

Effect of In-Medium Meson Masses on Nuclear Matter Properties

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Masses of hadrons (baryons as well as mesons) are modified in the nuclear medium because of their interactions. In this paper we investigate the effect of in-medium meson masses on the properties of the nuclear matter. The calculations are performed in the Walecka model. We find that with the inclusion of meson mass modification, the computed equation of state becomes softer. We also find that the decrease in the nucleon as well as meson masses in the medium is smaller than that obtained in the case when the in-medium meson masses are not taken into account.

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I. INTRODUCTION

Walecka model [1,2] and its extensions, which include nonlinear meson interactions [3] and derivative coupling [4], have been used extensively in the investigation of nuclear systems for quite some time now. One of the attractive features of these models is that these are relativistic models based on meson-nucleon interactions. The saturation properties of the nuclear systems is in-built in these models through scalar (σ) and vector (ω and ρ) mesons interacting with the nucleons. The scalar meson generates long range attraction and the vector mesons give rise to short range repulsion. Furthermore, the spin-orbit interaction which is essential in nuclear physics arises in a very natural way in these models. A number of nuclear structure calculations based on the extensions of the Walecka model [5] have been performed. These calculations have succeeded in explaining the properties of stable nuclei across the periodic table as well as those of nuclei far away from the stability line. In a nut-shell, the calculations in the Walecka model are performed in a mean field approximation where one solves the nucleon and meson (classical) field equations self-consistently. Thus, the calculations are done in Hartree approximation and Fock (exchange) terms are not included. Essentially, this amounts to saying that the meson fields develop nonzero vacuum expectation values in the presence of

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nucleons and these mean fields, in turn, generate scalar and vector potentials for the nucleons. The effective (in-medium) nucleon mass (m_n^*) is then the sum of free nucleon mass and the scalar potential.

The success of the Walecka-type models in explaining the nuclear properties has led to the application of these models to other areas. This essentially amounts to extrapolating the applicability of the model to large isospin, nuclear densities much larger than the normal nuclear matter density (ρ_{NM}) and nonzero temperatures. For example, the model has been used to compute the equation of state of neutron matter which is used in neutron star calculations [6]. It has also been employed in the calculations of nuclear equation of state at densities higher than ρ_{NM} [7]. Recently calculations of in-medium meson masses at high densities and temperatures in Walecka model have also been reported [8,9]. These calculations were motivated by the conjecture that the properties of hadrons may be modified in nuclear medium [10] and this may have significant consequences on the reactions occurring in the nuclear medium. These calculations show that ρ and ω meson masses are significantly altered in the nuclear medium. For example, ω mass may be as low as $630MeV$ even at ρ_{NM} which is about 0.8 times the free ω mass m_ω .

The Walecka model was initially introduced as an effective model for the calculation of properties of nuclei. Thus the scalar and vector mesons were effective fields introduced to account for the main features of the N-N interaction. It is true that the pion, which generates the long range NN interaction is not included. But it is argued that in the mean field approximation for spin and/or isospin zero systems one pion exchange does not contribute and the two pion exchange is incorporated through σ meson exchange. This argument breaks down at high temperatures where pionic excitations should dominate. If one takes this argument seriously, in the spirit of effective interactions of nuclear structure or the Fermi theory of weak interaction, then it is not meaningful to compute the in-medium meson masses in Walecka model. That is, since the parameters of the model (meson masses and the coupling constants) are fitted to satisfy the nuclear properties (binding energies, nuclear incompressibility, symmetry energy etc.), it does not make sense to compute the in medium meson masses again. In other words, the fitted meson masses are in medium masses and therefore should not be computed again.

The argument given in the preceding paragraph is reasonable if the variation of the effective masses as a function of nuclear density or temperature is weak. However, the calculations [8] show that the effective masses change quite a bit as a function of density. This means that the fitted value of the meson masses at ρ_{NM} will change significantly at higher densities and the computation of the nuclear matter properties at these densities will not be reliable.

Clearly, a better procedure would be to use the in-medium meson masses in Walecka model calculations particularly if the results are sensitive to the meson masses. We do believe that such self-consistent calculations would yield results different from the standard Walecka model calculations. To illustrate this point, let us consider nucleons interacting with scalar mesons of mass m . The nucleon-nucleon interaction potential generated by the scalar meson exchange is then $V(r) = g^2 \frac{e^{-mr}}{4\pi r}$ where g is the meson-nucleon coupling constant (see for details [11]). If we consider the meson mass in the medium is modified to m_{eff} , the nucleon-nucleon potential in the medium is obtained by replacing m by m_{eff} . Clearly, this change will affect the computation of energy/nucleon, effective nucleon mass etc. In particular, the decrease in the meson mass will lead to increase in the range of the potential and decrease in its strength. Further more, since the meson mass in the medium depends on the nuclear density, the strength and the range of the potential will be density-dependent. Thus, one expects that nuclear matter properties computed using this procedure are likely to differ significantly from the standard Walecka model calculations particularly at densities larger than ρ_{NM} .

