B decay anomalies in an effective theory

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We investigate how far a new physics scenario affecting primarily the third generation fermions can ameliorate the tension between B-decay observables and Standard Model expectations. Adopting a model-independent approach, we find that among the three observables that show signs of such a tension, *viz.* the branching fractions for $B^+ \to \tau\nu$, $B_d \to D(D^*)\tau\nu$, and the like-sign dimuon anomaly in neutral B decays, the first two can be explained adequately, while there is only a marginal improvement for the third. As a spin-off, it is shown that one can also accommodate a change in the branching fraction of the Higgs boson to a τ lepton pair from the SM expectation, if such a change is established in future data.

I. INTRODUCTION

While the purported discovery of the Higgs boson at the Large Hadron Collider (LHC) [1, 2] seems to vindicate the Standard Model (SM), there are enough reasons to believe that the latter is but an effective theory, valid only up to a certain energy scale, with a more complete theory lurking beyond that. One of the major reasons for such a belief is the fine-tuning problem associated with such an elementary scalar; also of considerable import are issues such as the existence of the dark matter, or the baryon asymmetry in the universe.

This acts as a strong motivation to look for signals, both direct and indirect, of such a new theory. While direct signals very often involve production of new particles, the indirect signals will, most probably, be manifested as modifications to SM observables by new effective operators, or even the same operators as in the SM, but with modified Wilson coefficients.

B meson observables have constituted a favourite hunting ground for indirect signals. Over the years, several experiments, including B-factories, Tevatron, and even the LHC, have reported observables that are not in good agreement with the SM. While the tension is not so overwhelming as to claim unquestioned evidence of New Physics (NP), the pattern is interesting. Here, one must remember to tackle the theoretical uncertainties carefully; some of the discrepancies, like the longitudinal polarization anomaly in the decay of a B meson to two vector mesons, vanished because of a more careful reappraisal of the SM effects.

Let us begin by considering a few observables which are not in full conformity with the SM expectations:

- the large branching ratio of $B \to D(D^*)\tau\nu$, with a combined tension of 3.4σ [3];

- the large branching ratio of $B^+ \to \tau^+\nu$, with a tension of 1.6σ [4] [1];

- the like-sign dimuon asymmetry, with a tension of 3.9σ [6].

It is interesting to note that the first two involve a τ lepton in the final state. This motivates us to ask if there exists one or more new effective operators involving the b quark and the τ lepton. Such a possibility was raised in Ref. [7], and further investigated in Refs. [8–13]. One might feel tempted to add to this list a hint of another anomaly: the branching ratio of $H \to \tau^+\tau^-$ seems to be a bit on the lower side than that expected in the SM [1].

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[1] The tension has come down very recently; it was about 2.8σ before the publication of the latest Belle result [5].

The lowest dimensional operators of interest can, generically, be expressed as $(\overline{b}\,\Gamma_A\,s)\,(\overline{\tau}\,\Gamma_B\,\tau)$ where $\Gamma_{A,B}$ are appropriate combinations of the Dirac matrices. As shown in Ref. [9], this four-fermion operator is relatively unconstrained. This leads to a new contribution to the B_s–$\overline{B}_s$ mixing amplitude, with a nonzero absorptive part: cutting the intermediate τ propagators can yield an on-shell τ-pair. Thus, one has a new contribution to $\Delta\Gamma_s$, the difference in the widths of the two B_s mass eigenstates, which, in turn, ameliorates the apparent discrepancy in the like-sign dimuon asymmetry. However, the strength of any such operator is ultimately constrained by the mass difference ΔM_s of the B_s mass eigenstates.

Considering the fact that there is hardly any tension in the data involving electrons or muons in the final state (except the dimuon anomaly, to which we will come shortly), one might feel tempted to invoke one or more effective operators involving only third generation quarks and leptons. While such an effective operator based study was undertaken in Refs. [10, 13], the constraints on ΔM_s were not correlated with those coming from the decay width difference $\Delta\Gamma_s$; they were assumed to be independent numbers. The authors of Ref. [11] discussed the effectiveness of ΔM_s as a possible constraint on the parameter space.

In this paper, we adopt a different approach. To begin with, we posit a single effective operator involving a third-generation quark current and a third-generation lepton current. As it involves only third-generation fields, the constraints on the Lorentz structure for the same, or on the magnitude of the corresponding Wilson coefficient is relatively weak. For example, just restricting the new couplings to the perturbative regime ensures that $\Upsilon(1S) \to \tau^+\tau^-$ does not receive a significant contribution over and above the SM amplitude, which is electromagnetic in nature. We will, however, not venture to discuss any particular models that might predict such an interaction, and adopt, instead, a bottom-up approach.

A theory of flavour would, generically, dictate that such an operator would be written in the weak basis. On the breaking of the electroweak symmetry, the quark fields would need to be rotated to the mass basis. This leads to a plethora of new operators, related to the original through the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Included, amongst others, are those leading to $b \to s\tau^+\tau^-$ and $b \to s\nu\overline{\nu}$ (the lepton fields are not rotated, so we will always write ν for ν_τ). We will see that, even with a moderate NP scale ~ 1 TeV, the constraints from ΔM_s are so strong for vector and axial-vector ($\{V, A\}$) operators that the effects on $\Delta\Gamma_s$ and $B_s \to \tau^+\tau^-$ are bound to be unobservably small. Thus, the explanation of the dimuon anomaly must lie elsewhere, while this scheme can successfully explain the charged-current decays. The outlook is better if the effective operators are of scalar and pseudoscalar ($\{S, P\}$) variety. Indeed, such a scenario predicts a rather strong enhancement of the branching fraction of $B_c \to \tau\nu$ over its SM prediction. As for the tensor current operators, the corresponding Wilson coefficients are severely constrained [11] from radiative decays like $b \to s\gamma$, and so we will not consider them any further.

The rest of the paper is arranged as follows. In the next section, we will briefly go through the existing data. In Section III, we will discuss the new operators; first, the $\{S, P\}$ type, and then the $\{V, A\}$ type. In Section IV, we show how these operators may help in bringing down the tension with the SM. We summarize and conclude in Section V.

II. EXISTING CONSTRAINTS

A. $B \to D(D^*)\tau\nu$

The importance of studying the $B \to D(D^*)\tau\nu$ modes for a possible signal of new physics has already been pointed out in the literature [14]. The BaBar Collaboration [3] measured the branching fractions for these two modes, and they are above the SM predictions. They are also not consistent with a type II two-Higgs doublet model (such as the minimal supersymmetric extension of the SM). The implications of the data as a possible hint of physics beyond the SM have been studied in [12].

It is particularly useful to consider the ratios $R(D)$ and $R(D^*)$, defined as

$$R(D^{(*)}) = \frac{\mathrm{Br}(B \to D^{(*)}\tau\nu)}{\mathrm{Br}(B \to D^{(*)}\ell\nu)} \tag{1}$$

as these are largely free of the uncertainties—*e.g.*, those in the form factors—that exclusive modes are often prey to. The SM predictions are

$$R(D) = 0.297 \pm 0.017\,, \qquad R(D^*) = 0.252 \pm 0.003\,, \tag{2}$$

while the BaBar Collaboration quotes [3]

$$R(D) = 0.440 \pm 0.058 \pm 0.042\,, \qquad R(D^*) = 0.332 \pm 0.024 \pm 0.018\,. \tag{3}$$

It should be noted that a recent calculation [16] in unquenched lattice QCD gives, in the SM, $R(D) = 0.316 \pm 0.012 \pm 0.007$. This is consistent with the earlier SM prediction, but cannot explain the tension with the data[2].

