $K_{\tilde{g}}$, $K_{\tilde{g}}^*$, $K_{\tilde{g}}^{**}$, $\eta_{\tilde{g}}$, $\eta_{\tilde{g}}'$, $\omega_{\tilde{g}}$ and $\omega_{\tilde{g}}'$ states.

With the hyperfine interaction turned off, the $1S$ $\tilde{g}g$ state has a mass $m_{1S}^{(2)} = m_{\tilde{g}} + \Lambda_{1S}^{(2)}$, where $\Lambda_{1S}^{(2)}$, the effective Λ for 2-body $1S$ bound states may

be estimatedfrom the known meson masses,e.g.,

$$\Lambda_{1S}^{(2)} = \frac{1}{4}(m_{D^0} + 3m_{D^*}) - m_c, \tag{6}$$

and also by

$$\Lambda_{1S}^{(2)} = \frac{1}{4}(m_{B^0} + 3m_{B^*}) - m_b, \tag{7}$$

which can be used as a cross-check. Likewise, the $1S$ meiktino masses are given by $m_{1S}^{(3)} = m_{\tilde{g}} + \Lambda_{1S}^{(3)}$, where similarly

$$\Lambda_{1S}^{(3)} = m_{\Lambda_c} - m_c. \tag{8}$$

The cross-check from Λ_b is ineffective as it has a large uncertainty in mass [6].

Though the most efficient way to find $\Lambda_{1P}^{(2)}$ is from the $D_1 - D_2^*$ splitting in the case of mesons, unfortunately, the analogous quantity for the baryons is not accurately determinable due to the fact that $1P$ baryons are still not very well studied. However, it turns out that $\Lambda_{1P}^{(2)}$ is nearly one order of magnitude smaller than $\Lambda_{1S}^{(2)}$. This is evident from the way the $c\bar{q}$ mesons are split. Hyperfine splitting due to the spin-spin interaction being a contact one, is naturally reduced for the $1P$ states as compared to that for the $1S$ states, and it is the spin-orbit interaction that gives the sizable contribution to the splitting.

As for the $1S-1P$ splitting, HQET (or HGET) cannot indicate its magnitude; but it may be estimated from the properties of the bound-state wavefunctions. For a linear confining potential, it can be shown [9] that the splitting is constant for all hadrons of a given type (meson or baryon) as

long as one can neglect the mass of the brown muck compared to the heavy constituent, quark or gluino. This fact is also verified experimentally (it is well known that the size of hadrons is nearly constant). Thus, we may take the $1S - 1P$ splitting of $c\bar{q}$ system to be constant throughout the heavy meson spectra, the splitting of $\Lambda_c^+(2625) - \Lambda_c^+$ to be constant throughout the heavy baryon spectra, and this may not be too unreasonable an assumption.

It is a phenomenological observation that the masses of the heavy hadrons increase by about 100 MeV when one replaces one of the u or d quarks by a s quark, and this is explained by considering the constituent quark masses. We assume this fact to be true for SUSY hadrons too.

4. Spectra of the SUSY hadrons

First, let us assume that the gluino is massive enough (say, $m_{\tilde{g}} = 200$ GeV) so that the hyperfine splitting is really negligible. The spectra is shown in fig. 1. We have assumed $m_c = 1.4$ GeV and $m_b = 4.7$ GeV. For $\tilde{g}g$ spectra, the positions of the $1S$ levels are determined by $\Lambda_{1S}^{(2)}$, which is nearly 575 MeV (we have rounded off the energy levels to a 5 MeV accuracy, which is more than sufficient). However, $\Lambda_{1S}^{(3)}$ is 885 MeV, which is reflected in the $I = 1$ meiktino spectra. Replacing one of the first generation quarks by a s quark enhances the heavy hadron mass by 100 MeV; these are shown in the last two columns of fig. 1. The $1S - 1P$ splittings are from ordinary mesonic and baryonic data. One notes that $I = 0$ and $I = 1$ $\tilde{g}u\bar{d}$ states, as is expected from HQET, are degenerate in mass.

When the hyperfine interaction is turned on, the levels are split, as shown

in fig. 2. The point to note is that the $1P$ levels split very little (as Λ_{1P} is small), and even for $m_{\tilde{g}} = 2$ GeV, would appear as a not-too-broad band (the width being ~ 10 MeV). All the $1S$ levels are fully resolved, even to allow strong transitions among them. Some typical strong decays (permitted by parity conservation) among the lowest lying levels are:

$$(1)\pi_{\tilde{g}} \to (g\tilde{g})_{3/2^+} + \pi; \quad (2)\rho_{\tilde{g}} \to (g\tilde{g})_{3/2^+} + \pi;$$

$$(3)\rho_{\tilde{g}}^* \to (g\tilde{g})_{1/2^+} + \pi; \quad (4)(g\tilde{g})_{5/2^-} \to \pi_{\tilde{g}} + \pi;$$

$$(5)(g\tilde{g})_{1/2^-} \to \rho_{\tilde{g}} + \pi; \quad (6)(g\tilde{g})_{3/2^-} \to \rho_{\tilde{g}}^* + \pi;$$

$$(7)(g\tilde{g})_{5/2^-} \to g\tilde{g}_{3/2^+} + \pi\pi; \quad (8)(g\tilde{g})_{3/2^-} \to g\tilde{g}_{1/2^+} + \pi\pi;$$

$$(9)(g\tilde{g})_{1/2^-} \to g\tilde{g}_{1/2^+} + \pi\pi.$$

With $m_{\tilde{g}} = 200$ GeV, the spin-spin and the spin-orbit splittings are negligible, so that all of the above mentioned decays are kinematically allowed. For a light gluino ($m_{\tilde{g}} = 2$ GeV, say, as in fig. 2), these splittings start to play significant roles, so that transitions 2 and 6 become kinematically forbidden. However, we stress again that the list is a typical one and by no means exhaustive.