The preceding discussion clearly brings out the problems associated with the computation of the meson masses in the medium blindly. The correct procedure should be to compute meson masses as well as nuclear matter properties self-consistently (upto certain order in perturbation) and fit the nuclear matter properties by adjusting the parameters of the Walecka model. Note that this self-consistency is over and above the self-consistency in the standard Walecka model calculations. We have done this in the present work. Here we have employed the Walecka model with σ and ω mesons coupling to nucleons. From our calculations we find that it is indeed possible to carry out such a program and fit the nuclear matter properties (binding energy/nucleon at nuclear matter density). We also find that the change in the nucleon and meson masses in the nuclear medium are less pronounced and the computed equation of state is softer. The computed nuclear matter incompressibility is about 460 MeV in comparison with 610 MeV, the value computed in standard the Walecka model.

Before we go on to the details of our calculation we would like to note that our calculations can be repeated for the extensions of the Walecka model [3,4]. We have not done this here since we want to focus on the effect of the variation of meson masses on the nuclear matter properties as a function of nuclear density. We believe one would obtain qualitatively similar results for the extensions of Walecka model.

II. MESON MASSES AND NUCLEAR MATTER PROPERTIES

The Walecka model Lagrangian, having nucleons interacting with scalar σ -meson and vector ω -meson is given by [2]

$$\begin{aligned} \mathcal{L}_W = & \bar{\psi}(i\gamma_\mu\partial^\mu - m_n)\psi - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) + g_\sigma\bar{\psi}\sigma\psi \end{aligned} \quad (1)$$

In the above equation ψ , σ and ω^μ are, respectively, the nucleon, the scalar and the vector meson fields; m_n, m_σ and m_ω are the corresponding masses; g_σ and g_ω are the couplings of the nucleon to scalar and vector mesons, respectively and $G_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$.

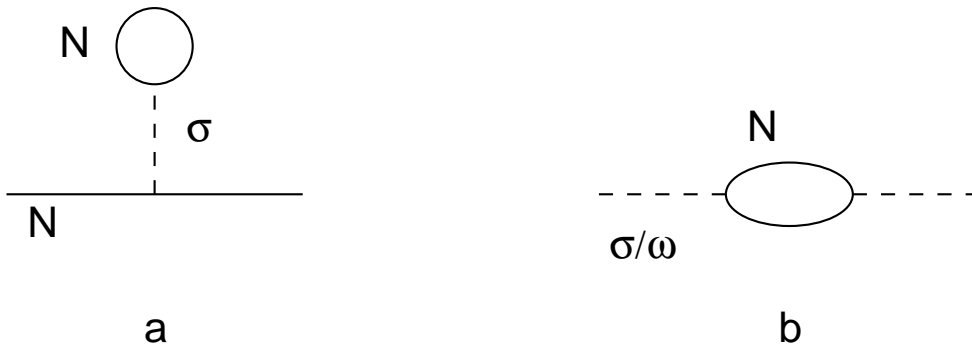


FIG. 1. Diagrams contributing to nucleon self energy (a) and σ and ω self energies (b).

In the following discussion we have described the study of the in-medium properties of hadrons using the above Lagrangian. The in medium nucleon mass, the energy density per baryon and the σ and ω masses are calculated self-consistently. The coupling constants are determined by fitting the nuclear matter saturation density to be $\rho_{NM} = 0.15 fm^{-3}$, the corresponding binding energy being $-15.75 MeV$ per baryon.

The effective nucleon mass in the Relativistic Hartree Approximation (RHA) can be calculated from figure 1a. This is given by [2]

$$\begin{aligned} m_n^* &= m_n - g_\sigma \sigma_0 \\ &= m_n - \frac{ig_\sigma^2}{m_\sigma^{*2}} \int \frac{d^4p}{(2\pi)^4} \text{Tr}.G(p) \end{aligned} \quad (2)$$

where G is the in-medium baryon propagator which is given by

$$G(p) = (\gamma^\mu p_\mu + m_n^*) \left[\frac{1}{p^2 - m_n^{*2} + i\epsilon} + 2i\pi\delta(p^2 - m_n^{*2})\theta(p_0 - k_F) \right] \quad (3)$$