Using Eqs. (2) and (3), and adding all errors in quadrature, we get

$$\frac{R(D)_{exp}}{R(D)_{SM}} = 1.481 \times (1 \pm 0.173), \quad \frac{R(D^*)_{exp}}{R(D^*)_{SM}} = 1.317 \times (1 \pm 0.091). \tag{4}$$

B. $B \to \tau\nu$ and $B_c \to \tau\nu$

The partial decay width $B \to \tau\nu$, in the SM, is given by

$$\Gamma(B \to \tau\nu) = \frac{1}{8\pi} G_F^2 |V_{ub}|^2 f_B^2 m_\tau^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2. \tag{5}$$

The world average is [4]

$$\mathrm{Br}(B \to \tau\nu) = (11.5 \pm 2.3) \times 10^{-5}, \tag{6}$$

while the theoretical prediction is

$$\mathrm{Br}(B \to \tau\nu)_{\mathrm{SM}} = \left(7.57^{+0.98}_{-0.61}\right) \times 10^{-5}, \tag{7}$$

which gives a tension at the level of 1.6σ [4]. The theoretical uncertainty comes from those in the decay constant f_B and the CKM matrix element V_{ub}. While the discrepancy has eased considerably, from 2.8σ to 1.6σ, after the publication of the new Belle result [5], there is still a non-negligible tension between the value of $|V_{ub}|$ determined from this decay, and that determined indirectly from the sides of the unitarity triangle, or an average of direct inclusive $(B \to X_u \ell \nu)$ and exclusive $(B \to \pi \ell \nu)$ measurements.

The discrepancy has led to several attempts in the literature to explain this as a possible NP signal. However, the explanations based on the existence of only a charged Higgs boson of type-II are ruled out at 95% confidence limit from the combined analysis of processes like $b \to s\gamma$, $Z \to b\bar{b}$, $B^0 - \overline{B}^0$ mixing [17]; the goodness-of-fit is hardly better than the fit with the SM alone. Models like R-parity violating supersymmetry fare much better and give a satisfactory explanation of the excess [18]. Note that all those analyses were performed using the pre-2012 data.

A similar expression as in Eq. (5) holds for $B_c \to \tau\nu$. For numerical evaluation, one might use

$$f_{B_c} = (395 \pm 15)\ \mathrm{MeV}, \quad \tau_{B_c} = 0.458 \pm 0.030\ \mathrm{ps}. \tag{8}$$

C. B_s–$\overline{B}_s$ mixing: ΔM_s, $\Delta\Gamma_s$, β_s and ϕ_s

While there are no apparent tensions in this sector at present, the data, as we will soon see, acts as a very tight constraint on NP operators. The mass splitting between two B_s mass eigenstates, $\Delta M_s \approx 2|M_{12s}|$, is extremely well-measured [19], namely

$$\Delta M_s = 17.719 \pm 0.043\ \mathrm{ps}^{-1}. \tag{9}$$

This agrees very well with the SM expectation [20], *viz.*

$$\Delta M_s\ (\mathrm{SM}) = (17.3 \pm 2.6)\ \mathrm{ps}^{-1}, \tag{10}$$

and acts as a very tight constraint on NP models. There are two relevant phases in the B_s–$\overline{B}_s$ system. The first one, the mixing phase, is defined as

$$\beta_s = \arg\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right), \tag{11}$$

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[2] The Belle collaboration measurements[15] viz.

$$R(D) = 0.70^{+0.19+0.11}_{-0.18-0.09}, \qquad R(D^*) = 0.47^{+0.11+0.06}_{-0.10-0.07}.$$

while being even further away from the SM expectations, nonetheless are consistent with these as well as the BaBar results. This agreement, though, is but a consequence of the large error margins.

while the second one, responsible for semileptonic asymmetries, is given by

$$\phi_s = \arg\left(-\frac{M_{12s}}{\Gamma_{12s}}\right).$$ (12)

The SM predictions [19] are

$$\phi_s = 0.0041 \pm 0.0007 \,, \quad -2\beta_s = -0.038 \pm 0.002 \,.$$ (13)

The experimental numbers are

$$-2\beta_s = -0.040^{+0.090}_{-0.085}$$ (14)

from direct determination, and

$$-2\beta_s = -0.0363^{+0.0016}_{-0.0015}$$ (15)

from an indirect global fit [19]. We will use the former number.

Note that B_s lifetime is rather ill-defined, as the two mass eigenstates have a significant lifetime difference, namely

$$\tau_{B_{sL}} = 1.408 \pm 0.017 \text{ ps} \,, \qquad \tau_{B_{sH}} = 1.626 \pm 0.023 \text{ ps} \,.$$ (16)

Averaging over the two, we have

$$\tau_{B_s}(\text{average}) = \frac{2}{\Gamma_L + \Gamma_H} = 1.509 \pm 0.012 \text{ ps} \,,$$ (17)

which should be compared with $\tau_{B_d} = 1.519 \pm 0.007$ ps. Thus,

$$\frac{\tau_{B_s}}{\tau_{B_d}} = 0.993 \pm 0.009 \,,$$ (18)

while the SM expectation for this ratio lies between 0.99 and 1.01 [19].

The width difference $\Delta\Gamma_s$ is given by

$$\Delta\Gamma_s = 2\,|\Gamma_{12s}|\cos(\phi_s)\,.$$ (19)

While the SM predicts $\Delta\Gamma_s > 0$, there was a sign ambiguity earlier in its determination. Recently LHCb, from the decay $B_s \to J/\psi K^+ K^-$, found $\Delta\Gamma_s > 0$ with a 4.7σ confidence level [21]. The experimental number, an average over various measurements [19],

$$\Delta\Gamma_s = 0.095 \pm 0.014 \text{ ps}^{-1}$$ (20)

is to be compared with the SM prediction [20]

$$\Delta\Gamma_s \text{ (SM)} = 0.087 \pm 0.021 \text{ ps}^{-1}\,.$$ (21)

D. The like-sign dimuon asymmetry

The like-sign dimuon asymmetry, defined as

$$A_{sl}^b = \frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)}\,,$$ (22)

and measured with 9.0 fb^{-1} of data at the DØ Collaboration is [6]

$$A_{sl}^b = (-7.87 \pm 1.96) \times 10^{-3}\,.$$ (23)

This can be expressed as individual flavour-specific (fs) semileptonic asymmetries coming from B_d and B_s:

$$A_{sl}^b = (0.595 \pm 0.022)\,a_{fs}^d + (0.405 \mp 0.022)\,a_{fs}^s\,,$$ (24)

where the numbers in the parentheses are the production fractions for B_d and B_s. The SM expectations are

$$a_{fs}^d = (-4.1 \pm 0.6) \times 10^{-4}, \quad a_{fs}^s = (1.9 \pm 0.3) \times 10^{-5}, \tag{25}$$

which give the SM prediction

$$(A_{sl}^b)_{SM} = (-2.4 \pm 0.4) \times 10^{-4}. \tag{26}$$

Comparing Eqs. (23) and (26), one finds a 3.9σ discrepancy between theoretical prediction and experiment. a_{fs}^d has already been measured by BaBar and Belle; the average [19]

$$a_{fs}^d = (-3.3 \pm 3.3) \times 10^{-3} \tag{27}$$

is consistent with the SM. This gives an indirect prediction for a_{fs}^s, viz.

$$a_{fs}^s = (-1.52 \pm 1.04) \times 10^{-2}, \tag{28}$$

where the error has been symmetrized. We have neglected the correlation between a_{fs}^d and a_{fs}^s, but have taken the uncertainties in the production fractions into account. Recently, the DØ Collaboration directly measured $a_{fs}^s = (-1.08 \pm 0.72\,(\text{stat}) \pm 0.17\,(\text{syst})) \times 10^{-2}$ [22] which is also consistent with the SM expectation. The HFAG collaboration averages over several direct measurements of a_{fs}^s and quotes [19]

$$a_{fs}^s = (-1.05 \pm 0.64) \times 10^{-2} \tag{29}$$

but this has a nonzero correlation with a_{fs}^d.

