

## Constraints on R-parity violating supersymmetry from neutral meson mixing

Anirban Kundu

*Department of Physics, University of Calcutta,  
92 A.P.C. Road, Kolkata 700009, India\**

Jyoti Prasad Saha

*Department of Physics, Jadavpur University, Kolkata 700032, India<sup>†</sup>*

(Dated: August 17, 2018)

Upper bounds at the weak scale are put on all  $\lambda'_{ijk}\lambda'_{lmn}$  type products of R-parity violating supersymmetry that may affect  $K^0 - \overline{K}^0$  and  $B^0 - \overline{B}^0$  mixing. We constrain all possible products, including some not considered before, using next-to-leading order QCD corrections to the mixing amplitudes. Constraints are obtained for both real and imaginary parts of the couplings. We also discuss briefly some correlated decay channels which should be investigated in future experiments.

PACS numbers: 11.30.Hv, 12.60.Jv, 14.40.Nd

## I. INTRODUCTION

Is there any new physics (NP) beyond the Standard Model (SM)? Probably yes, if high-energy physics does not want itself to be lost in a mire of unanswered questions. Supersymmetry (SUSY) is one of the most widely discussed options of NP, in both its R-parity conserving (RPC) and R-parity violating (RPV) incarnations. However, SUSY introduces a plethora of new particles, and even in its most constrained version, a few more arbitrary input parameters over and above to that of the SM. Thus, it has become imperative to constrain the SUSY parameter space as far as possible from existing data.

There are, of course, direct bounds on the sparticle masses from collider data. These bounds are typically weaker for RPV SUSY than for the RPC version, since in the former case the final-state signal is radically different and more possible channels are open. Apart from them, there are bounds on the parameter space from low-energy processes, whose amplitudes can be affected by intermediate sparticle states. For RPC SUSY, the new amplitudes must be at least at one-loop level, so that they can compete with the SM amplitude and show up only if the SM amplitude is also suppressed. The new amplitudes may appear at tree-level if R-parity is violated, and hence there is a greater chance of constraining RPV SUSY models from low-energy data. There is another good reason to focus upon the RPV version: most of the precision low-energy processes from which one can obtain bounds are flavor-changing neutral current (FCNC) type. Now, there are some elegant mechanisms of suppressing FCNC effects in RPC SUSY, but absolutely none (except putting some flavor-changing Yukawa-type couplings equal to zero, or vanishingly small, by hand) if we have RPV. Thus, the bounds on the parameter space of RPV SUSY coming from FCNC processes are in a sense

more robust.

In this paper we use the data from  $K^0 - \overline{K}^0$  and  $B^0 - \overline{B}^0$  mixing to constrain the relevant couplings for RPV SUSY. For the latter we use both  $\Delta m_B$  and  $\sin(2\beta)$  constraints, while for the former we use the results on  $\Delta m_K$  and  $\varepsilon_K$ . We do not discuss other CP violating parameters like  $\varepsilon'/\varepsilon$ , since that has large theoretical uncertainty. We have also discussed some correlated channels leading to leptonic and semileptonic decays. We do not consider the  $B_s$  system since there is only a lower bound on  $\Delta M_{B_s}$ . This, in turn, means that there is no such upper bound on the relevant NP couplings; we can only have a lower bound, which is consistent with zero. The situation should change dramatically once the hadronic B machines, producing copious  $B_s$  mesons, come on line.

Do we have any motivation to invoke NP for the K and the B systems? In other words, is there any inconsistency of the experimental data with the SM predictions? The answer is yes for the B system, though the error bars are still large to draw any definite conclusion (but we have reasons to be hopeful). The sore thumbs are (i) the abnormally high branching ratios (BR) for the generic channels  $B \rightarrow \eta'K, \eta K^*$  [1] [32], (ii) the direct CP-asymmetry in the channel  $B_d \rightarrow \pi^+\pi^-$  as found by Belle [2], (iii) the discrepancy in the extracted value of  $\sin(2\beta)$  from  $B_d \rightarrow J/\psi K_S$  and  $B_d \rightarrow \phi K_S$  [3], and (iv) the so-called “ $\pi K$ ” puzzle: the abnormal enhancement of electroweak penguins in  $B \rightarrow \pi K$  decays [1] [33]. However, one must not be over-enthusiastic since these channels are nonleptonic and QCD uncertainties are yet to be fully understood. But one may hope more such anomalies from leptonic and hadronic B-factories. For the K system, the nonleptonic channels are notoriously difficult for any systematic analysis of NP effects [4], but for the first time we are having precise data (or bound) on leptonic and semileptonic K decay channels from Brookhaven and DaΦne. It is always better to be ready for any unexpected result.

Another motivation for supersymmetry comes from the neutrino mass. It has been shown that the existence of nonzero neutrino mass may have observable signatures in

\*Electronic address: akundu@cucc.ernet.in

<sup>†</sup>Electronic address: jyotip@juphys.ernet.in

B-physics [5, 6] like the enhancement of the  $b \rightarrow s\bar{s}s$  amplitude and also  $\Delta M_s$ , the mass difference between two  $B_s$  mass eigenstates, if one assumes some unified RPC SUSY theory. In RPV SUSY, one may have a definite texture at the GUT scale (only a few RPV couplings are nonzero) and can generate the whole lot of nonzero RPV couplings at the weak scale through renormalization group evolution and CKM-type mixing [7]. One expects a signature of those RPV couplings relevant for neutrino mass generation in the B-factory data.

Must the RPV couplings be complex? The answer is that there is no a priori reason why they should be all real. Even if there is some real GUT texture, the weak scale couplings may turn out to be complex. The phase of one single coupling can be absorbed in the sfermion field, but a nontrivial phase should be there in a product of two such couplings.

Effects of RPV SUSY on K and B physics have been discussed extensively in the literature [8–12]. Constraints coming from  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  mixing have been discussed in [13–15]. This paper is a culmination of the effort in the following sense: (i) This is the first time that both SM and all RPV contributions (including their phases) have been taken into account for the systems under consideration; (ii) the short-distance QCD corrections have been implemented upto next-to-leading order (NLO) accuracy (there is scope for questioning this procedure for the K system; this will be discussed later); (iii) Both CP-conserving and CP-violating constraints have been discussed for the  $B^0 - \bar{B}^0$  and  $K^0 - \bar{K}^0$  systems; (iv) Some discrepancies in the relevant formulae used in some of the earlier works have been corrected; and (v) Correlated signals for leptonic and semileptonic decays have been discussed.

The paper is arranged as follows. In Section 2 we outline the relevant formulae necessary for the analysis. Section 3 deals with the numerical inputs and Section 4 with the analysis of the results. Decay channels mediated by the same couplings are touched upon in Section 5, while we conclude and summarize in Section 6. Some calculational details have been relegated to the two appendices.

## II. BASIC INPUTS

### A. Neutral meson mixing

Let the neutral mesons be generically denoted by  $M^0$  and  $\bar{M}^0$ , with the valence quark content  $\bar{q}d$  and  $q\bar{d}$  respectively. For the cases under study,  $q$  can be either  $d$  or  $s$ . For  $B_s$  system, replace  $q$  by  $b$  and  $d$  by  $s$ .

The off-diagonal element in the  $2 \times 2$  effective Hamiltonian causes the  $M^0 - \bar{M}^0$  mixing. The mass difference between the two mass eigenstates  $\Delta M$  is given by (following the convention of [16])

$$\Delta M = 2|M_{12}|, \quad (1)$$

with the approximation  $|M_{12}| \gg |\Gamma_{12}|$ . This, however, is true for the B system only. Let the SM amplitude be

$$|M_{12}^{SM}| \exp(-2i\theta_{SM}) \quad (2)$$

where  $\theta_{SM} = \beta(\phi_1)$  for the  $B^0 - \bar{B}^0$  system and approximately zero for the  $K^0 - \bar{K}^0$  (and also for  $B_s - \bar{B}_s$ ) system. We follow the  $(\alpha, \beta, \gamma)$  convention for the unitarity triangle [16].

If we have  $n$  number of NP amplitudes with weak phases  $\theta_n$ , one can write

$$M_{12} = |M_{12}^{SM}| \exp(-2i\theta_{SM}) + \sum_{i=1}^n |M_{12}^i| \exp(-2i\theta_i). \quad (3)$$

This immediately gives the effective mixing phase  $\theta_{eff}$  as

$$\theta_{eff} = \frac{1}{2} \arctan \frac{|M_{12}^{SM}| \sin(2\theta_{SM}) + \sum_i |M_{12}^i| \sin(2\theta_i)}{|M_{12}^{SM}| \cos(2\theta_{SM}) + \sum_i |M_{12}^i| \cos(2\theta_i)}, \quad (4)$$

and the mass difference between mass eigenstates as

$$\begin{aligned} \Delta M &= 2[|M_{12}^{SM}|^2 + \sum_i |M_{12}^i|^2 \\ &+ 2|M_{12}^{SM}| \sum_i |M_{12}^i| \cos 2(\theta_{SM} - \theta_i) \\ &+ 2 \sum_i \sum_{j>i} |M_{12}^j| |M_{12}^i| \cos 2(\theta_j - \theta_i)]^{1/2}. \end{aligned} \quad (5)$$

These are going to be our basic formulae. The only task is to find  $M_{12}^i$  and  $\theta_i$ .