So we get

$$\begin{aligned} m_n^* &= m_n - \frac{g_\sigma^2}{\pi^2 m_\sigma^{*2}} m_n^* \left[k_F E_F + m_n^{*2} \ln \left(\frac{k_F + E_F}{m_n^*} \right) \right] + \frac{g_\sigma^2}{\pi^2 m_\sigma^{*2}} \left[m_n^{*3} \ln \left(\frac{m_n^*}{m_n} \right) \right. \\ &\quad \left. + \frac{1}{4}(m_n^{*3} - m_n^3) + \frac{7}{4}m_n^2(m_n - m_n^*) - \frac{13}{4}m_n(m_n - m_n^*)^2 + \frac{25}{12}(m_n - m_n^*)^3 \right] \end{aligned} \quad (4)$$

The effective mass of a meson can be calculated from the solution of the Dyson-Schwinger equation which relates the free and the full propagator as follows :

$$D(p) = D_0(p) + D_0(p)\Pi D(p) \quad (5)$$

where $D_0(p)$ is the free Green's function, $D(p)$ is the full Green's function and Π is the polarisation function (at zero three momentum). The pole of the full propagator then gives the effective mass of the meson.

We calculate the meson masses in Random Phase Approximation (RPA). The corresponding diagram is given in figure 1b. The effective mass of the σ meson is given by [12]

$$m_\sigma^{*2} = m_\sigma^2 + \Pi_\sigma \quad (6)$$

where

$$\begin{aligned} \Pi_\sigma &= -ig_\sigma^2 \int \frac{d^4p}{(2\pi)^4} G(p+q)G(p) \\ &= -\frac{8g_\sigma^2}{\pi^2} \int_0^{k_F} \frac{p^4 dp}{E_p^*} \frac{1}{(m_\sigma^{*2} - 4E_p^{*2})} + \frac{g_\sigma^2}{\pi^2} \left[\frac{3}{2} (m_n^{*2} - m_n^2) \int_0^1 dx \ln \left(1 - \frac{m_\sigma^2}{m_n^2} x(1-x) \right) \right. \\ &\quad \left. + \frac{3}{2} \int_0^1 dx (m_n^{*2} - m_\sigma^{*2} x(1-x)) \ln \left\{ \frac{m_n^{*2} - m_\sigma^{*2} x(1-x)}{m_n^2 - m_\sigma^2 x(1-x)} \right\} - \frac{m_\sigma^2 - m_\sigma^{*2}}{4} \right] \end{aligned} \quad (7)$$

The polarisation function for the σ -meson has two parts. First part is the in-medium contribution and the second part is the vacuum contribution. In calculating the vacuum part we have renormalised the contribution according to ref. [13].

For the omega meson we have

$$m_\omega^{*2} = m_\omega^2 + \Pi_{\omega T} \quad (8)$$

where $\Pi_{\omega T}$ is the transverse part of the polarisation tensor $\Pi_{\mu\nu}$ (in the static limit the transverse and the longitudinal components are the same) which is given by

$$\Pi_{\mu\nu} = -ig_\omega^2 \int \frac{d^4p}{(2\pi)^4} \gamma_\mu G(p+q) \gamma_\nu G(p) \quad (9)$$

The transverse part is given by [8]

$$\Pi_{\omega T} = -\frac{8g_{\omega}^2}{3\pi^2} \int_0^{k_F} \frac{p^2 dp}{E_p^*} \frac{(2p^2 + 3m_n^{*2})}{(m_{\omega}^{*2} - 4E_p^{*2})} - \frac{g_{\omega}^2 m_{\omega}^{*2}}{\pi^2} \int_0^1 dx \ln \frac{m_n^{*2} - m_{\omega}^{*2} x(1-x)}{m_n^2 - m_{\omega}^2 x(1-x)} \quad (10)$$

The binding energy per baryon, calculated in RHA, is given by,

$$\epsilon = \frac{\mathcal{E}}{\rho_{NM}} - m_N \quad (11)$$

where \mathcal{E} is the energy density which is $\langle T^{00} \rangle$ and $T^{\mu\nu}$ is the energy momentum tensor. The energy density in the RHA is given by

$$\begin{aligned} \mathcal{E} = & \frac{g_{\omega}^2}{2m_{\omega}^{*2}} \rho_{NM}^2 + \frac{m_{\sigma}^{*2}}{2g_{\sigma}^2} (m_n - m_n^*)^2 + \frac{2}{\pi^2} \int_0^{k_F} dp p^2 \sqrt{(p^2 + m_n^{*2})} - \frac{1}{4\pi^2} \left[m_n^{*4} \ln \left(\frac{m_n^*}{m_n} \right) \right. \\ & \left. + m_n^3 (m_n - m_n^*) - \frac{7}{2} m_n^2 (m_n - m_n^*)^2 + \frac{13}{3} m_n (m_n - m_n^*)^3 - \frac{25}{12} (m_n - m_n^*)^4 \right] \end{aligned} \quad (12)$$