This gives a weak constraint on ϕ_s:

$$\tan \phi_s = a_{fs}^s \frac{\Delta M_s}{\Delta \Gamma_s}. \tag{30}$$

If there is some NP contributing to both M_{12s} and Γ_{12s}, one can parametrize the NP contribution as

$$\begin{aligned}
M_{12} &= M_{12}^{SM} + M_{12}^{NP} &\equiv M_{12}^{SM} R_M \exp(i\phi_M), \\
\Gamma_{12} &= \Gamma_{12}^{SM} + \Gamma_{12}^{NP} &\equiv \Gamma_{12}^{SM} R_\Gamma \exp(i\phi_\Gamma),
\end{aligned} \tag{31}$$

resulting in [8]

$$\phi_s = \phi_s^{SM} + \phi_M - \phi_\Gamma. \tag{32}$$

Thus, there are two ways to have a large a_{fs}^s; either a large contribution to $\Delta \Gamma_s$ or a large $\phi_s \sim \pi/2$. But $\phi_s^{SM} \approx 0$ and $\phi_M \equiv -2\beta_s$ is known to be small, so a large ϕ_s necessarily warrants a large ϕ_Γ, and hence a large contribution to Γ_{12}.

Taking all the existing constraints into account, it was shown [7, 8, 11] that $b \to s\tau^+\tau^-$ is a viable option to generate a large Γ_{12}. However, such a new channel decreases the lifetime of B_s compared to B_d; moreover, one does not expect $\text{Br}(B_s \to \tau^+\tau^-)$ to be more than 3-3.5% [11]. The inclusive mode $B(B_d \to X_s\tau\tau)$ is constrained to be less than 5% [23], while BaBar gives a 90% limit [24]

$$B(B^+ \to K^+\tau^+\tau^-)|_{q^2 > 14.23\ \text{GeV}^2} < 3.3 \times 10^{-3}. \tag{33}$$

However, not all Lorentz structures that contribute to a new absorptive part in B_s–$\overline{B}_s$ mixing contribute simultaneously to $B_s \to \tau^+\tau^-$ or $B^+ \to K^+\tau^+\tau^-$. At the same time, there can be significant long-distance effects in B_s–$\overline{B}_s$ mixing, through meson loops, and they can have a non-negligible contribution in $\Delta \Gamma_s$ [25, 26]. While we will discuss these issues in detail later, the crucial point to note is that the NP contributing to Γ_{12} should, in general, contribute to M_{12} also, and the mass difference ΔM_s is so tightly constrained that this leaves only a very small room for any NP.

E. Di-tau suppression

Looking for the SM Higgs in the $H \to \tau\tau$ mode, the CMS collaboration has failed to see [1] an unambiguous excess over the background. Indeed, for the preferred mass of $m_H = 125$ GeV (corresponding to the much-touted diphoton and four-lepton excesses), it is only able to impose a 95% C.L. upper limit on the ditau excess. With the result being similar for the ATLAS collaboration [2] as well, the $pp \to H \to \tau\tau$ cross-section is, in fact, consistent with zero, viz., $\sigma/\sigma_{SM} = 0.100^{+0.714}_{-0.699}$ [27].

Observable	SM	Expt
$-2\beta_s$	-0.038 ± 0.002	$-0.040^{+0.090}_{-0.085}$
τ_{B_s}		1.509 ± 0.012 ps
τ_{B_s}/τ_{B_d}	$0.99 - 1.01$	0.993 ± 0.009
$\Delta M_s \approx 2\|M_{12s}\|$	(17.3 ± 2.6) ps^{-1}	(17.719 ± 0.043) ps^{-1}
$\Delta\Gamma_s \approx 2\|\Gamma_{12s}\|\cos\phi_s$	(0.087 ± 0.021) ps^{-1}	(0.095 ± 0.014) ps^{-1}
A^b_{sl}	$(-2.4 \pm 0.4) \times 10^{-4}$	$(-7.87 \pm 1.96) \times 10^{-3}$
$a^d_{fs} \equiv a^d_{sl}$	$(-4.1 \pm 0.6) \times 10^{-4}$	$(-3.3 \pm 3.3) \times 10^{-3}$
$a^s_{fs} \equiv a^s_{sl}$	$(1.9 \pm 0.3) \times 10^{-5}$	$(-1.05 \pm 0.64) \times 10^{-2}$
$R(D)$	0.297 ± 0.017	0.440 ± 0.072
$R(D^*)$	0.252 ± 0.003	0.332 ± 0.030
$\mathrm{Br}(B^+ \to \tau^+\nu)$	$(7.57^{+0.98}_{-0.61}) \times 10^{-5}$	$(11.5 \pm 2.3) \times 10^{-5}$

TABLE I: Inputs used in our analysis. For details, see text.

F. Numbers used for the analysis

Apart from the numbers shown in the previous subsections, a summary of which is given in Table 1, we also use the following for our analysis:

$$m_{B^+} = m_{B_d} = 5.279 \text{ GeV}, \quad \tau_{B^+} = 1.641 \text{ ps}, \quad \tau_{B_d} = 1.519 \text{ ps}, \quad m_{B_s} = 5.367 \text{ GeV}, \tag{34}$$

and

$$|V_{td}| = (8.67^{+0.29}_{-0.31}) \times 10^{-3}, \quad |V_{ts}| = 0.0404^{+0.0011}_{-0.0005}, \quad |V_{cb}| = 0.0412^{+0.0011}_{-0.0005},$$
$$|V_{ub}| = (3.49 \pm 0.13) \times 10^{-3}, \quad \gamma \approx \arg(V^*_{ub}) = 77°. \tag{35}$$

Note that $|V_{ts}|$ is measured from B_s–$\overline{B}_s$ mixing, but if we talk about new physics in the mixing and hence ΔM_s, we should, instead, use $|V_{ts}|$ as determined from the unitarity constraints. The central value as determined from the unitarity is 0.0404; purely from ΔM_s measurement, this comes out to be 0.0429 ± 0.0026. The error margin in γ is not important for our analysis, and so we use the central value [19]. Note that only the difference of γ and the weak phase coming in the NP amplitude is relevant for our purpose; the latter is *a priori* unknown and must be treated as a free parameter of the theory. For the evaluation of ΔM_s, we have used the unquenched lattice value

$$f_{B_s}\sqrt{B_{B_s}} = 248 \pm 15 \text{ MeV}. \tag{36}$$

III. THE NEW EFFECTIVE OPERATORS

Let us, now, consider a set of possible operators involving third-generation fermions, satisfying both Lorentz and gauge invariance. These might be of the form

$$\mathcal{O}_S = A(\overline{Q}_{3L}d_{3R})(\overline{e}_{3R}L_{3L}) + B(\overline{Q}_{3L}u_{3R})(\overline{e^c}_{3R}L'^c_{3L}) + \text{h.c.} \tag{37}$$

with $L'_3 \equiv i\sigma_2 L_3$, or

$$\mathcal{O}_V = C\left[\overline{Q}_{3L}\gamma^\mu\tau_a Q_{3L}\right]\left[\overline{L}_{3L}\gamma_\mu\tau_a L_{3L}\right], \tag{38}$$

where Q_3, L_3, u_3, d_3, and e_3 stand for the doublet quark, doublet lepton, singlet up-type, singlet down-type, and singlet charged lepton of the third generation respectively. In view of the experimental measurements that we seek to address, we limit ourselves, here, to only those operators that admit charged-current interactions. Furthermore, we do not consider tensor operators as their Wilson coefficients are very tightly constrained from radiative decays. In