For the  $K^0 - \bar{K}^0$  system [17],  $|\Gamma_{12}|$  is non-negligible, and we can write

$$\begin{aligned} \Delta M &= 2 \operatorname{Re} \left[ \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right) \right]^{1/2}, \\ \Delta \Gamma &= -4 \operatorname{Im} \left[ \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right) \right]^{1/2}, \end{aligned} \quad (6)$$

so that  $\Delta M = -(1/2)\Delta\Gamma$ . Since the dominant decay is to the  $I = 0$  final state, one can neglect  $\operatorname{Im}\Gamma_{12}$  and write

$$\Delta M = 2 \operatorname{Re} M_{12}, \quad \Delta \Gamma = 2 \operatorname{Re} \Gamma_{12}. \quad (7)$$

The CP-violating parameter  $\varepsilon_K$  is given by

$$|\varepsilon_K| = \frac{1}{2\sqrt{2}} \frac{\operatorname{Im} M_{12}}{\operatorname{Re} M_{12}}, \quad (8)$$

which can be written as

$$|\varepsilon_K| = \frac{1}{\sqrt{2}} \frac{\operatorname{Im} M_{12}}{\Delta M}. \quad (9)$$

Note that  $\operatorname{Re} M_{12}$  has both short-distance (SD) and long-distance (LD) contributions. The LD contribution is not calculable; what one calculates from the box amplitude

is the SD part. That is why one generally uses the experimental value of  $\Delta M_K$  in the denominator of eq. (9).

For  $K^0 - \bar{K}^0$  system, the short-distance SM amplitude is

$$M_{12}^{SM} \equiv \frac{\langle \bar{K}^0 | H_{eff} | K^0 \rangle}{2m_K} \approx \frac{G_F^2}{6\pi^2} (V_{cd}V_{cs}^*)^2 \eta_K m_K f_K^2 B_K m_W^2 S_0(x_c), \quad (10)$$

where generically  $x_j = m_j^2/m_W^2$ ,  $f_K$  is the K meson decay constant, and  $\eta_K$  (also called  $\eta_{cc}$  in the literature) and  $B_K$  parametrize the short- and the long-distance QCD corrections, respectively ( $B_K$  is a phenomenological parameter, thrown in to parametrize the LD contribution). The tiny top-quark loop dependent part responsible for CP violation has been neglected. The function  $S_0$  is given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}. \quad (11)$$

For the  $B^0 - \bar{B}^0$  system, we have an analogous equation, dominated by the top quark loop:

$$M_{12}^{SM} \equiv \frac{\langle \bar{B}^0 | H_{eff} | B^0 \rangle}{2m_B} = \frac{G_F^2}{6\pi^2} (V_{td}V_{tb}^*)^2 \eta_B m_B f_B^2 B_B m_W^2 S_0(x_t). \quad (12)$$

There is not enough motivation to consider the  $B_s - \bar{B}_s$  system right now, since there exists only a lower bound on  $\Delta M_s (\geq 14.4 \text{ps}^{-1})$  [1]. This can accommodate arbitrarily large NP couplings, starting from zero. However, if the NP amplitude has a nonzero phase, then there will be an effective phase in  $B_s - \bar{B}_s$  mixing amplitude, whose presence may be tested in the hadronic B factories. Whether there is any detectable new physics in  $B_s - \bar{B}_s$  mixing (this is particularly relevant since the  $b \rightarrow s$  penguin transition, at least for nonleptonic decays, shows hint of an anomalous behaviour) can be effectively tested in hadronic B machines, which will measure  $\Delta M_s$  as well as the CP asymmetries in  $B_s \rightarrow J/\psi\phi$  and hopefully  $B_s \rightarrow \phi(\eta')\phi(\eta')$ . Detection of NP signals in the double vector meson modes requires angular analysis of the decay products but hopefully can be done in future colliders [18].

In the presence of NP, the general  $\Delta F = 2$  effective Hamiltonian can be written as

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^5 c_i(\mu) O_i(\mu) + \sum_{i=1}^3 \tilde{c}_i(\mu) \tilde{O}_i(\mu) + H.c. \quad (13)$$

where  $\mu$  is the regularization scale, and

$$\begin{aligned} O_1 &= (\bar{q}\gamma^\mu P_L d)_1 (\bar{q}\gamma_\mu P_L d)_1, \\ O_2 &= (\bar{q}P_R d)_1 (\bar{q}P_R d)_1, \\ O_3 &= (\bar{q}P_R d)_8 (\bar{q}P_R d)_8, \\ O_4 &= (\bar{q}P_L d)_1 (\bar{q}P_R d)_1, \\ O_5 &= (\bar{q}P_L d)_8 (\bar{q}P_R d)_8, \end{aligned} \quad (14)$$

where  $q$  is either  $b$  or  $s$ , and  $P_{R(L)} = (1 + (-)\gamma_5)/2$ . The subscripts 1 and 8 indicate whether the currents are in color-singlet or in color-octet combination. The  $\tilde{O}_i$ s are obtained from corresponding  $O_i$ s by replacing  $L \leftrightarrow R$ . The Wilson coefficients  $c_i$  at  $q^2 = m_W^2$  include NP effects, coming from couplings and internal propagators. However, for most of the NP models, and certainly for the case we are discussing here, all NP particles are heavier than  $m_W$  and hence the running of the coefficients between  $m_W$  and  $\mu = \mathcal{O}(m_s \text{ or } m_b)$  are controlled by the SM Hamiltonian alone. In other words, NP determines only the boundary conditions of the renormalization group (RG) equations. For the evolution of these coefficients down to the low-energy scale, we follow Ref. [19], which uses, for  $B^0 - \bar{B}^0$  mixing,  $\mu = m_b = 4.6 \text{ GeV}$ . The low-scale Wilson coefficients, using the NLO-QCD corrections, are

$$c_i(\mu) = \sum_r \sum_s (b_r^{i,s} + \eta d_r^{i,s}) \eta^{ar} c_s(M_S) \quad (15)$$

where  $\eta = \alpha_s(M_S)/\alpha_s(m_t)$ ,  $M_S$  being the scale of NP, which, for SUSY, may be taken to be the average of the squark and the gluino masses. We will use  $M_S = 500 \text{ GeV}$  throughout the paper. For the numerical values of  $a$ ,  $b$  and  $d$  matrices we refer the reader to eq. (10) of Ref. [19] (to avoid confusion with the Wilson coefficients, we use the symbol  $d$  for the matrix denoted by  $c$  in [19]). All the numbers are not relevant for our discussion; the modification of the NP operators due to the short-distance QCD corrections is discussed in Section 4. It will be shown that only operators  $\tilde{O}_1$  and  $O_4$  are relevant at the scale  $q^2 = m_W^2$ , while there is a slight admixture of  $O_5$  at the low-energy scale.

The operators  $O_i$  are also to be renormalized at the scale  $\mu$ . The expectation values of these operators between  $\bar{B}^0$  and  $B^0$  at the scale  $\mu$  are given by

$$\begin{aligned} \langle O_1(\mu) \rangle &= \frac{2}{3} m_B^2 f_B^2 B_1(\mu), \\ \langle O_2(\mu) \rangle &= -\frac{5}{12} S_B m_B^2 f_B^2 B_2(\mu), \\ \langle O_3(\mu) \rangle &= \frac{1}{12} S_B m_B^2 f_B^2 B_3(\mu), \\ \langle O_4(\mu) \rangle &= \frac{1}{2} S_B m_B^2 f_B^2 B_4(\mu), \\ \langle O_5(\mu) \rangle &= \frac{1}{6} S_B m_B^2 f_B^2 B_5(\mu), \end{aligned} \quad (16)$$

where

$$S_B = \left( \frac{m_B}{m_b(m_b) + m_d(m_b)} \right)^2 \quad (17)$$

The  $B$ -parameters, whose numerical values are given in Section 3, have been taken from [20]. Note that the expectation values are scaled by factor of  $2m_B$  over those given in some literature due to our different normalization of the meson wavefunctions. It is trivial to check that

both conventions yield the same values for physical observables. Analogous expressions follow for the  $K^0 - \bar{K}^0$  system, with  $m_B, f_B \rightarrow m_K, f_K$ , and

$$S_K = \left( \frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2, \quad (18)$$

with  $\mu = 2$  GeV. The running of the Wilson coefficients is given by an equation which is exactly analogous to eq. (15) [21] but with different  $a, b$  and  $d$  matrices, which are evaluated at  $\mu = 2$  GeV. The  $B$ -parameters, which include all nonperturbative effects below the scale  $\mu$ , are different too.

It is noteworthy that the NLO corrections as discussed above cannot be applied when there are light quarks like  $u$  flowing in the loop, since the operator product expansion takes a different form [21]. This will be the case when we switch on the RPV interaction. There should be an enhancement coming from large logarithms; however, this puts on a tighter constraint on the RPV couplings. Since these QCD corrections are not known, we use the same procedure as adopted for heavy particles, and probably deduce somewhat lenient constraints on the parameter space for those particular couplings.

It is clear from equations (10) and (12) that at least in the SM, one can either evaluate the matrix element of  $O_1$  at  $m_W$  and compute the running to the low energy scale, or alternatively can use the parameters  $B_{K,B}$  and  $\eta_{K,B}$ , to be determined from the lattice studies. We adopt the latter method for calculating the SM amplitudes, scanning over the whole range of these parameters, so that even the nonperturbative effects can be successfully taken into account, including all the uncertainties. This gives us the most conservative bounds on the NP parameters. The NP amplitude is calculated using the former method.