We now solve the above equations self-consistently. The parameter values that we get, from the nuclear matter saturation properties, are given by $g_{\sigma} = 6.716$ and $g_{\omega} = 7.64$. In case we do not consider the in-medium meson masses the corresponding values are 8.69 and 9.9 respectively. The results are shown in figures 2-3. In figure 2 we have plotted the masses of nucleon, scalar meson and vector meson as a function of density. In order to make a comparison, both the cases *i.e.* with and without self-consistency have been plotted. In figure 3 the energy per baryon has been plotted for both cases.

III. RESULTS AND DISCUSSION

We now come to the discussion of the results. Figure 2 displays the plots of hadron masses as a function of nuclear density. The dashed lines correspond to the standard Walecka model calculation where the modification of meson masses are not included in the calculation of nuclear matter properties and the continuous lines correspond to our (self-consistent) calculation. We find that in a self-consistent calculation the change in the masses is somewhat less dramatic. In particular, the change in the in-medium σ and ω masses is less than 15% in a self-consistent calculation whereas the maximum change is more than 20% for the standard Walecka model. Hence at nuclear matter density the σ and ω masses are $471 MeV$ and $686 MeV$ respectively for the self-consistent case, whereas the corresponding values for the case where there is no self-consistency are $445 MeV$ and $632 MeV$ respectively. This in terms of the $N - N$ interaction potential implies that the potential is more attractive for the self-consistent case. As a result the nuclear matter will be more compressible *i.e.* the incompressibility will be lower. Such changes, in general, will have bearing on the binding energy as well as on the equation of state of the system. The behavior of the nucleon mass appears to be similar for both the cases but the effective nucleon mass is slightly larger in a self-consistent calculation ($0.79 m_N$ vs $0.73 m_N$ at ρ_{NM}). From the behavior of the masses one can conclude that in order to obtain a reliable estimate of masses of hadrons in nuclear medium it is essential to do a self-consistent calculation.

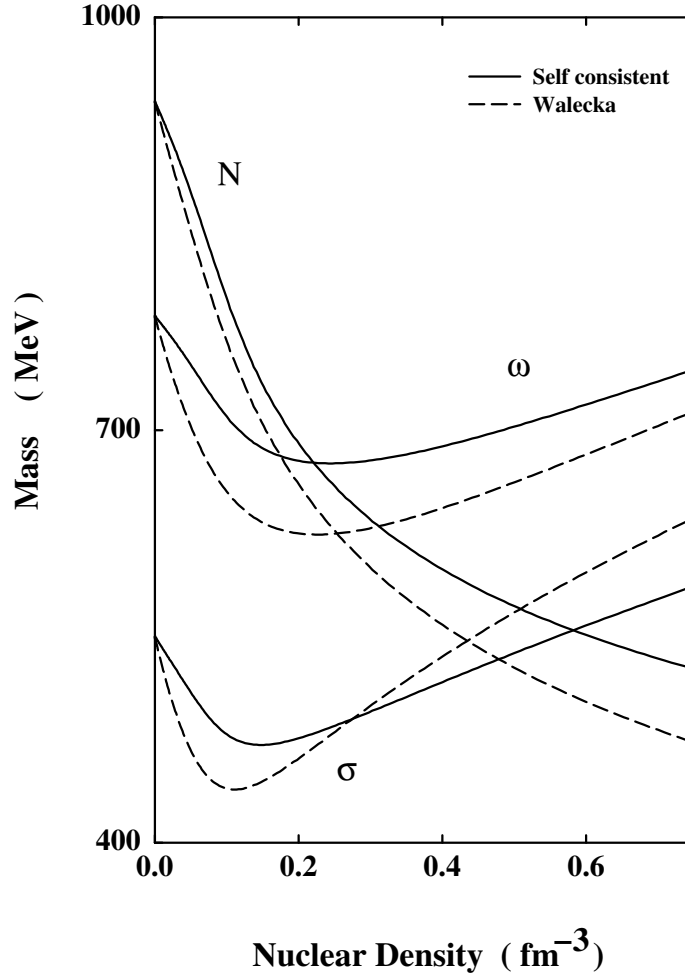


FIG. 2. Nucleon, ω and σ masses effective masses. Continuous and dashed lines are for with and without self-consistency.