terms of component fields, we can write the scalar-pseudoscalar operators as

$$
\begin{aligned}
\mathcal{O}_S &= A\left[(\bar{b}P_Rb)(\bar{\tau}P_L\tau) + (\bar{t}P_Rb)(\bar{\tau}P_L\nu) + (\bar{b}P_Lb)(\bar{\tau}P_R\tau) + (\bar{b}P_Lt)(\bar{\nu}P_R\tau)\right] \\
&\quad + B\left[(\bar{t}P_Rt)(\bar{\tau}P_L\tau) - (\bar{b}P_Lt)(\bar{\nu}P_R\tau) + (\bar{t}P_Lt)(\bar{\tau}P_R\tau) - (\bar{t}P_Rb)(\bar{\tau}P_L\nu)\right], \\
&= A\left[(\bar{b}P_Rb)(\bar{\tau}P_L\tau) + \text{h.c.}\right] + B\left[(\bar{t}P_Rt)(\bar{\tau}P_L\tau) + \text{h.c.}\right] \\
&\quad + (A-B)\left[(\bar{t}P_Rb)(\bar{\tau}P_L\nu) + (\bar{b}P_Lt)(\bar{\nu}P_R\tau)\right] \\
&= \frac{1}{2}A\left[(\bar{b}b)(\bar{\tau}\tau) - (\bar{b}\gamma_5 b)(\bar{\tau}\gamma_5\tau)\right] + \text{similar terms}.
\end{aligned}
\tag{39}
$$

Eq. (39) shows that the neutral current operators have a coefficient different from that for the charged current operators. In fact, there are two neutral current operators now, one involving scalar currents and the other involving pseudoscalar currents. While we will discuss later the consequences of such an operator structure, note that the τ Yukawa coupling can, in principle, be significantly modified by a top loop. Corrections to the Yukawa couplings of other third generation fermions are negligible.

In a similar vein,

$$
\begin{aligned}
\mathcal{O}_V &= C\left[(\bar{b}\gamma^\mu P_L t)(\bar{\nu}\gamma_\mu P_L \tau) + (\bar{t}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu P_L \nu)\right. \\
&\quad \left. + \frac{1}{2}\left(\bar{b}\gamma^\mu P_L b - \bar{t}\gamma^\mu P_L t\right)(\bar{\tau}\gamma_\mu P_L \tau - \bar{\nu}\gamma_\mu P_L \nu)\right].
\end{aligned}
\tag{40}
$$

To make explicit the higher-dimensional nature of the couplings, we denote

$$
A = a/\Lambda^2, \quad B = b/\Lambda^2, \quad C = c/\Lambda^2,
\tag{41}
$$

where a, b, and c are dimensionless couplings.

If Eqs. (39) and/or (40) are all we have, the phenomenology is straightforward, and only a subset of that we would consider below. One might think that $\Upsilon \to \tau\tau$ will put a tight constraint on the coefficients, but, in actuality, that constraint is far too weak. The reason is that the SM decay is an electromagnetic one, and the width is given by [28]

$$
\Gamma_{\Upsilon(1S)\to\ell\ell} = 4\,\alpha^2\,Q_b^2\,M_\Upsilon^{-2}\,|R(0)|^2\,(1+2x)\,\sqrt{1-4x},
\tag{42}
$$

where $x = M_\ell^2/M_\Upsilon^2$ and $R(0)$ is the radial part of the non-relativistic wave function at the origin.

A. The rotation

Assuming that the operators in question have arisen on account of some flavour physics operative at scales higher than the weak scale, we now put forward the Ansatz that the fields in Eqs. (39) and (40) are in the weak basis, and should be rotated to the stationary or mass basis. Let us, for simplicity, assume that right-chiral fields are not rotated, and for the left-chiral fields,

$$
b_{wk} \to x_1 d + x_2 s + x_3 b, \quad t_{wk} \to y_1 u + y_2 c + y_3 t,
\tag{43}
$$

where the right-hand side fields are in the mass basis.

If $\mathcal{U}$ and $\mathcal{D}$ matrices are responsible for the rotation of $T_3 = +1/2$ and $T_3 = -1/2$ fields from the weak basis to the mass basis, so that the CKM matrix $V = \mathcal{U}^\dagger\mathcal{D}$, one notes that (x_1, x_2, x_3) and (y_1, y_2, y_3) are just the third rows of $\mathcal{D}$ and $\mathcal{U}$ respectively. If we assume the rotation matrices to be almost diagonal, the only constraint is

$$
y_3^* x_3 \approx V_{tb}.
\tag{44}
$$

As for other combinations, we can, at most, use order-of-magnitude arguments to yield

$$
y_3^* x_1 \sim V_{td}, \; y_3^* x_2 \sim V_{ts}, \; y_2^* x_3 \sim V_{cb}, \; y_1^* x_3 \sim V_{ub},
\tag{45}
$$

although there can be significant deviations. Note that this is a rather conservative constraint and one can build models to bypass this. However, one has to be extremely careful about constraints coming from flavour physics, in particular those involving fermions of the first two generations. Furthermore, such models involve some degree of fine-tuning between the rotations in the right-chiral and left-chiral quark sectors.
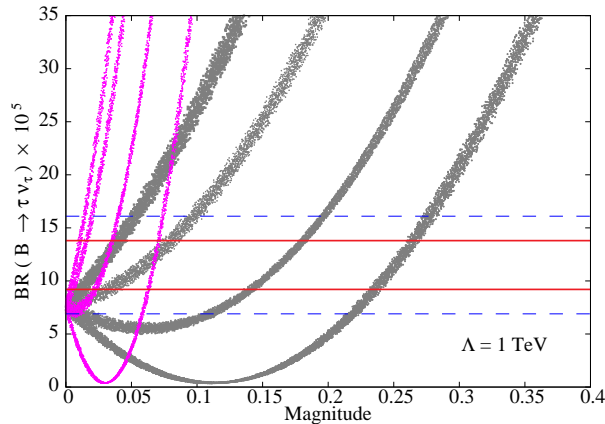
FIG. 1: The dependence of $\text{Br}(B \to \tau\nu)$ on the magnitude of the new physics couplings $|cy_1^* x_3|$ (thick black bands) and $|(a-b)y_1^*|$ (thin purple/light grey bands). In each case, the different bands from left to right correspond to differing values of the phase of the new physics coupling, namely, $-\pi/2, 0, \pi/4$ and $\pi/2$ respectively and the thickness of the individual band reflects the errors in $|V_{ub}|$ and f_B at the 1σ level. The experimental data on $\text{Br}(B \to \tau\nu)$ at 1σ (red/dark grey solid lines) and 2σ (blue/light grey broken lines) intervals are shown as horizontal bands.

IV. THE OBSERVABLES

A. Leptonic and semileptonic decay channels: $B^+ \to \tau^+\nu$, $B_c \to \tau^+\nu$, $B \to D(D^*)\tau\nu$

The relevant new operators are

$$\mathcal{O}_S \supset \frac{1}{\Lambda^2}(a-b)\left[(\{y_1^*\overline{u} + y_2^*\overline{c}\}P_R b)\ (\overline{\tau}P_L\nu) + \text{h.c.}\right],$$

$$\mathcal{O}_V \supset \frac{1}{\Lambda^2}c\,x_3\left[(\{y_1^*\overline{u} + y_2^*\overline{c}\})P_L b)\ (\overline{\tau}P_L\nu) + \text{h.c.}\right], \tag{46}$$

and their effect on the amplitudes of interest can be obtained by simple replacements in the corresponding SM expressions, namely,

$$
\begin{aligned}
\frac{G_F}{\sqrt{2}}V_{ub} &\to \frac{G_F}{\sqrt{2}}V_{ub} + \frac{1}{4}\frac{c}{\Lambda^2}y_1^* x_3 &&\text{for } \mathcal{O}_V, \\
\frac{G_F}{\sqrt{2}}V_{ub}m_\tau &\to \frac{G_F}{\sqrt{2}}V_{ub}m_\tau - \frac{1}{4}\frac{a-b}{\Lambda^2}y_1^*\frac{m_B^2}{m_b + m_u} &&\text{for } \mathcal{O}_S,
\end{aligned}
\tag{47}
$$

where the latter follows from

$$\langle 0|\overline{u}(1-\gamma_5)b|B^-\rangle = -if_B\frac{m_B^2}{m_b + m_u}. \tag{48}$$

For the B_c decay, one has to make the following substitutions: $\{u, V_{ub}, y_1, m_B, m_u, f_B\} \to \{c, V_{cb}, y_2, m_{B_c}, m_c, f_{B_c}\}$.