## B. R-Parity violating SUSY

R-parity is a global quantum number, defined as  $(-1)^{3B+L+2S}$ , which is  $+1$  for all particles and  $-1$  for all superparticles. In the minimal version of supersymmetry and some of its variants, R-parity is assumed to be conserved *ad hoc*, which prevents single creation or

annihilation of superparticles. Such models have a natural candidate for dark matter, and have telltale signatures in collider searches as large missing energy. However, models with broken R-parity can be constructed naturally, and such models have a number of interesting phenomenological consequences [22, 23]. Some of these R-parity violating models can be motivated from an underlying GUT framework [24].

It is well known that in order to avoid rapid proton decay one cannot have both lepton number and baryon number violating RPV couplings, and we shall work with a lepton number violating model. This leads to both slepton (charged and neutral) and squark mediated decays, and new amplitudes for  $M^0 - \bar{M}^0$  mixing, where particles flowing inside the box can be (i) charged sleptons and up-type quarks, (ii) sneutrino and down type quarks, (iii) squarks and leptons. One or both of the scalar particles inside the box can be replaced by  $W$  bosons, charged Higgs bosons and Goldstone bosons (in a non-unitary gauge) (see Fig. 1). We follow the usual practice of avoiding the so-called ‘‘pure SUSY’’ contributions to the box amplitudes, *i.e.*, those coming from charginos, neutralinos or gluinos inside the loop. Not only the strongly interacting superparticles are expected to be heavier than the electroweak ones (and hence the contribution being suppressed), but also one can choose SUSY models where these contributions become negligible (*e.g.*, alignment in the squark sector, or Higgsino-dominated lighter chargino, to kill off the respective boxes.) Since the current lower bound on the slepton mass is generally weaker than that on squark mass by a factor 2-3, the slepton mediated boxes have greater chance to be numerically significant.

We start with the superpotential

$$\mathcal{W}_{\lambda'} = \lambda'_{ijk} L_i Q_j D_k^c, \quad (19)$$

where  $i, j, k = 1, 2, 3$  are quark and lepton generation indices;  $L$  and  $Q$  are the  $SU(2)$ -doublet lepton and quark superfields and  $D^c$  is the  $SU(2)$ -singlet down-type quark superfield respectively. Written in terms of component fields, this superpotential generates six terms, plus their hermitian conjugates:

$$\mathcal{L}_{LQD} = \lambda'_{ijk} \left[ \tilde{\nu}_L^i \bar{d}_R^k d_L^j + \tilde{d}_L^j \bar{d}_R^k \nu_L^i + (\bar{d}_R^k)^* \bar{\nu}_L^i d_L^j - \tilde{e}_L^i \bar{d}_R^k u_L^j - \tilde{u}_L^j \bar{d}_R^k e_L^i - (\bar{d}_R^k)^* \bar{e}_L^i u_L^j \right] + H.c. \quad (20)$$

With such a term, one can have two different kind of boxes, shown in Fig. 1, that contribute to  $M^0 - \bar{M}^0$  mixing: first, the one where one has two sfermions flowing inside the loop, alongwith two SM fermions [14], and secondly, the one where one slepton, one  $W$  (or charged Higgs or Goldstone) and two up-type quarks complete

the loop [13]. It is obvious that the first amplitude is proportional to the product of four  $\lambda'$  type couplings, and the second to the product of two  $\lambda'$  type couplings times  $G_F$ . We call them L4 and L2 boxes, respectively, for brevity, where L is a shorthand for  $\lambda'$ .

We will constrain only products of two  $\lambda'$ -type cou-

plings at a time, and assume a hierarchical structure, *i.e.*, only one product is, for all practical purpose, simultaneously nonzero (but can have a nontrivial phase). This may not be physically the most appealing scenario but keeps the discussion free from unnecessary complications. For any product, there are individual bounds on each of them, the product of which we call the direct product bound (DPB). Interesting bounds are those which are numerically smaller, and hence stronger, than the corresponding DPBs, the more the better. This turns out to be the case for almost all the products. A list of the individual bounds at the weak scale can be found in [7].

At this point, let us clarify how the bounds are obtained. There are, generically, three boxes: SM, L2 and L4. For L2 boxes, one can have same or different up-type quarks flowing inside the loop. For the former case, there is also an L4 counterpart, *i.e.*, the coupling that gives rise to the L2 box can also generate an L4 box. The operator for the L2 box is  $O_4$ , while that for the L4 box is  $\tilde{O}_1$ . The RG evolution generates a small  $O_5$  admixture to  $O_4$  at the low-energy scale, while  $\tilde{O}_1$  is just multiplicatively renormalized.

The effective Hamiltonian is obtained from a coherent sum of these three amplitudes, and there can be intricate interference patterns depending on the magnitudes and the phases of them. A general trend, however, is easy to follow. The pure SM part is proportional to  $G_F^2$  times some CKM factor. The L2 box is proportional to  $G_F \lambda'^2 / \tilde{m}^2$ , and the L4 box goes as  $\lambda'^4 / \tilde{m}^4$ . For small values of the  $\lambda'\lambda'$  coupling, L2 contribution dominates over L4, and the “bound” (as obtained by [13]) is controlled by L2. We will see that if the phases are included, this procedure does not give the bounds, so the numbers obtained in this way are not listed in tables 1 and 2. For larger values of the coupling, L4 becomes dominant, and if there happens to be a cancellation between SM and L2 amplitudes, L4 controls the show. That is, the solution describes a contour in the  $Re(\lambda'\lambda') - Im(\lambda'\lambda')$  plane. There are certain points, far from the origin, where there happens to be a complete destructive interference between SM and L2 boxes. Thus, if the product has an arbitrary phase, it is always the L4 term that gives the bound when the quarks inside are the same. We will see explicit examples of this later.

If the up-type quarks in the box are different, there is no corresponding L4 contribution (actually, there is, but that comes from four different  $\lambda'$  type couplings which we neglect systematically), and the treatment becomes simpler.

A note of caution here. In the literature, most of the bounds on the RPV couplings are not absolute, but scale with the corresponding sfermion mass that comes in the propagator. For a product coupling, the dependence is typically  $\lambda'\lambda'/m_{\tilde{f}}^2$ . The results are usually quoted for  $m_{\tilde{f}} = 100$  GeV, and bounds for higher  $m_{\tilde{f}}$  are straightforward to obtain. However, for the L4 boxes, there can be two distinct type of amplitudes, one through a quark-

slepton box, and the other through a squark-lepton box. For our analysis, we take all sleptons to be degenerate at 100 GeV, and all squarks to be degenerate at 300 GeV (100 GeV squarks are already ruled out). Thus, it is not easy to scale the bounds quickly; a good way is to remember (and scale accordingly) that at the amplitude level, the squark contribution is about 10% of the slepton contribution, and the second way, to get a more rough scaling, is to neglect the squark contribution altogether.

The effective Hamiltonian for the L4 box has been computed in [14], with degenerate sfermions. For a nondegenerate case, the expression reads ( $q = 2$  for  $K^0 - \bar{K}^0$  and  $q = 3$  for  $B^0 - \bar{B}^0$  boxes)

$$\mathcal{H}_{L4} = \frac{(\lambda'_{ik1} * \lambda'_{ikq})^2}{128\pi^2} \times \left[ \frac{1}{\tilde{m}_l^2} \left\{ I \left( \frac{m_{q_k^u}^2}{\tilde{m}_l^2} \right) + I \left( \frac{m_{q_k^d}^2}{\tilde{m}_l^2} \right) \right\} + 2 \frac{1}{\tilde{m}_q^2} \right] \tilde{O}_1 \quad (21)$$

where

$$I(x) = \frac{1 - x^2 + 2x \log x}{(1 - x)^3} \quad (22)$$

with  $I(0) = 1$ , which justifies the omission of this function in the last term, where we have explicitly assumed all leptons to be massless. In fact, this is also a reasonable assumption for all quarks except the top quark.

There may be another L4-type amplitude, with left-handed quarks as external legs and right-handed down-type squarks and neutrinos (and quarks and sneutrinos too) flowing inside the box. Neglecting the fermion masses, the effective Hamiltonian reads

$$\mathcal{H}'_{L4} = \frac{(\lambda'_{i1k} * \lambda'_{iqk})^2}{128\pi^2} \left[ \frac{1}{\tilde{m}_l^2} + \frac{1}{\tilde{m}_q^2} \right] \tilde{O}_1. \quad (23)$$

Note that this bounds a different combination. As we will see soon, there is a corresponding L2 box with the same coupling as in  $\mathcal{H}_{L4}$ , but none for  $\mathcal{H}'_{L4}$ . (The reason is simple:  $W$ -bosons couple only to left-chiral quarks, and scalar couplings are negligible.) Thus, all  $\lambda'_{i1k} \lambda'_{iqk}$  combinations are expected to yield the same bound.

