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$\rho - \omega$ MIXING AT HIGH TEMPERATURE AND DENSITY

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Abstract

The temperature and density dependence of the $\rho - \omega$ mixing amplitude has been studied from a purely hadronic model. The in-medium baryon masses and chemical potentials have been obtained at the Mean Field (MF) Level and these results have been used to calculate the mixing amplitude. It has been observed that the mixing amplitude changes substantially at high temperature and density.

Charge Symmetry Violation (CSV) is a very well established and interesting sphere of research. Though this field of research is quite old it is still quite challenging as the finer corrections from the experiments are still coming. The cross section for the process $e^+e^- \to \pi^+\pi^-$, in the $\rho - \omega$ resonance region, reveals an interference shoulder which results from the superposition of the narrow resonant ω and broad resonant ρ amplitudes [1]-[3]. Thus a G-parity violating process $\omega \to \pi^+\pi^-$ is observed. This observation motivates the study of $\rho - \omega$ mixing. Different authors have studied the $\rho - \omega$ mixing from different models and prescriptions [4]-[9]. In this article the temperature and density dependence of the mixing amplitude will be studied. The

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$\rho - \omega$ mixing may be important in the context of hot and dense hadronic matter which can be observed in high energy heavy ion collisions. So it is interesting to see whether this amplitude is changed substantially at high temperature/density.

After a brief discussion of the model the baryon masses and chemical potentials, in the hot and dense matter, will be calculated in the Mean Field Approximation (MFA) self consistently. This result will be used to calculate the temperature and density dependence of the mixing amplitude.

The description of hadronic matter at high temperature should be based on a reliable relativistic model [10]. The model should reproduce the basic features of the strong interaction *i.e.* short range repulsive and long range attractive forces. Here we will use non-linear Walecka model to calculate the mixing amplitude.

The non-linear Walecka model Lagrangian with nucleons, scalar mesons and vector (ω and ρ) mesons can be written as [11]:

$$\begin{aligned}
\mathcal{L} &= \sum_B \bar{\psi}_B(\gamma^\mu p_\mu - m_B + g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu)\psi_B + \frac{1}{2}(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) \\
&- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{3} b m_N (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 \\
&- \frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2 + g_\rho \bar{\psi} \gamma^\mu \tau.\psi \rho_\mu + f_\rho \bar{\psi} \sigma^{\mu\nu} \tau.\psi \frac{\partial_\mu}{2m} \rho_\nu
\end{aligned} \tag{1}$$

In the above expression ψ_B, σ, ω and ρ are respectively the baryon, the σ-meson, the ω-meson and the ρ-meson fields; m_B, m_σ, m_ω and m_ρ are the corresponding masses; $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $G_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]$. Here we have considered two baryons: neutron(n) and proton(p). Their masses are $m_n = 939.5731$ and $m_p = 938.2796$. In equation (1), $m_N = (m_n + m_p)/2$. The other parameter values used here are given by: $m_\rho = 770.0 MeV$, $m_\omega = 783.0 MeV$, $g_\rho^2/4\pi = 0.74$, $g_\omega^2/4\pi = 5.48$, $g_\sigma^2/4\pi = 5.35$ and $C_\rho = f_\rho/g_\rho = 6.1$ [5, 12].

In ref. [5] similar interactions of ρ and ω have been used to calculate the momentum dependence of the $\rho - \omega$ mixing amplitude. The advantage of using this prescription, as pointed out in ref. [5], is that one can write down the contribution of the $\rho-\omega$ mixing to the NN potential in a parameter free form:

$$V_{NN}^{\rho\omega}(q) = -\frac{g_\rho g_\omega \langle\rho|H|\omega\rangle}{(q^2-m_\rho^2)(q^2-m_\omega^2)} \tag{2}$$

where the $\rho-\omega$ mixing amplitude is given by:

$$\langle\rho|H|\omega\rangle = g_\rho g_\omega \Pi(q) \tag{3}$$

In the above equation $\Pi(q)$ is the transverse part of the polarisation function at finite temperature (the definetion of of $\Pi(q)$ is somewhat different compared to that in ref [5]).

The task ahead is now to calculate $\Pi(q)$ at finite temperature and density keeping in mind that the ρ-meson has both vector and tensor coupling to the nucleons. So one has [5]

$$\Pi^{\mu\nu}(q,T) = \Pi_{vv}^{\mu\nu}(q,T) + C_\rho \Pi_{vt}^{\mu\nu}(q,t) \tag{4}$$

where

$$\begin{aligned} i\Pi_{vv}^{\mu\nu} &= \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^\mu G(k+q)\gamma^\nu \tau_z G(k)\right] \\ i\Pi_{vt}^{\mu\nu} &= \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^\mu G(k+q)\frac{i\sigma^{\nu\lambda}q_\lambda}{2m}\tau_z G(k)\right] \end{aligned} \tag{5}$$

where G is the finite temperature nucleon propagator.