Taking only one set of new physics couplings, $cx_3 y_1^*$ or $(a-b)y_1^*$, to be non-zero, in Fig. 1 we show the variation of $\text{Br}(B \to \tau\nu)$ with the magnitude of the coupling. We have set the scale of the new physics $\Lambda = 1$ TeV, and used $\phi_3 = \gamma = 77°$ [19]. Understandably, the phase of the NP coupling plays a significant role with positive values allowing for destructive interference with the SM amplitude. This results in the different bands (one for each representative value of the phase). The width of the bands is due to the uncertainty in V_{ub} and to a lesser extent, that in f_B. The two horizontal bands correspond to 1σ (red/dark grey lines) and 2σ (blue/light grey lines) intervals of experimental data on $\text{Br}(B \to \tau\nu)$. Their intersection with the NP bands determine the allowed ranges for the couplings. Note that if $|y_1|$ is indeed $\mathcal{O}(|V_{ub}|)$, then $|cy_1^* x_3| > \mathcal{O}(0.1)$ would indicate a significant departure from the expectations in naive dimensional analysis.

For $B \to D(D^*)\tau\nu$ and $B_c \to \tau\nu$, the SM effective Lagrangian is

$$\mathcal{L}_{eff} = \frac{4G_F V_{cb}}{\sqrt{2}}\left(\overline{c}\gamma^\mu P_L b\right)\left(\overline{\tau}\gamma_\mu P_L\nu\right). \tag{49}$$
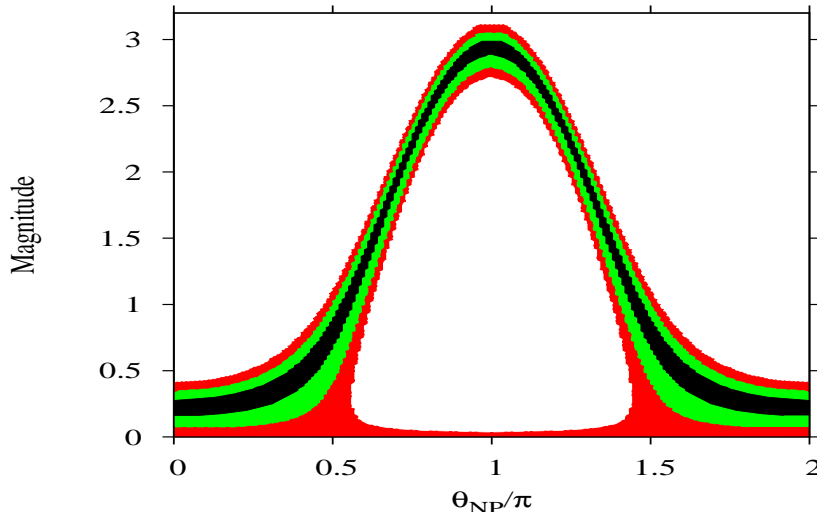
FIG. 2: Allowed values of the magnitude and phase of the new physics coupling $cy_2^*x_3$. The black, green (light grey), and red (dark grey) regions denote the 1σ, 2σ, and 3σ consistency bands respectively, where both theoretical and experimental errors are taken into account and added in quadrature.

The two NP operators $\mathcal{O}_S$ and $\mathcal{O}_V$ modify this to

$$\mathcal{L}_{NP} = \frac{4G_F V_{cb}}{\sqrt{2}} \left[(1+g') \left(\bar{c}\gamma^\mu P_L b \right) \left(\bar{\tau}\gamma_\mu P_L \nu \right) + g_R \left(\bar{c} P_R b \right) \left(\bar{\tau} P_L \nu \right) \right], \tag{50}$$

where the g' (g_R) term emanates from $\mathcal{O}_V$ ($\mathcal{O}_S$), namely

$$\frac{4G_F V_{cb}}{\sqrt{2}} g' = C y_2^* x_3, \quad \frac{4G_F V_{cb}}{\sqrt{2}} g_R = (A-B) y_2^*. \tag{51}$$

If $g' \neq 0$ but $g_R = 0$, one can write

$$R(D) = R_{SM}(D) |1 + g'|^2 \tag{52}$$

and a similar equation for $R(D^*)$, assuming that the new interaction does not contribute to the electron or muon channel. On the other hand, if $g_R \neq 0$ but $g' = 0$, we get

$$\begin{aligned} R(D) &= R_{SM}(D) \left[1 + 1.5\,\mathrm{Re}(g_R) + |g_R|^2 \right], \\ R(D^*) &= R_{SM}(D^*) \left[1 + 0.12\,\mathrm{Re}(g_R) + 0.05\,|g_R|^2 \right]. \end{aligned} \tag{53}$$

The same couplings also contribute to the leptonic decay $B_c \to \tau\nu$, and depending on the phase of the coupling, can increase or decrease the branching ratio.

In Figure 2, we show the allowed values of the coupling $cy_2^*x_3$, with $\Lambda = 1$ TeV, at different confidence levels. The intervals are calculated with individual error margins and not with a combined χ^2-fit. Corresponding to these three cases, the branching ratio of $B_c \to \tau\nu$ are

$$\mathrm{Br}(B_c \to \tau\nu) \in [2.05 - 2.40]\% \ (1\sigma), \quad [1.80 - 2.60]\% \ (2\sigma), \quad [1.60 - 2.80]\% \ (3\sigma), \tag{54}$$

which should be compared with the SM value of 1.68%.

For the $\{S, P\}$ couplings, there is no region in the parameter space compatible with both $R(D)$ and $R(D^*)$ at 1σ or 2σ level. This is because of the small contribution of g_R to $R(D^*)$. Only at 3σ, does one get an allowed region in the parameter space. However, $\mathrm{Br}(B_c \to \tau\nu)$ can be quite large because of the chiral enhancement. If we assume, as a conservative estimate, $\mathrm{Br}(B_c \to \tau\nu) < 10\%$, this translates to

$$| (a-b) y_2^* | < 1.05 \tag{55}$$

for $\Lambda = 1$ TeV. Note that the limits one obtains from neutral current mediated processes, like B_s–$\overline{B}_s$ mixing or $B_s \to \tau^+\tau^-$ [11], are not valid in these cases as the couplings are different.