The case for the L2 box is more complicated. A simplified treatment, keeping all bosons (sleptons,  $W$ s, and Higgs bosons) to be degenerate at 100 GeV, neglecting the QCD corrections, and saturating the experimental number with RPV amplitude alone, was given in [13]. Only real RPV couplings were considered since there was no data for CP violation in the B system at that time. We improve the calculation by incorporating all the above factors, but note that the bounds quoted in [13] for  $\tan \beta = 1$  are still surprisingly close to the mark. In fact, with the introduction of RPV phases, the possibility of destructive interference between the amplitudes opens up, and one would have expected a weaker bound than in

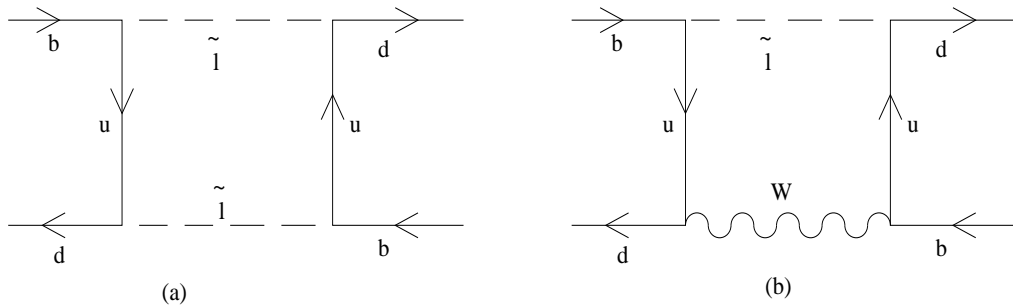


FIG. 1: *R*-parity violating contributions to  $B^0 - \overline{B}^0$  mixing. Figure (a) corresponds to  $L_4$ , while figure (b) to  $L_2$  amplitudes (see text for their meanings). For  $L_4$ , there are similar diagrams with squarks and leptons (both charged and neutral), as well as diagrams with left-chiral quarks as external legs and quarks and sneutrinos flowing in the box. For  $L_2$ , there are diagrams where the  $W$  is replaced by the charged Higgs or the charged Goldstone. The internal slepton can be of any generation, and so can be the internal charge  $+2/3$  quarks, generically depicted as  $u$ .

[13]. Consideration of the CP violating parameters  $\sin 2\beta$  and  $\varepsilon_K$  helps us to put a tighter set of constraints. The effective Hamiltonian, in this approximation, is given by

$$\mathcal{H}_{L2} = -\frac{G_F \lambda'_{ik1} \lambda'_{ikq}}{4\sqrt{2}\pi^2} V_{kq}^* V_{kd} \times \left[ (1 + \cot^2 \beta) x_k^2 J(x_k) + I(x_k) \right] O_4, \quad (24)$$

where  $x_k = m_{u_k}^2 / m_l^2$ , and

$$J(x) = \frac{-2(x-1) + (x+1) \log x}{(x-1)^3}, \quad (25)$$

and  $\cot \beta = v_d / v_u$ , the ratio of the vacuum expectation values of the two Higgs bosons that give mass to the down- and the up-type quarks respectively (not to be confused with the phase of  $V_{td}$ ). For nondegenerate masses the expression is expectedly more complicated:

$$\mathcal{H}_{L2} = -\frac{G_F \lambda'_{ik1} \lambda'_{ikq}}{4\sqrt{2}\pi^2} V_{kq}^* V_{kd} \times \left[ m_W^2 \mathcal{A}_{L2}^W + \cot^2 \beta m_k^4 \mathcal{A}_{L2}^H + m_k^4 \mathcal{A}_{L2}^G \right] O_4, \quad (26)$$

where  $\mathcal{A}_{L2}^{W,H,G}$ s are factors that come out of the box diagram integration (the superscripts indicate the SM boson in the loop). Note that as expected, the Higgs- and Goldstone-box contributions are negligible for  $k = 1, 2$ . The expressions for these factors in 't Hooft-Feynman gauge are given in Appendix A.

The Hamiltonian is slightly modified if we consider two different up-type quarks  $k$  and  $p$  inside the loop:

$$\mathcal{H}_{L2} = -\frac{G_F \lambda'_{ik1} \lambda'_{ipq}}{4\sqrt{2}\pi^2} V_{kq}^* V_{pd} \times \left[ m_W^2 \mathcal{B}_{L2}^W + \cot^2 \beta m_k^2 m_p^2 \mathcal{B}_{L2}^H + m_k^2 m_p^2 \mathcal{B}_{L2}^G \right] O_4, \quad (27)$$

and again the integration factors  $\mathcal{B}_{L2}^{W,G,H}$  are shown explicitly in Appendix A. Note that the Hamiltonian is evaluated at  $q^2 = m_W^2$  and the matrix elements are to be evaluated at the proper low-energy scale.

Apart from these new amplitudes, there are a number of other boxes, a detailed list of which is given in Appendix B, which are proportional to four  $\lambda'$  type couplings. They may be important in a scheme with a definite texture for such couplings, but for our present study, we do not consider them any further.

### III. NUMERICAL INPUTS

The major sources of the numerical inputs are: (i) the Heavy Flavor Averaging Group (HFAG) website [1] for the latest (summer 2003) updates on B physics; (ii) Particle Data Group 2002 edition [25] and update for 2004 available on the web [26]; and (iii) the inputs used in the CKMfitter package [27]. The quark masses and Wilson coefficients have been taken from [19, 21]. We use the following numbers.

Masses (all in GeV) [19, 21, 27]:

$$\begin{aligned} m_B &= 5.2794; & m_K &= 0.494; & m_b &= 4.23; \\ m_t &= 167 \pm 5; & m_c &= 1.3; & m_s(2 \text{ GeV}) &= 0.125; \\ m_d &= 0.007. \end{aligned} \quad (28)$$

The quark masses have been evaluated in the  $\overline{MS}$  scheme. The pole mass for the top quark is about 5 GeV higher and the mass for the bottom quark is 4.6 GeV. The strange quark mass, however, is a major source of error for the chirally enhanced operators  $O_4$  and  $O_5$  evaluated for the  $K^0 - \overline{K}^0$  system. The function  $S_K$  increases by a factor of 2 if we take  $m_s = 95$  MeV, which in turn tightens the bound on the  $\lambda' \lambda'$  couplings.  $m_s = 125$  MeV

is almost at the uppermost limit of the allowed range and hence generates the most conservative bound.

The mass differences, in  $\text{ps}^{-1}$ , are given by [1, 25]

$$\Delta m_B = 0.502 \pm 0.006; \quad \Delta m_K = (5.31 \pm 0.01) \times 10^{-3}. \quad (29)$$

The CP-violating parameter  $\sin(2\beta)$  is taken as the average over all charmonium modes [1], since we have reasons to suspect that there may be hints of new physics in  $b \rightarrow s$  transition. On the other hand, the theoretical prediction of  $\sin(2\beta)$ , obtained from a fit excluding the direct experimental results, is taken from [28]:

$$\begin{aligned} \sin(2\beta)_{exp} &= 0.736 \pm 0.049; \\ \sin(2\beta)_{th} &= 0.685 \pm 0.052. \end{aligned} \quad (30)$$

We also use [27]

$$|\varepsilon_K| = (2.282 \pm 0.017) \times 10^{-3}. \quad (31)$$

This brings us to the only possible caveat of our analysis. If we assume the existence of NP in the  $B^0 - \bar{B}^0$  mixing amplitude, one should not use  $\Delta m_B$  data for extracting  $V_{td}$ . In fact, what is needed is a reevaluation of the bounds excluding all inputs that may be affected from NP (*e.g.*,  $\Delta m_B$ ,  $\varepsilon_K$ , etc.). We do not venture into that here. Rather, we take the 90% confidence limit (CL) bound on  $V_{td}$  assuming unitarity of the CKM matrix and considering tree-level processes only [26]:

$$|V_{td}| = 0.0094 \pm 0.0046. \quad (32)$$

Rest of the CKM elements are taken from [26], most of them at their central value:

$$\begin{aligned} |V_{ud}| &= 0.9745, |V_{us}| = 0.224, |V_{ub}| = 0.0037(8), \\ |V_{cd}| &= 0.224, |V_{cs}| = 0.9737, |V_{cb}| = 0.0415, \\ |V_{ts}| &= 0.040(3), |V_{tb}| = 0.99913. \end{aligned} \quad (33)$$

The angle  $\gamma$  is taken to lie between  $50^\circ$  and  $72^\circ$  [27]. The analysis is fairly insensitive to its precise value.

The long- and short-distance QCD corrections to the box amplitudes are mostly determined from lattice studies. We use [27]

$$\begin{aligned} B_K &= 0.86 \pm 0.14 \pm 0.06, \\ \sqrt{B_B} f_B &= (0.228 \pm 0.033) \text{ GeV}, \\ f_K &= 159.8 \text{ MeV}, \\ \eta_K &= 1.38 \pm 0.53; \quad \eta_B = 0.55. \end{aligned} \quad (34)$$

The leptonic and semileptonic BRs for the K meson, which are of interest to us, are as follows [26]:

$$\begin{aligned} \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) &= 1.6_{0.8}^{+1.8} \times 10^{-10}; \\ \text{Br}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) &= (8.1 \pm 1.4) \times 10^{-8}; \\ \text{Br}(K^+ \rightarrow \pi^+ e^+ e^-) &= (2.88 \pm 0.13) \times 10^{-7}; \\ \text{Br}(K_L \rightarrow \mu^+ \mu^-) &= (7.24 \pm 0.14) \times 10^{-9}. \end{aligned} \quad (35)$$

There are stringent bounds on  $K_L \rightarrow e^+ e^-$  too but that is chirally suppressed compared to the  $\mu^+ \mu^-$  mode. For the B mesons, the relevant numbers may be found in [1]. We will not analyze them here (see [29] for such an analysis) but the bounds on RPV couplings obtained from B decays are not compatible with those obtained here, particularly for a 300 GeV squark.