The decay amplitude for strong decay transitions can be written in HQET as [3]

$$
\begin{aligned}
\mathcal{A}(s \to s' + J_h) &= \langle \, || \, || \, \rangle [(2s'+1)(2s_l+1)]^{1/2}(-)^{s_Q+s_l'+J_h+s} \\
&\times \begin{Bmatrix} s_Q & s_l' & s' \\ J_h & s & s_l \end{Bmatrix}
\end{aligned}
\tag{9}
$$

where the symbols s, s', s_l, s_l', s_Q and J_h respectively stand for total angular momentum of the initial and the final hadron, angular momentum of the light

degrees of freedom of the initial and the final hadron, spin of the heavy quark (=1/2) and total angular momentum of the light quanta emitted; $\langle \ || \ || \ \rangle$ stands for the reduced matrix element. We have considered three distinct sets of transitions: $s_l = 0$ to $s'_l = 0$ (1 to 3); 2-body $s_l = 1$ to $s'_l = 0$ (4 to 6) and 3-body $s_l = 1$ to $s'_l = 0$ (7 to 9). The reduced matrix elements are the same within a particular set but should differ among different sets. Without taking into account the phase space factors, the reduced partial widths work out to be

$$\Gamma_1^r : \Gamma_2^r : \Gamma_3^r \ = \ 4 : 2 : 1$$

$$\Gamma_4^r : \Gamma_5^r : \Gamma_6^r \ = \ 1 : 1 : 0.6 \qquad (10)$$

$$\Gamma_7^r : \Gamma_8^r : \Gamma_9^r \ = \ 1 : 0.17 : 1$$

where the suffixes on Γ^r indicate the serial number of the transitions enlisted above. However, for low $m_{\tilde{g}}$, these results get seriously modified due to the kinematic factor of $(|\vec{p}_\pi|/m_{\tilde{g}})^{2J_h+1}$ which is associated with the decay width. For large gluino mass, the kinematic factor is the same for all members in a particular set, and so the ratios remain unaltered. Still the $J_h = 2$ decay modes (set 2) will be highly suppressed compared to $J_h = 1$ modes (set 1) due to an extra factor of $1/m_{\tilde{g}}^2$. It is easier to show the results for 2-body decays; for 3-body decays one can only compute the ratios at some fixed point of the relevant Dalitz plots. For $m_{\tilde{g}} = 2$ GeV, one gets

$$\Gamma_1 : \Gamma_3 \ = \ 1 : 8.3$$

$$\Gamma_4 : \Gamma_5 \ = \ 2 \times 10^{-4} : 1 \qquad (11)$$

9

which is in sharp contrast with the näive estimate of eq. (10), and shows that some decay widths depend sensitively on $m_{\tilde{g}}$.

5. Discussions and conclusion

In this paper, we have obtained the energy levels and the ratios of decay amplitudes of the SUSY hadrons, exploiting certain symmetries of an effective-theory approach. Essentially, we have compared the SUSY hadrons with their analogous ordinary counterparts and have eliminated, from this comparison, the unknown factors in the determination of the spectra. Of course, phenomenological inputs not directly available in HQET were also used, $e.g.$, the fact that $1S-1P$ splitting is nearly constant for heavy hadrons with a linearly confining potential, or the increase of mass of the heavy hadrons by approximately 100 MeV when a strange quark is substituted for a lighter one (u , d).

The fact that the gluino is a Majorana particle hardly affects our results, since all of them were obtained in the static limit of the gluino where the spin symmetry is restored. However, we have not taken into account the QCD dressing of the spectra. Most of the QCD corrections can be taken into account with the phenomenological choice of Λ, as the levels of ordinary hadrons, too, undergo QCD dressing; a small part, which is really negligible, remains as the exponents of the Wilson coefficients have some Majorana contributions. This will be discussed in detail in our next paper [2].

According to us, the most impressive feature of an effective-theory approach is the simplicity and elegance with which one obtains the predictions,

compared to the other approaches. The results are also more or less in conformity with those obtained elsewhere [4, 5]. For example, the $1S - 1P$ splitting for $\tilde{g}g$ ($\tilde{g}q\bar{q}$) system is 600 (300) MeV in the Bethe-Salpeter approach, while we obtain 475 (340) MeV. The parity of the $1S$ $\tilde{g}g$ and $1S$ $\tilde{g}q\bar{q}$ are same, as in the Bethe-Salpeter or the bag model approach (however, in the limit $m_g \to 0$, the bag model $\tilde{g}g$ $1S$ and $1P$ turn over, changing the parity of the ground state).

If the gluino is discovered at the future colliders, it may be possible to study these spectra. For a heavy gluino, it may also be possible to test how far the effective-theory approach can be successfully extrapolated, and how large the QCD dressing effects turn out to be. We hope that this will provide the most severe test for such an approach.

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Figure Captions

1. The energy levels of SUSY hadrons for $m_{\tilde{g}} = 200$ GeV, $m_c = 1.4$ GeV and $m_b = 4.7$ GeV.

2. The energy levels of SUSY hadrons for $m_{\tilde{g}} = 2$ GeV, $m_c = 1.4$ GeV and $m_b = 4.7$ GeV.