B. $B_s \to \tau^+\tau^-$

The term from $\mathcal{O}_S$ that we would be interested in is

$$A\left[(\{x_1^*\overline{d} + x_2^*\overline{s} + x_3^*\overline{b}\}P_R b)(\overline{\tau}P_L\tau) + (\overline{b}P_L\{x_1 d + x_2 s + x_3 b\})(\overline{\tau}P_R\tau)\right] , \tag{56}$$

and the relevant operator is

$$A x_2^*(\overline{s}P_R b)(\overline{\tau}P_L\tau) = \frac{1}{2}A x_2^*\left[(\overline{s}P_R b)(\overline{\tau}\tau) - (\overline{s}P_R b)(\overline{\tau}\gamma_5\tau)\right] . \tag{57}$$

This gives

$$\mathrm{Br}(B_s \to \tau^+\tau^-) = \frac{G_F^2 \alpha^2 m_{B_s}^5 f_{B_s}^2 \tau_{B_s}}{256\pi^3}\sqrt{1 - \frac{4m_\tau^2}{m_{B_s}^2}}\left[\left(1 - \frac{4m_\tau^2}{m_{B_s}^2}\right)\mathcal{A} + \mathcal{B}\right] \tag{58}$$

where

$$\mathcal{A} = \left|\frac{\zeta}{m_b + m_s}\right|^2 ,$$

$$\mathcal{B} = \left|\frac{\zeta}{m_b + m_s} + \frac{2m_\tau}{m_{B_s}^2}[(V_{tb}V_{ts}^*)C_{10}]\right|^2 \approx \mathcal{A} ,$$

$$\zeta = \frac{a x_2^*}{\Lambda^2}\frac{\sqrt{2}}{8G_F}\frac{4\pi}{\alpha} , \tag{59}$$

where C_{10} is the Wilson coefficient for the corresponding SM operator, and is too small to be of any consequence. If we take the upper limit of $\mathrm{Br}(B_s \to \tau^+\tau^-)$ to be 3.5%, we get a bound, depending on definition used for m_b. For example,

$$|a x_2^*| < 1.52\,(1.34)\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\left(\frac{\mathrm{Br}(B_s \to \tau^+\tau^-)}{3.5\%}\right)^{1/2} , \tag{60}$$

for $m_b = m_b^{pole} = 4.8$ GeV ($m_b = m_b(m_b) = 4.2$ GeV). This agrees with Ref. [11]. Thus, potentially, a can be large.

For the $\{V, A\}$ couplings coming from $\mathcal{O}_V$, one has

$$\mathrm{Br}(B_s \to \tau^+\tau^-) = \frac{f_{B_s}^2 \tau_{B_s} m_\tau^2 m_{B_s}}{32\pi}\sqrt{1 - \frac{4m_\tau^2}{m_{B_s}^2}}\left|\frac{1}{4}\frac{c}{\Lambda^2}x_2^*x_3\right|^2 . \tag{61}$$

Note that this is further suppressed by a factor of $m_\tau^2/m_{B_s}^2$, which results in a weaker bound:

$$|c x_2^*x_3| < 6.0\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^2\left(\frac{\mathrm{Br}(B_s \to \tau^+\tau^-)}{3.5\%}\right)^{1/2} . \tag{62}$$

C.   Width difference $\Delta\Gamma_s$

Following Ref. [11], let us quote the relevant expressions for the width difference $\Delta\Gamma_s$:

$$\begin{aligned}
\mathcal{O}_S &\Rightarrow \Gamma_{12,NP}^s = 3\,N\,x\,\sqrt{1 - 4x}\,\langle Q_S^R\rangle\,C_S^2 , \\
\mathcal{O}_V &\Rightarrow \Gamma_{12,NP}^s = N\left[\left\{1 + (1 - x)\sqrt{1 - 4x}\right\}\langle Q_V^L\rangle \right. \\
&\qquad\qquad\qquad \left. + \left\{1 + (1 + 2x)\sqrt{1 - 4x}\right\}\langle Q_S^R\rangle\right]C_V^2 ,
\end{aligned} \tag{63}$$
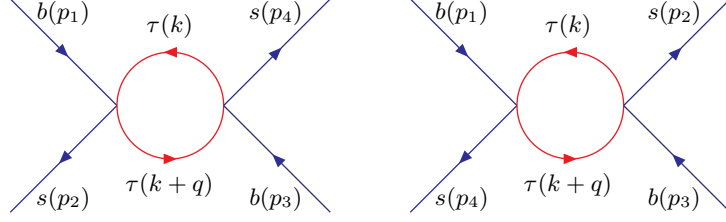
FIG. 3: Typical one-loop corrections to B_s–$\overline{B}_s$ mixing originating from four-fermion operators.

where

$$x = m_\tau^2/m_{B_s}^2,$$

$$N = -\frac{G_F^2 m_b^2}{6\pi m_{B_s}}(V_{ts}^* V_{tb})^2,$$

$$\langle Q_S^R \rangle = -\frac{5}{12} f_{B_s}^2 m_{B_s}^2 B_S,$$

$$\langle Q_V^L \rangle = \frac{2}{3} f_{B_s}^2 m_{B_s}^2 B_V,$$

$$C_S = \frac{\sqrt{2}}{4 G_F} \frac{1}{V_{ts}^* V_{tb}} \frac{a x_2^*}{2 \Lambda^2},$$

$$C_V = \frac{\sqrt{2}}{4 G_F} \frac{1}{V_{ts}^* V_{tb}} \frac{c}{2\Lambda^2} x_2^* x_3. \tag{64}$$

Note that the second equation of (63) has to be augmented by the inclusion of the ν_τ loop, which can be obtained from the corresponding τ contribution by putting $x = 0$. For numerical evaluation, we use the lattice values $f_{B_s} = 0.231$ GeV, $B_S = 1.3$, $B_V = 0.84$. From $B \to K\tau\tau$, there is a (scale-independent) bound, namely $C_V < 0.8$, which translates to

$$\frac{1}{2} c x_2^* x_3 < 1.05 \left(\frac{\Lambda}{1 \text{ TeV}} \right)^2. \tag{65}$$

D. ΔM_s and the mixing phase ϕ_M

The aim of this subsection is to show how and why the constraints coming from ΔM_s measurements are so restrictive in nature. Here, we will start from $\mathcal{O}_V$. There can be two sets of possible diagrams, one with the τ lepton (see Fig.3) and the other with the neutrino. As the amplitudes are not chirality-suppressed, both the diagrams contribute equally. The exact amplitudes cannot be calculated unless we know about the ultraviolet completion of the effective theory. If we use a cut-off regularization, the leading term, which is divergent, should match with the leading term of the full theory. The leading term of the loop amplitude is quadratically divergent, so we can safely neglect the subleading terms.

The relevant part of the effective operator is

$$\mathcal{O}_V \supset \frac{1}{2} C(\overline{b}\gamma^\mu P_L b)(\overline{\tau}\gamma_\mu P_L \tau) \to \frac{1}{2} C x_2^* x_3 (\overline{s}\gamma^\mu P_L b)(\overline{\tau}\gamma_\mu P_L \tau). \tag{66}$$

This gives rise to a mixing amplitude

$$\begin{aligned} iM_{12s} &= \left(\frac{1}{2\Lambda^2} c x_2^* x_3 \right)^2 \langle O_1 \rangle (i\Gamma_2) \times 16 \\ &= \frac{i}{\pi^2} (c x_2^* x_3)^2 \Lambda^{-2} \langle O_1 \rangle \\ &= \frac{i}{3\pi^2} (c x_2^* x_3)^2 \Lambda^{-2} \eta_{B_s} M_{B_s} f_{B_s}^2 B_{B_s}, \end{aligned} \tag{67}$$