To evaluate the QCD corrections, we take  $\alpha_s(m_Z^2) = 0.1172 \pm 0.0020$  [25], and take the SUSY scale  $M_S = 500$  GeV. The precise value of this scale is not important, however, and we can take it to be at the squark mass scale (300 GeV) without affecting the final results. The exact evolution matrix can be found in [21] and [19]; for our purpose, it is sufficient to note that for the K system, the operator  $\tilde{O}_1$  is multiplicatively renormalized by a factor 0.794 at the scale  $\mu = 2$  GeV, and the operator  $O_4$  at  $m_W$  changes to  $(3.965O_4 + 0.149O_5)$ . For the B system, the respective numbers are  $\tilde{O}_1 \rightarrow 0.820\tilde{O}_1$ ,  $O_4 \rightarrow (2.83O_4 + 0.077O_5)$ . We again stress that theoretically the procedure is questionable for boxes with light quarks flowing in the loop. However, the numbers that we obtain are fairly robust and one can very well drop the NLO corrections altogether, if necessary, without compromising the results.

The relevant  $B$ -parameters (eq. (16)) are [20, 21]

For B :

$$\begin{aligned} B_1(m_b) &= 0.87(4)_{-4}^{+5}, \\ B_4(m_b) &= 1.16(3)_{-7}^{+5}, \\ B_5(m_b) &= 1.91(4)_{-7}^{+22}, \end{aligned}$$

For K :

$$\begin{aligned} B_1(\mu) &= 0.60(6), \\ B_4(\mu) &= 1.03(6), \\ B_5(\mu) &= 0.73(10). \end{aligned} \quad (36)$$

Since the  $O_5$  admixture is small, one can take the central values for these parameters without introducing too much error.

In the supersymmetry sector, we assume only one RPV product coupling to be nonzero at a time. We take all sleptons to be degenerate at 100 GeV, and all squarks at 300 GeV. We also take  $\tan\beta (\equiv v_2/v_1) = 5$  (very low values are excluded by LEP, and the numbers are not sensitive to the precise choice of  $\tan\beta$ ), and the charged Higgs boson mass as 200 GeV (lower values are disfavored from  $b \rightarrow s\gamma$ ).

#### IV. ANALYSIS

Our bounds are summarized in tables 1 and 2, for the K and the B systems respectively. The procedure has been outlined in Section 2, and is identical for both systems. However, for the  $K^0 - \bar{K}^0$  system, the SD contribution may not be the full story. If the SD contribution is smaller, the contribution to  $\Delta m_K$  is also smaller, and

$\lambda'\lambda'$ combination	Only real	Complex, real part	Complex, imag. part	Previous bound
(i31)(i32) $\spadesuit$	$4.5 \times 10^{-6}$	$1.0 \times 10^{-5}$	$5.0 \times 10^{-6}$	0.203
(i21)(i22)	$3.9 \times 10^{-7} \dagger$	$4.0 \times 10^{-8}$	$1.7 \times 10^{-9}$	1.254
(i11)(i12)	$3.9 \times 10^{-7} \dagger$	$4.9 \times 10^{-8}$	$1.6 \times 10^{-9}$	0.109
(i31)(i22)	$3.3 \times 10^{-5} \dagger$	$5.5 \times 10^{-6}$	$1.4 \times 10^{-7}$	0.504
(i21)(i32)	$5.1 \times 10^{-7}$	$2.7 \times 10^{-6}$	$1.1 \times 10^{-6}$	0.504
(i21)(i12) $\ddagger$	$9.0 \times 10^{-8} \dagger$	$1.6 \times 10^{-8}$	$4.0 \times 10^{-10}$	$10^{-9}$
(i11)(i22)	$1.7 \times 10^{-6} \dagger$	$4.9 \times 10^{-7}$	$7.1 \times 10^{-9}$	0.370
(i31)(i12)	$7.5 \times 10^{-6} \dagger$	$2.6 \times 10^{-6}$	$2.9 \times 10^{-8}$	0.149
(i11)(i32)	$2.2 \times 10^{-6}$	$3.5 \times 10^{-5}$	$1.1 \times 10^{-5}$	0.149
(i1j)(i2j) $\spadesuit$	$2.7 \times 10^{-3} \dagger$	$2.3 \times 10^{-3}$	$6.0 \times 10^{-4}$	0.370

TABLE I: Bounds on  $\lambda'\lambda'$  combinations from  $K^0 - \overline{K^0}$  mixing and  $\varepsilon_K$ , shown for cases when the product is strictly real and when the product may be complex. The table displays the magnitudes only, and not the signs. Weakest DPBs, displayed in the last column, occur for  $i = 3$ . They are from [7] except that we scale them for 300 GeV squarks wherever necessary, unless they hit the perturbative bound. The entries marked with a dagger are bounded from  $\Delta m_K$ , see text. The one marked with a double dagger is bounded from tree-level  $K^0 - \overline{K^0}$  mixing, and those marked with a  $\spadesuit$  have bounds of the order of  $10^{-5}$  from squark-mediated  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  (see text).

hence the analysis runs for a smaller  $\Delta m_K$  and gives tighter bounds on the relevant RPV couplings. The bounds we obtain are, therefore, the most lenient ones.

It appears that the constraints on the  $K^0 - \overline{K^0}$  system are essentially controlled by  $\varepsilon_K$ , particularly when we allow for complex RPV couplings. In the SM, the CP violating part is suppressed by the CKM factors; with RPV there is no such suppression, and thus one gets tight constraints. They are shown in the third and fourth columns of table 1. The second column shows the constraints for real RPV couplings. Unless the L2 box contains a CKM phase, the imaginary part coming from RPV is zero, and there is no bound coming from  $\varepsilon_K$ ; the only constraint is that coming from  $\Delta m_K$ . These entries have been marked with a dagger.

For the  $B^0 - \overline{B^0}$  system, one noteworthy thing is that the bounds on the real and the imaginary parts of any product coupling are almost the same. (That is why we show only the real part; the same bound applies to the imaginary part too. We found it to differ by at most 10% in a few cases.) This is, of course, no numerical accident. To understand this, let us analyze the origin of these bounds. There are two main constraints for the B system:  $\Delta m_B$  and  $\sin(2\beta)$ . There will be a region, centered around the origin (since  $\Delta m_B$  can be explained by the SM alone) of  $Re(\lambda'\lambda') - Im(\lambda'\lambda')$  plane, where  $|\lambda'\lambda'|$  is small and the phase can be arbitrary. This is the SM-dominated region, where RPV creeps in to whatever place is left available. A conventional analysis, taking both SM and RPV but assuming incoherent sum of amplitudes, should generate this region only.

However, there is always scope for fully constructive or destructive interference. Consider a situation where the RPV contribution is large, so large that even af-

$\lambda'\lambda'$ combination	Only real	Complex, real part	Previous bound
(i31)(i33)	$1.7 \times 10^{-3}$	$1.7 \times 10^{-3}$	0.203
(i21)(i23)	$5.0 \times 10^{-5}$	$1.6 \times 10^{-3}$	1.254
(i11)(i13)	$8.0 \times 10^{-4}$	$2.6 \times 10^{-3}$	0.109
(i31)(i23)	$6.0 \times 10^{-6}$	$1.6 \times 10^{-4}$	0.504
(i21)(i33)	0.13	0.11	0.504
(i21)(i13)	$1.2 \times 10^{-5}$	$2.9 \times 10^{-4}$	0.370
(i11)(i23)	$2.6 \times 10^{-3}$	$1.6 \times 10^{-2}$	0.370
(i31)(i13) $\dagger$	$2.8 \times 10^{-6}$	$2.5 \times 10^{-5}$	$8 \times 10^{-8}$
(i11)(i33)	$2.7 \times 10^{-2}$	$2.7 \times 10^{-2}$	0.149
(i1j)(i3j)	$2.0 \times 10^{-3}$	$4.1 \times 10^{-3}$	0.149

TABLE II: Bounds on  $\lambda'\lambda'$  combinations from  $B^0 - \overline{B^0}$  mixing. Rest is same as table 1, except that the entry marked with a dagger tree-level  $B^0 - \overline{B^0}$  mixing.

ter a destructive interference with the SM amplitude, enough is left to saturate  $\Delta m_B$ . This RPV-dominated region (this is true for all NP models in general) gives us the bounds, and in the limit where the SM can be neglected, the bounds on  $Re(\lambda'\lambda')$  are almost the same as on  $Im(\lambda'\lambda')$ . The bounds are, however, slightly different for the case where the RPV couplings are all real to the case where they can be complex; the reason is the SM phase in mixing, which chooses a particular direction in the  $Re(\lambda'\lambda') - Im(\lambda'\lambda')$  plane not coinciding with the real axis. This complication is absent for the K system; the SM amplitude is almost real. The pattern can be understood from Figures 2(a)-2(b). It is obvious that the bound is controlled by the  $\Delta m_B$  data. In Fig. 2(a) for the black boxes, the central cross is the region where L2 is important (and the interference with SM); however, the L4-dominated region gives the bound. Fig. 2(b) shows a similar scan plot for the  $K^0 - \overline{K^0}$  box driven by  $\lambda'_{i31}\lambda'_{i32}$ , but without  $\varepsilon_K$  (this also gives the reader an idea how strong the  $\varepsilon_K$  constraint can be). However, this analysis holds even for the cases with only L2 boxes.