Figure 3 shows a plot of the binding energy/nucleon (B/A) as a function of the nuclear density. Both the curves have a minimum at the nuclear matter density, simply because the binding energy at the saturation density is fitted (-15.75MeV at 0.15fm^{-3}). One should, however, note that the second derivative of B/A , which is related to the nuclear incompressibility, is smaller for the self-consistent calculation. The computed incompressibility for the self-consistent calculation is 460 MeV in comparison with the standard Walecka model result of 610 MeV. In other words, the self-consistent calculation drives the incompressibility towards lower values. The binding energy per baryon is much lower at higher densities for the self-consistent case. This implies that the equation of state obtained in the self-consistent calculation is softer. This is an important result in the context of neutron star physics. In the standard Walecka model the equation of state is stiffer resulting a value of the neutron star mass which is higher than the experimental bound. Our results indicate that a self-consistent calculation may yield a better estimate for the neutron star masses.

So, to conclude, we have calculated the nuclear matter properties as well as the in-medium meson masses self-consistently by including the change in the meson masses due to medium effect in the calculation of nuclear matter properties. Our study gives rise to two important conclusions. First of all, we find that

the change in the meson masses due to the effect of the medium is smaller in a self-consistent calculation. This means that a calculation which does not impose the self-consistency over estimates the meson masses. The second result is that the self-consistent calculation yields a softer equation of state and lower values of incompressibility. We believe that this qualitative behaviour will persist for asymmetric nuclear matter and will have a bearing on neutron star properties.

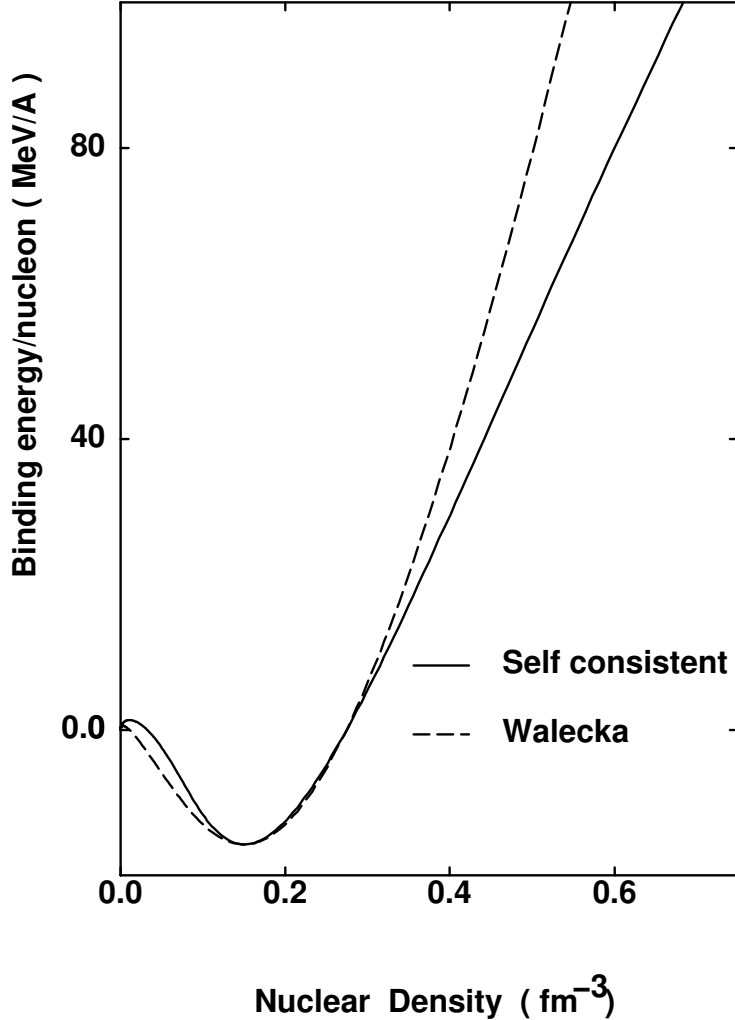


FIG. 3. Binding energy/nucleon (B/A) as a function of nuclear density. The continuous and dashed curves are for with and without self-consistency

We would like to emphasize again that, even if the meson fields are considered as effective fields, the self-consistency described in the present work is necessary if one wants to apply the Walecka model at higher densities. On the other hand if these fields are considered as real fields, it is essential to do a self-consistent calculation to obtain a reliable estimate of meson masses in the medium. It would be interesting to investigate the behaviour of ρ meson mass in the medium since it has a bearing on the quark-gluon plasma diagnostics. Such a calculation is in progress.

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