where we have used

$$\Gamma_2 = \frac{\Lambda^2}{4\pi^2}, \quad \langle O_1 \rangle = \frac{1}{3} \eta_{B_s} M_{B_s} f_{B_s}^2 B_{B_s}, \tag{68}$$
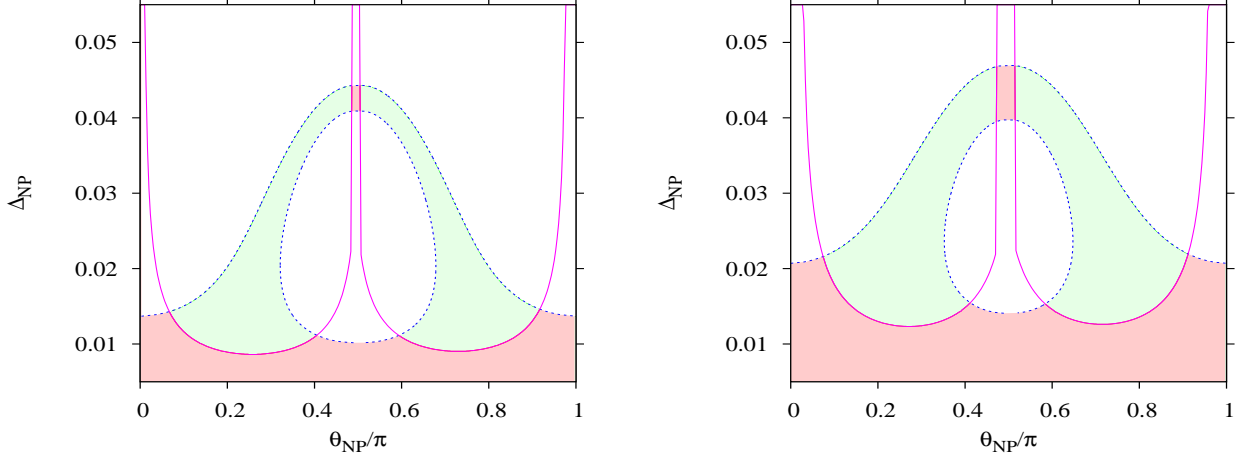
FIG. 4: The allowed magnitude and phase of the coupling $cx_2^*x_3$ from the measurement of ΔM_s and $2\beta_s$, the effective mixing phase from the box amplitude. The light (green) shaded area between the dashed (blue) curves is allowed by ΔM_s measurement, whereas the area between the solid (purple) curves is allowed by the data on $2\beta_s$. Thus, only the two patches of dark (pink) shaded area is finally allowed. For the left (right) plot, the experimental errors are taken at $1(2)\sigma$ level.

Γ_2 being the leading term of the loop amplitude, and $O_1 = [\bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha][\bar{s}_\beta \gamma_\mu (1 - \gamma_5) b_\beta]$, α and β being colour indices. The factor of 16 can be understood in the following way: there is another crossed box, which, in an effective theory, is something like a t-channel amplitude. This gives a factor of 2. The initial meson can pick up a $\bar{b}$ from O_1 in two ways, and an s in two ways, so the symmetry factor is 4. The neutrino mediated amplitude gives another factor of 2.

Comparing with iM_{12s}^{SM}, we find

$$\frac{M^{NP}}{M^{SM}} = \frac{4(cx_2^*x_3)^2 \Lambda^{-2}}{G_F^2 m_W^2 (V_{tb} V_{ts}^*)^2 S_0(x_t)} \,, \tag{69}$$

where $x_t = m_t^2 / m_W^2$ and $S_0(x_t)$ is the relevant Inami-Lim function. The SM amplitude is GIM suppressed whereas there is no such suppression for the NP amplitude, and thus Eq. (69) puts a fairly tight constraint on $cx_2^*x_3$. If we want the latter to be large, the phase must be opposite to that of the SM amplitude, so that there is a destructive interference: $M^{NP} \sim -2M^{SM}$. Taking the errors on the ΔM_s prediction in the SM and the measurement of the same quantity both at 2σ, we get

$$|cx_2^*x_3| < 0.048 \left(\frac{\Lambda}{1\ \text{TeV}}\right) \,. \tag{70}$$

The allowed region is shown in Fig. 4. For $\Lambda = 1$ TeV, the limits on a_{fs}^s are

$$-6.3(-11.2) \times 10^{-4} < a_{fs}^s < 2.2(6.9) \times 10^{-4} \tag{71}$$

at $1(2)\sigma$. Thus, by themselves, such operators are unable to explain the dimuon anomaly, and the explanation must lie somewhere else. The maximum value of $\text{Br}(B_s \to \tau^+\tau^-)$ is about $3 \times 10^{-4}\%$.

The situation is marginally better for a chiral coupling in the scalar sector; *i.e.*, either $S - P$ or $S + P$. For such operators, the leading term in the B_s–$\overline{B}_s$ mixing amplitude is proportional to $m_\tau^2 \log \Lambda^4$, and there is no effective constraint from ΔM_s and mixing phase. However, the major constraint comes from $\text{Br}(B_s \to \tau^+\tau^-)$, and also partially from $\Delta\Gamma_s$. We find

$$|a_{fs}^s| < 6 \times 10^{-4} \tag{72}$$

for $\text{Br}(B_s \to \tau^+\tau^-) < 4\%$ and taking all the errors at 2σ level. Thus, none of these schemes are enough to explain the dimuon anomaly completely.
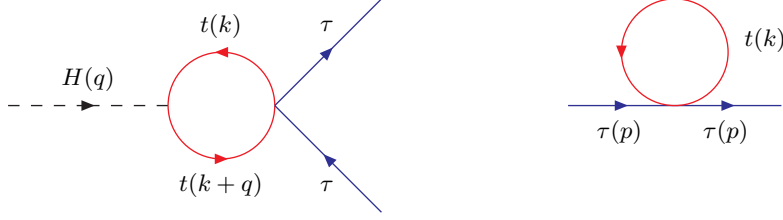
FIG. 5: Typical one-loop corrections to (left) the $H\overline{\tau}\tau$ vertex and (right) τ self energy.

E. $H \to \tau^+\tau^-$

Let us begin by parametrizing the tree-level Higgs-tau coupling by

$$\mathcal{L}_{\text{tree}}^{(H\overline{\tau}\tau)} = h_\tau \overline{\tau} \tau H . \tag{73}$$

As in any quantum theory, this interaction Lagrangian receives quantum corrections. We neglect here all the SM corrections and concentrate solely on that wrought by the four-fermion operators. The scalar-pseudoscalar operators give rise to an effective interaction of the form

$$\frac{b}{2\Lambda^2}\text{Re}(y_3) \left[(\overline{t}t)(\overline{\tau}\tau) - (\overline{t}\gamma_5 t)(\overline{\tau}\gamma_5\tau) \right] + \frac{b}{2\Lambda^2}\text{Im}(y_3) \left[(\overline{t}t)(\overline{\tau}\gamma_5\tau) - (\overline{t}\gamma_5 t)(\overline{\tau}\tau) \right] . \tag{74}$$

Each of these terms generates, at one-loop, a two-point diagram contributing to the effective $H\overline{\tau}\tau$ coupling (see Fig. 5). The said diagrams are manifestly quadratically divergent and need to be regularized. Given that our basic theory is only an effective one, we may use a momentum cut-off regularization scheme, to yield the following correction to the Lagrangian of Eq. (73):

$$\delta\mathcal{L}_{1-\text{loop}} = \frac{3bh_t}{8\pi^2} \frac{\Lambda_{\text{cutoff}}^2}{\Lambda^2} \left[\text{Re}(y_3) \, \overline{\tau}\tau + \text{Im}(y_3) \, \overline{\tau}\gamma_5\tau \right] H + \cdots , \tag{75}$$

where the ellipsis denote subleading terms. It is natural to consider $\Lambda_{\text{cutoff}} = \Lambda$, for the two are expected to be similar. The appearance of a divergent correction to the pseudoscalar coupling (one that did not exist at the tree level) might seem disconcerting at first. However, it should be realised that we are dealing with a nonrenormalizable theory and the existence of such a divergence only reflects the fact that a large correction to $H\tau\tau$ is not prevented by the symmetries of the theory extant on admitting the general four-fermion interaction. On inclusion of the ultraviolet completion, such divergences would disappear identically. Clearly the two Lorentz structures contribute incoherently to $\Gamma(H \to \tau\tau)$. Formally though, the contribution of the scalar coupling correction may be larger as it can interfere with the SM amplitude, and thus can enter at an earlier order in the perturbation theory. For simplicity, though, let us assume that y_3 is real. Then, we can parametrize the effective $H\tau\tau$ vertex, upto one-loop by

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(H\overline{\tau}\tau)} &= h_\tau \left(1 + \xi\right) \overline{\tau}\tau H + \cdots \\ \xi &= \frac{3bh_t y_3}{8\pi^2 h_\tau} . \end{aligned} \tag{76}$$

Similarly, several other Yukawa couplings also receive corrections, but these are suppressed on account of the particular structure of NP. For example, the bottom quark Yukawa coupling receives a correction on account of a tau-loop, and this change can be expressed as $h_b \to h_b + bh_\tau y_3/8\pi^2$.