The relative magnitude of the bounds is also easy to understand. Consider, for example, the bounds on  $\lambda'_{i31}\lambda'_{i23}$  vis-a-vis  $\lambda'_{i21}\lambda'_{i33}$ . The relevant box diagrams have the same particle content; but the first one is proportional to  $V_{ib}V_{cd}$  ( $\sim \lambda$ ), and the second one to  $V_{td}V_{cb}$  ( $\sim \mathcal{O}(\lambda^5)$ ). The relative suppression in  $\lambda$  enhances the limit on the RPV coupling.

It is interesting to note that in most of the cases, the bounds are far better than the DPBs. The exception is the product bounds responsible for tree-level  $K^0 - \overline{K^0}$  and  $B^0 - \overline{B^0}$  mixing [9]. Apart from them,  $\lambda'_{i11}\lambda'_{i13}$  has a comparable bound from  $B^0 \rightarrow \pi^+\pi^-$  [30].

Though mostly of the same orders of magnitude, these bounds are theoretically an improvement over those obtained earlier [13–15]. We have taken into account all possible amplitudes (and the interference patterns play a nontrivial role), including the SM one, but have systematically neglected the pure supersymmetric boxes coming from gaugino exchange. The reason is that those boxes decouple in the heavy squark limit, and one can always



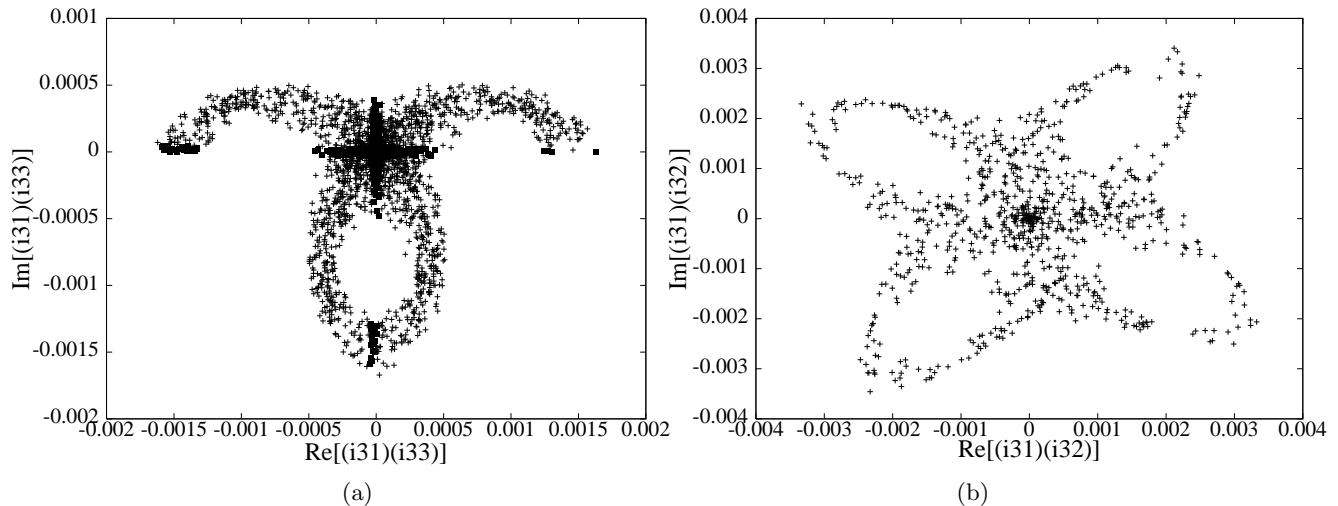


FIG. 2: (a) Allowed parameter space for  $\lambda'_{i31}\lambda'_{i33}$ . The crosses indicate the allowed parameter space switching the  $\sin(2\beta)$  constraint off, and the black boxes do the same by keeping it, along with  $\Delta m_B$ . (b) The parameter space for  $\lambda'_{i31}\lambda'_{i32}$ , which drives the  $K^0 - \bar{K}^0$  mixing. Only  $\Delta m_K$  constraint is used.

take an RPV model embedded in a minimal supersymmetric theory where such FCNC processes are somehow forbidden. It was shown in [15] that the bounds are fairly robust even if one takes into account such SUSY contributions. Furthermore, the QCD corrections are implemented upto NLO. We have also corrected a sign mistake (which, however, did not affect the result of the earlier papers since there was no interference to be considered) in the respective effective Hamiltonians. Lastly, we have also put bounds on the imaginary parts of the couplings. As we have stressed earlier, the role of the  $\sin(2\beta)$  data is to banish a sizable portion of the  $Re(\lambda'\lambda') - Im(\lambda'\lambda')$  plane, rather than constraining the absolute bounds, see Fig. 2(a). For the K system,  $\varepsilon_K$  chooses the region about the origin.

If we take a baryon-number violating RPV model, the only combinations that can be constrained from the box amplitudes are  $\lambda''_{i12}\lambda''_{i23}$  (from  $B^0 - \bar{B}^0$  mixing) and  $\lambda''_{i13}\lambda''_{i23}$  (from  $K^0 - \bar{K}^0$  mixing). The bounds are weaker by about two orders of magnitude from those on their  $\lambda'$  counterpart.

## V. CORRELATED CHANNELS

Though this paper is mainly on constraints on the  $\lambda'\lambda'$  product couplings coming from  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  boxes, let us also mention that such couplings are also responsible for squark-mediated semileptonic ( $b \rightarrow d\ell^+\ell^-$ ,  $s \rightarrow d\ell^+\ell^-$ ) and purely leptonic ( $B^0 \rightarrow \ell^+\ell^-$ ,  $K^0 \rightarrow \ell^+\ell^-$ ) B and K decays (and slepton-mediated nonleptonic decays) wherever kinematically possible.

For the B mesons, no such leptonic mode has yet been observed. The corresponding upper limits on the BRs are of the order of  $10^{-7}$  for  $\ell = e, \mu$  and  $10^{-5}$  for the  $\tau$  modes. With 300 GeV squarks, from the bounds that one obtains

here, a BR at most of the order of  $10^{-8}$  can be expected. Thus, we do not envisage to see such leptonic channels before the next-generation hadronic or super  $e^+e^-$  B factories. The semileptonic modes  $B \rightarrow K^{(*)}\ell^+\ell^-$  have been observed. However, the BRs are at the SM ballpark, and the RPV contributions are expected to be smaller by at least one order of magnitude.

We have not discussed the  $B_s$  system. That will evidently warrant a detailed analysis once the hadronic machines start running. Let us mention that the channel  $B_s \rightarrow \mu^+\mu^-$  can be mediated by a tree-level squark exchange diagram, and with the present bound of  $BR(B_s \rightarrow \mu^+\mu^-) < 0.95 \times 10^{-6}$  [1], the relevant upper limit on the RPV product coupling  $\lambda'_{2i2}\lambda'_{2i3}$  is about  $6 \times 10^{-4}$  (for 300 GeV squarks). The limit obtained from the  $B_s - \bar{B}_s$  mixing should be at the same level after a few years of data. Thus, one can have an interesting situation where there is a possibility of NP in both mixing and decay.

The situation in the K meson system is better as far as the leptonic and semileptonic modes are concerned. First, note that the four-Fermi effective Hamiltonian for leptonic and semileptonic decays is of the form [29]

$$\begin{aligned} \mathcal{H}_{RPV} = & \frac{1}{2} B_{jklm} [\bar{\ell}_j \gamma^\mu P_L \ell_i] [\bar{d}_m \gamma_\mu P_R d_k] \\ & + \frac{1}{2} B_{jklm} [\bar{\nu}_j \gamma^\mu P_L \nu_l] [\bar{d}_m \gamma_\mu P_R d_k] \\ & - \frac{1}{2} C_{jklm} [\bar{\nu}_j \gamma^\mu P_L \nu_l] [\bar{d}_k \gamma_\mu P_L d_m] + H.c., \end{aligned} \quad (37)$$

where

$$B_{jklm} = \sum_{i=1}^3 \frac{\lambda'_{jik} \lambda'_{ilm}}{m_{u/d_{L_i}}^2}, \quad C_{jklm} = \sum_{i=1}^3 \frac{\lambda'_{jki} \lambda'_{lmi}}{m_{\bar{d}_{R_i}}^2}. \quad (38)$$

We take any one to be nonzero at a time. Assuming the leptonic phase-space to be the same, and the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  to be dominated by RPV SUSY, one obtains [10]

$$\frac{Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{Br(K^+ \rightarrow \pi^0 e^+ \nu)} = \frac{|\lambda'_1 \lambda'_2|^2}{4G_F m_d^2 V_{us}^2}. \quad (39)$$

Taking  $Br(K_{e3}) = 0.0481$  (lower limit) and  $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 3.4 \times 10^{-10}$  ( $1\sigma$  upper limit), one obtains

$$|\lambda'_{ij1} \lambda'_{ij2}|, |\lambda'_{i1j} \lambda'_{i2j}| < 3.9 \times 10^{-5}. \quad (40)$$

It is clear that the modes  $\pi 2e$  or  $\pi 2\mu$  will yield a less severe bound, since the upper limit is weaker by orders of magnitude.

The decay  $K_L \rightarrow \mu^+ \mu^-$  bounds the imaginary part of the product  $\lambda'_{2i1} \lambda'_{2i2}$ . The reason is that in the limit of CP conservation,  $K_L = (K^0 - \bar{K}^0)/\sqrt{2}$ , and from eq. (37), it is easy to see that the real parts cancel out if we take the Hermitian conjugate term into account. The BR is

$$Br(K_L \rightarrow \mu^+ \mu^-) = \frac{Im(\lambda'_{2i1} \lambda'_{2i2})^2}{64\pi} \times \left(\frac{f_K m_\mu}{m_{\bar{u}_i}^2}\right)^2 \sqrt{1 - 4\frac{m_\mu^2}{m_K^2} m_K \tau_{K_L}}, \quad (41)$$

where  $\tau_{K_L}$  is the lifetime of  $K_L$ . Putting the values, the bound is

$$Im(\lambda'_{2i1} \lambda'_{2i2}) < 3.5 \times 10^{-5}. \quad (42)$$

This is, of course, an approximate bound, since the SM has not been included, and vacuum insertion is questionable for a t-channel squark exchange diagram.

In the K system, the nonleptonic decays are riddled with theoretical uncertainties. That is why one can always evade the bounds obtained from, say,  $\epsilon'/\epsilon$  [11]. On the other hand, we do not have enough nonleptonic data from the B factories to put comparable bounds on the couplings we have considered here. The only exception is  $\lambda'_{i11} \lambda'_{i13}$ , which can be bounded from the BR and the CP-asymmetry data of  $B \rightarrow \pi^+ \pi^-$ . Other affected modes are listed in Table 3. However, we note that unless a product coupling is at least of the order of  $10^{-3}$ , the RPV contribution is unlikely to affect the SM amplitude. Still, a systematic study of NP is worthwhile; for example, the CP-asymmetry in  $B \rightarrow K_S \pi^0$ , which is supposed to yield  $\sin(2\beta)$ , gives a smaller central value [31], notwithstanding the fact that the experimental errors are large:

$$\sin(2\beta)(B \rightarrow K_S \pi^0) = 0.48_{-0.47}^{+0.38} \pm 0.06. \quad (43)$$

Note that this channel is mediated by a coupling which has a bound of the order of  $10^{-2}$  only.

$\lambda' \lambda'$ combination	Quark level	Meson level	UT angle
(i21)(i23)	$b \rightarrow c \bar{c} d$	$B \rightarrow D^+ D^-$	$\beta$
	$b \rightarrow s \bar{s} d$	$B \rightarrow K \bar{K}$	$\beta$
(i11)(i13)	$b \rightarrow u \bar{u} d$	$B \rightarrow \pi(\rho)\pi(\rho)$	$\alpha$
(i21)(i13)	$b \rightarrow u \bar{c} d$	$B \rightarrow D\pi$	$\gamma$
	$b \rightarrow d \bar{s} d$	$B \rightarrow \pi \bar{K}$	SM suppressed
(i11)(i23)	$b \rightarrow c \bar{u} d$	$B \rightarrow \bar{D}\pi$	$\gamma$
	$b \rightarrow s \bar{d} d$	$B \rightarrow K_S \pi$	$\beta$
(i1j)(i3j)	$b \rightarrow d \bar{s} s$	$B \rightarrow \phi \pi$	

TABLE III: Some of the possible nonleptonic transitions mediated by the RPV couplings discussed in the paper. We show only the more interesting quark-level decays in column 2, and some typical decays in a generic manner in column 3. The last column shows the UT angle that may be affected due to the presence of RPV.

## VI. SUMMARY AND CONCLUSIONS

In this paper, we have computed the bounds on the product of two RPV couplings of the type  $\lambda' \lambda'$ , coming from  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  mixing, as well as from the  $\sin(2\beta)$  constraint. Though such a calculation is not new, we have implemented several features in the analysis which have not been taken into account in earlier studies. We have considered the exact expression for the box amplitudes, and have taken all possible amplitudes, including that of the SM, into consideration. The QCD corrections to the amplitudes have been taken upto the NLO level. We have considered the possibility that the RPV product couplings may be complex. The analysis is done in the benchmark point  $m_{H^\pm} = 200$  GeV,  $\tan\beta = 5$ , all sleptons degenerate at 100 GeV and all squarks degenerate at 300 GeV, and neglecting the pure MSSM contribution to the box amplitudes (by possibly applying to some underlying FCNC suppression principle, like alignment of the squark mass matrices).

It is to be observed that in some cases, our bounds are actually weaker than those obtained earlier by saturating the mass difference with RPV alone. The reason is that destructive interference with the SM amplitude plays a very crucial role in determining the bounds, particularly when the phase of the RPV coupling is arbitrary. There is an intricate interplay among different amplitudes as can be seen in Fig. 2(a). All in all, we consider these bounds to be the most conservative (or in other words most robust) ones.

Some of these bounds can, however, be bettered if we consider the leptonic and semileptonic K decays. Of course, one can enhance the squark mass to a limit where these bounds become weaker than those obtained from the box (the latter is not much affected by decoupling the squarks), but such extremely massive squarks are not interesting probably even for the Large Hadron Collider (LHC). There is no such competitive bound for the B system, and one must wait for the hadronic B machines, or the super  $e^+e^-$  B factories. However, some of these cou-

plings may affect the nonleptonic decay modes (which, being slepton mediated, cannot be suppressed by decoupling the squarks) of  $B$ , most important of them being  $B \rightarrow \pi^+\pi^-$ , which has been dealt with earlier [30].

The  $B_s$  system is a different proposition. With the first data on  $\Delta M_s$ , one should try to compute all the possible ways NP can affect this. Furthermore, one should also try to see whether such contributions to mixing also naturally lead to sizable (and comparable with the SM) contributions in decays like  $B_s \rightarrow J/\psi\phi$ ,  $\phi\phi$ ,  $\phi\eta'$ ,  $\mu^+\mu^-$  et cetera. In this respect RPV emerges as an excellent prospective candidate.

### Acknowledgments

A.K. has been supported by the BRNS grant 2000/37/10/BRNS of DAE, Govt. of India, and by the grant F.10-14/2001 (SR-I) of UGC, India. J.P.S. has been supported by a SERC research fellowship.

### APPENDIX A: THE KINEMATIC FUNCTIONS COMING FROM THE BOX INTEGRALS

The kinematic functions for the L2 boxes have been computed in the 't Hooft Feynman gauge, and we have explicitly checked that it is gauge independent.

First, let us deal the case where there is only one type of charge  $+2/3$  quark in the loop. We denote it by  $q$ . The bosons inside the box are a slepton (generically denoted as  $\tilde{l}$ ) and one among the W boson, the charged Higgs boson (of mass  $m_h$ ) and the charged Goldstone boson (of mass  $m_W$  in the 't Hooft-Feynman gauge).

To start with, let us define some shorthand notations:

$$Q \equiv m_q^2, \quad L \equiv m_{\tilde{l}}^2, \quad W \equiv m_W^2, \quad H \equiv m_h^2. \quad (\text{A1})$$

The kinematic factors are

$$\begin{aligned} \mathcal{A}_{L2}^W &= \frac{1}{(L-W)(L-Q)^2(W-Q)^2} \times \\ &\quad [Q(L-W)(2LW-Q(L+W)) \log Q + L^2(W-Q)^2 \log L \\ &\quad + (L-Q)\{Q(W-Q)(L-W) - W^2(L-Q) \log W\}], \\ \mathcal{A}_{L2}^G &= \frac{[W(L-Q)^2 \log W + (L-W)(Q^2-LW) \log Q - (W-Q)\{(L-W)(L-Q) + L(W-Q) \log L\}]}{(L-W)(W-Q)^2(L-Q)^2}, \\ \mathcal{A}_{L2}^H &= \mathcal{A}_{L2}^G(W \rightarrow H). \end{aligned} \quad (\text{A2})$$

The Hamiltonian is slightly modified if we have two different up-type quarks  $q_1$  and  $q_2$ . Using the shorthand  $Q_1(Q_2) \equiv m_{q_{1,2}}^2$ , one has

$$\begin{aligned} \mathcal{B}_{L2}^W &= \frac{1}{D} [Q_1 F_1 \log Q_1 + Q_2 G_1 \log Q_2 + L J_1 \log L - W K_1 \log W], \\ \mathcal{B}_{L2}^G &= -\frac{1}{D} [F_1 \log Q_1 + G_1 \log Q_2 + J_1 \log L + K_1 \log W], \\ \mathcal{B}_{L2}^H &= \mathcal{B}_{L2}^G(W \rightarrow H). \end{aligned} \quad (\text{A3})$$

Here

$$\begin{aligned} D &= (Q_1 - Q_2)(L - Q_1)(L - Q_2)(W - Q_1)(W - Q_2)(L - W), \\ F_1 &= Q_1(L - Q_2)(W - Q_2)(L - W), \\ G_1 &= -F_1(Q_1 \leftrightarrow Q_2), \\ J_1 &= (Q_1 - Q_2)(W - Q_1)(W - Q_2)L, \\ K_1 &= -J_1(L \leftrightarrow W). \end{aligned} \quad (\text{A4})$$

We have checked that in the limit  $Q_1 = Q_2$ , the  $\mathcal{B}$  functions reduce to corresponding  $\mathcal{A}$ s.