It might be argued that our calculation of ξ is somewhat naive, and it is indeed true. However, an exact calculation necessitates a knowledge of the ultraviolet completion of the theory, and, in a sense, goes against the spirit of an effective theory. Nonetheless, ξ does encapsulate the leading correction, and in Fig. 6, we show the variation of the branching fraction of $H \to \tau^+\tau^-$ as a function of the real variable ξ for $m_H = 125$ GeV. From Fig. 6, it is very clear that even a moderate value of $\xi \sim -0.3$ is enough to give a 50% suppression in the BR($H \to \tau^+\tau^-$) . On the other hand, from the observed upper limit of $\sigma_{H\to\tau\tau}/\sigma_{SM} \approx 1.1$ from the LHC [1], we get an upper limit of $\xi \approx 0.05$.

For the $\{V, A\}$ current, once again, both scalar and pseudoscalar couplings appear at one-loop. However, the loop is convergent as the current structure demands that extra powers of the fermion masses must be picked up. Hence, the corresponding corrections are too small to be of any consequence.
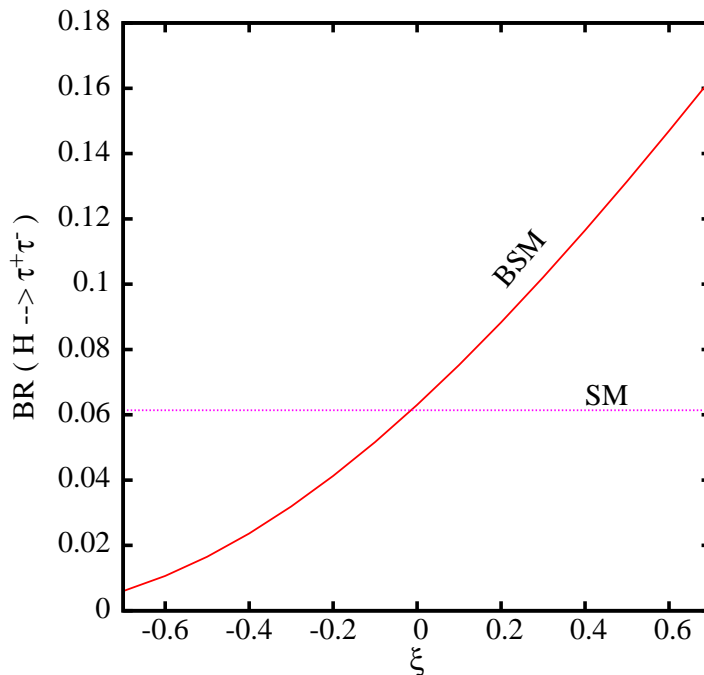
FIG. 6: The $H \to \tau^+\tau^-$ branching fraction as a function of ξ, which we take to be real for $m_H = 125$ GeV. For comparison, we also show the SM branching ratio.

F. Anomalous top decays

One might ask whether the new couplings will lead to observable rates for FCNC top decays, *e.g.* $t \to c\tau^+\tau^-$. Unfortunately though, even if we use values of the couplings $b\, y_2^*$ (for scalar operators) or $c\, y_2^*$ (for vector operators) significantly larger than what we need to explain the anomalies under investigation, the rates for this decay are still much smaller than the LHC sensitivity limits. For example, we might naively use the limit on the branching ratio of $t \to cZ$ [31], namely

$$\mathrm{Br}(t \to cZ) < 0.24\%, \tag{77}$$

along with the measurement [30] of the decay width of the top

$$\Gamma_t = 2.00^{+0.47}_{-0.43} \text{ GeV} \tag{78}$$

to yield the very weak limit of $\Lambda \gtrsim 0.5\, m_t$ when the couplings are only restricted to be perturbative. Furthermore, even the use of Eq.(77) is over-optimistic, for the CMS limits have been derived requiring that the Z-mass can be reconstructed from its decay products. In the current case, this does not apply and the signal to noise ratio is lower than that assumed to obtain Eq.(77). In other words, the actual limit is much weaker than that quoted above.

V. CONCLUSIONS

In this paper, we have investigated the possible implications of a scenario that involves some new interactions involving the third generation fields. Any model that treats the third generation differently from the first two generations may lead to such a scenario. Without attempting to prescribe an ultraviolet-complete theory, we rather consider an effective theory valid below some cutoff scale Λ, above which the full theory takes over. A possible motivation for such a scenario is the fact that there are excesses over the SM predictions for the charged current B-decays, namely $B \to D(D^*)\tau\nu$ and $B^+ \to \tau\nu$, while the predictions for the processes involving the first two generations of leptons do not show any tension with the data.

In the effective theory, there can be several four-fermion operators involving the third generation fields and several possible choices for the Lorentz structures of the currents. With the Wilson coefficients for tensor operators being severely constrained by the data on radiative decays, we preclude these from our discussions. With Λ being larger

than the electroweak scale, it is quite likely that such four-fermion operators in the effective Lagrangian should be written in the weak basis, and for reasons of economy, we consider only one such operator at a time. Rotating the fields to the mass basis generates new operators involving first and second generation quark fields, albeit suppressed by the corresponding entries of the quark mixing matrix.

Once we have a set of such operators, we study their implications on several B-decay observables. In particular, we show that the apparent excesses in the B-decay channels mentioned above can be accommodated satisfactorily in this scenario; complementary observables lead to nontrivial constraints on the model parameters. The vector-axial vector operators successfully explain the excesses in both $B \to D\tau\nu$ and $B \to D^*\tau\nu$ channels, apart from leading to a sizable enhancement to the $B_c \to \tau\nu$ branching ratio as a testable prediction. The scalar-pseudoscalar couplings are not that successful in explaining both the excesses, but there is a definite improvement over the SM predictions. The excess in the channel $B^+ \to \tau\nu$ can have a satisfactory explanation too, although the tension is no longer worrying.

The operators leading to B_d–$\overline{B}_d$ and B_s–$\overline{B}_s$ mixing are more constrained. They have identical Lorentz structures as those discussed before, but with different quark fields and different Wilson coefficients. While these coefficients are constrained from the measured mass differences ΔM_d and ΔM_s, the restrictions are not strong enough to rule out any observable enhancement in the $B_s \to \tau^+\tau^-$ channel, which should be investigated more carefully as one of the best windows to new physics. Unfortunately though, the anomalously large dimuon asymmetry receives only a marginal improvement over the SM prediction. One might need other operators to explain this, but it is not easy given the tight constraints from ΔM_s measurements.

One thing that still remains unobservably small in this class of models is anomalous top decay, like $t \to c\tau^+\tau^-$. The other side of the coin is that if such decays are observed, the new physics must be something different from those described here, as the expectations will be in conflict with the B-decay observables.

It is not yet certain whether there is a deficiency in the $H \to \tau^+\tau^-$ channel, but at the 1σ level, the cross-section is slightly below the SM prediction. While it is too early to say anything about this channel, we would like to point out that the interactions discussed in this paper can potentially modify the predictions for this channel, without disturbing those for other channels. Further data from LHC will be eagerly anticipated.

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