### APPENDIX B: THE FOUR- $\lambda'$ BOXES

Apart from the boxes that we have discussed, there may be boxes whose amplitudes are proportional to the

$4\lambda'$ combination	Internal propagators	Operator
$(ijq)(kjq)(ij1)^*(kj1)^*$	same $u$ , different $\tilde{\ell}$	$\tilde{O}_1$
	same $d$ , different $\tilde{\nu}$	$\tilde{O}_1$
	same $\tilde{u}$ , different $\ell$	$\tilde{O}_1$
	same $\tilde{d}$ , different $\nu$	$\tilde{O}_1$
$(i1j)(k1j)(iqj)^*(kqj)^*$	same $d$ , different $\tilde{\nu}$	$O_1$
	same $\tilde{d}$ , different $\nu$	$O_1$
$(ijq)(ikq)(ij1)^*(ik1)^*$	same $\ell$ , different $\tilde{u}$	$\tilde{O}_1$
	same $\nu$ , different $\tilde{d}$	$\tilde{O}_1$
	same $\tilde{\ell}$ , different $u$	$\tilde{O}_1$
	same $\tilde{\nu}$ , different $d$	$\tilde{O}_1$
$(i1j)(i1k)(iqj)^*(ikq)^*$	same $\nu$ , different $\tilde{d}$	$O_1$
	same $\tilde{\nu}$ , different $d$	$O_1$
$(ijq)(lkq)(ik1)^*(lj1)^*$	different $u$ and $\tilde{\ell}$	$\tilde{O}_1$
	different $d$ and $\tilde{\nu}$	$\tilde{O}_1$
	different $\ell$ and $\tilde{u}$	$\tilde{O}_1$
	different $\nu$ and $\tilde{d}$	$\tilde{O}_1$
$(i1k)(l1j)(iqj)^*(lkq)^*$	different $d$ and $\tilde{\nu}$	$O_1$
	different $\nu$ and $\tilde{d}$	$O_1$

TABLE IV: Different  $4\lambda'$  combinations, and the corresponding propagators.

product of four different  $\lambda'$  couplings. They are not important if we assume a hierarchical coupling scheme where one and only one product is numerically significant, but may appear in a scheme with a given texture at a high scale which results in a number of nonzero couplings at the weak scale. Some of these couplings have

been shown in, *e.g.*, [14].

The effective Hamiltonian is always proportional to, apart from the four  $\lambda'$  couplings, a factor of  $1/64\pi^2$  times a momentum-space integral  $F(p^2)$  of the form

$$F(p^2) = \int \frac{p^4}{\prod_{i=1}^4 (p^2 + m_i^2)} dp^2 \quad (\text{B1})$$

where the denominator indicates the four propagators inside the box. To obtain a quick numerical estimate, we may put all SM fermions except the top quark to be massless, all sleptons (neutral and charged) to be degenerate (say at 100 GeV) and all squarks to be degenerate too (say at 300 GeV). The operator is either  $O_1$  or  $\tilde{O}_1$ , see eq. (14). If the sneutrinos are not their own antiparticles, there cannot be any operators of the form  $O_4$  or  $O_5$ .

In the following table, we show all possible combinations of the  $4\lambda'$  product, with the propagators. Expanded, they look like, *e.g.*, for same up-type quarks but different sleptons:

$$\mathcal{H}_{L4} = \frac{\lambda'_{ijq}\lambda'_{kjq}\lambda'^*_{ij1}\lambda'^*_{kj1}}{64\pi^2}\tilde{O}_1 F(p^2) \quad (\text{B2})$$

where  $F(p^2)$  is to be evaluated with two quarks and two different sleptons.

Note that if these combinations are present, bounds coming from L2 boxes alone have to be reevaluated.

- 
- [1] See <http://www.slac.stanford.edu/xorg/hfag/>, the website of the Heavy Flavor Averaging Group, for the Particle Data Group 2003 update of the rare decay (hadronic, charmless) data, averaged over BaBar, Belle and CLEO collaborations.
- [2] See Ref. [1]; also K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **93**, 021601 (2004).
- [3] B. Aubert *et al.* (BaBar Collaboration), hep-ex/0207070; K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 261602 (2003); K. Hagiwara *et al.* (Particle Data Group Collaboration), Phys. Rev. D **66**, 010001 (2002) and 2004 update (URL: <http://pdg.lbl.gov>). Also see Ref. [1] for Summer 2003 update.
- [4] There are some attempts to introduce new physics in  $\varepsilon'/\varepsilon$ , *e.g.*, see A. Masiero and H. Murayama, Phys. Rev. Lett. **83**, 907 (1999).
- [5] D. Chang, A. Masiero and H. Murayama, Phys. Rev. D **67**, 075013 (2003).
- [6] S. Jäger and U. Nierste, hep-ph/0312145.
- [7] B.C. Allanach, A. Dedes and H.K. Dreiner, Phys. Rev. D **60**, 075014 (1999); Phys. Rev. D **69**, 115002 (2004).
- [8] R. Barbieri and A. Masiero, Nucl. Phys. **B267**, 679 (1986).
- [9] R. Barbieri, A. Strumia and Z. Berezhiani, Phys. Lett. **B407**, 250 (1997).
- [10] K. Agashe and M. Graesser, Phys. Rev. D **54**, 4445 (1996).
- [11] S.A. Abel, Phys. Lett. **B410**, 173 (1997).
- [12] J.-H. Jang, Y.G. Kim and J.S. Lee, Phys. Lett. **B408**, 367 (1997); Phys. Rev. D **58**, 035006 (1998); J.-H. Jang, J.K. Kim and J.S. Lee, Phys. Rev. D **55**, 7296 (1997); D. Guetta, Phys. Rev. D **58**, 116008 (1998); K. Huitu *et al.*, Phys. Rev. Lett. **81**, 4313 (1998); D. Choudhury, B. Dutta and A. Kundu, Phys. Lett. **B456**, 185 (1999); G. Bhattacharyya and A. Datta, Phys. Rev. Lett. **83**, 2300 (1999); G. Bhattacharyya, A. Datta and A. Kundu, Phys. Lett. **B514**, 47 (2001); D. Chakraverty and D. Choudhury, Phys. Rev. D **63**, 075009 (2001); D. Chakraverty and D. Choudhury, Phys. Rev. D **63**, 112002 (2001); H. Dreiner, G. Polesello and M. Thormeier, Phys. Rev. D **65**, 115006 (2002); J.P. Saha and A. Kundu, Phys. Rev. D **66**, 054021 (2002); A. Datta, Phys. Rev. D **66**, 071702 (2002); A. Akeroyd and S. Recksiegel, Phys. Lett. **B541**, 121 (2002); B. Dutta, C.S. Kim and S. Oh, Phys. Rev. Lett. **90**, 011801 (2003); A. Kundu and T. Mitra, Phys. Rev. D **67**, 116005 (2003); B. Dutta *et al.*, hep-ph/0312388, hep-ph/0312389.
- [13] G. Bhattacharyya and A. Raychaudhuri, Phys. Rev. D **57**, 3837 (1998).
- [14] B. de Carlos and P. White, Phys. Rev. D **55**, 4222 (1997).
- [15] J.P. Saha and A. Kundu, Phys. Rev. D **69**, 016004 (2004).
- [16] A.J. Buras and R. Fleischer, hep-ph/9704376, also in

- Heavy Flavours II, World Scientific, Singapore (1997), ed. A.J. Buras and M. Lindner.
- [17] E.A. Paschos and U. Türke, Phys. Rep. **178**, 145 (1989).
- [18] D. London, N. Sinha and R. Sinha, Phys. Rev. D **69**, 114013 (2004).
- [19] D. Bećirević *et al.*, Nucl. Phys. **B634**, 105 (2002).
- [20] D. Bećirević *et al.*, J. High Energy Physics **0204**, 025 (2002).
- [21] M. Ciuchini *et al.*, J. High Energy Physics **9810**, 008 (1998).
- [22] G. Farrar and P. Fayet, Phys. Lett. **B76**, 575 (1978); S. Weinberg, Phys. Rev. D **26**, 287 (1982); N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982); C. Aulakh and R. Mohapatra, Phys. Lett. **B119**, 136 (1982).
- [23] For recent reviews, see G. Bhattacharyya, Nucl. Phys. (Proc. Suppl.) **B52A**, 83 (1997); hep-ph/9709395; H. Dreiner, hep-ph/9707435; R. Barbier *et al.*, hep-ph/9810232.
- [24] L.J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984); D.E. Brahm and L.J. Hall, Phys. Rev. D **40**, 2449 (1989); T.G. Rizzo, Phys. Rev. D **46**, 5102 (1992); K. Tamvakis, Phys. Lett. **B382**, 251 (1996); G.F. Giudice and R. Rattazzi, Phys. Lett. **B406**, 321 (1997);
- [25] K. Hagiwara *et al.* (Particle Data Group Collaboration), Phys. Rev. D **66**, 010001 (2002).
- [26] S. Eidelman *et al.* (Particle Data Group Collaboration), Phys. Lett. **B592**, 1 (2004), also available at <http://pdg.lbl.gov>.
- [27] See <http://ckmfitter.in2p3.fr> for the latest version of the CKMfitter package. For earlier versions, see S. Laplace, hep-ph/0209188; A. Höcker *et al.*, Eur. Phys. J. C **21**, 225 (2001).
- [28] M. Ciuchini *et al.*, hep-ph/0307195.
- [29] J.P. Saha and A. Kundu, in [12].
- [30] G. Bhattacharyya, A. Datta and A. Kundu, hep-ph/0212059.
- [31] B. Aubert *et al.* (BaBar Collaboration), hep-ex/0403001.
- [32] We use  $B^0$  and  $\overline{B}^0$  for flavor eigenstates,  $B_d$  for a state whose flavor composition is immaterial, and B for a generic  $B^0/\overline{B}^0/B^\pm$  meson.
- [33] This may not be a puzzle. See, *e.g.*, S. Nandi and A. Kundu, hep-ph/0407061.