The nucleon propogator can be splitted into its isoscalar and isovector components as:

$$G(k) = \frac{1}{2}G_p(k)(1+\tau_z) + \frac{1}{2}G_n(k)(1-\tau_z) \tag{6}$$

Once this splitting is introduced in the polarisation function and the isospin trace is carried out one can easily see that the $\rho-\omega$ mixing amplitude is proportional to the difference between the proton and the neutron loops (fig. 1) [5] *i.e.*

$$\Pi_{\mu\nu}(q) = \Pi_{\mu\nu}^{(p)}(q) - \Pi_{\mu\nu}^{(n)}(q) \tag{7}$$

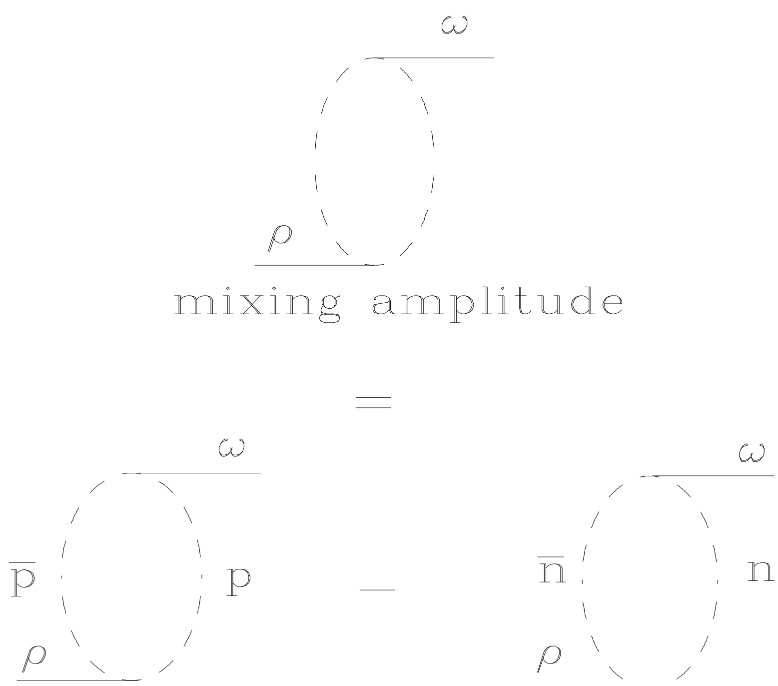

Fig. 1 : Feynmann diagram for the $\rho-\omega$ mixing amplitude.

In the above set of equations the propagator for the baryons can be given by

$$G_B(p) = (\gamma^\mu p_\mu + m_B^*)\left[\frac{1}{(p^2 - m_B^{*2} + i\epsilon)} + 2\pi i\delta(p^2 - m_B^{*2}) sin^2\phi_{p_0}\right] \tag{8}$$

where

$$\begin{aligned} sin\phi_{p_0} &= \frac{e^{-x/2}\theta(p_0)}{(1+e^{-x})^{\frac{1}{2}}} - \frac{e^{x/2}\theta(-p_0)}{(1+e^{x})^{\frac{1}{2}}} \\ x &= (p_0 - \mu_B)/T \end{aligned} \tag{9}$$

B is either p or n, T is the temperature and μ_B is the chemical potential.

Once we use this propagator the calculation of the polarisation function is really straight forward and we have [5, 10]

$$\begin{aligned} \Pi_{vv}^{(p)}(q) &= -\frac{q^2}{2\pi^2}\left[\frac{1}{6\epsilon} - \frac{\gamma}{6} - \int dx x(1-x)\left[\frac{m_p^2 - x(1-x)q^2}{\Lambda^2}\right]\right] \\ &- 16q^2 \int \frac{d^3k}{(2\pi)^3}\frac{[f_p^+(E_p) + f_p^-(E_p)]}{2E_p(k)}\left[\frac{(E_p^2(k) - |k|^2(1-cos^2\theta)/2)}{q^4 - 4(k.q)^2}\right] \\ \Pi_{vt}^{(p)}(q) &= -\frac{q^2}{8\pi^2}\left[\frac{1}{\epsilon} - \gamma - \int dx\left[\frac{m_p^2 - x(1-x)q^2}{\Lambda^2}\right]\right] \end{aligned}$$

$$+ \quad 8q^4 \int \frac{d^3k}{(2\pi)^3} \frac{[f_p^+(E_p) + f_p^-(E_p)]}{2E_p(k)} \left[\frac{1}{q^4 - 4(k.q)^2}\right] \tag{10}$$

and similar expressions for the neutron loops. In the above expression $f_B^{\pm}(E_B) = \frac{1}{1+exp((E_B \mp \mu_B)/T)}$ is the thermal distribution function, Λ is an arbitary renormalisation cutoff and γ is the Euler-Mascheroni constant. Since we need to have the difference between the p and n loop contributions we get

$$\begin{aligned}
\Pi_{vv}(q) &= \Pi_{vv}^{(p)} - \Pi_{vv}^{(n)} \\
&= q^2 \frac{1}{2\pi^2} \int dx x(1-x) \left[\frac{m_p^2 - x(1-x)q^2}{m_n^2 - x(1-x)q^2}\right] \\
&- 16q^2 \int \frac{d^3k}{(2\pi)^3} \left[\frac{[f_p^+ + f_p^-]}{2E_p} \times \frac{(E_p^2 - |k|^2(1-cos^2\theta)/2)}{q^4 - 4(k.q)^2}\right. \\
&- \left.\frac{[f_n^+ + f_n^-)]}{2E_n} \times \frac{(E_n^2 - |k|^2(1-cos^2\theta)/2)}{q^4 - 4(k.q)^2}\right] \\
\Pi_{vt}(q) &= \Pi_{vt}^{(p)} - \Pi_{vt}^{(n)} \\
&= q^2 \frac{1}{8\pi^2} \int dx \left[\frac{m_p^2 - x(1-x)q^2}{m_n^2 - x(1-x)q^2}\right] \\
&+ 8q^4 \int \frac{d^3k}{(2\pi)^3} \left[\frac{[f_p^+ + f_p^-]}{2E_p} - \frac{[f_n^+ + f_n^-]}{2E_n}\right] \times \frac{1}{q^4 - 4(k.q)^2}
\end{aligned} \tag{11}$$

where $E_B = \sqrt{k^2 + m_B^{*2}}$ is the effective energy of the baryon.

From equation (11) it is clear that the mixing amplitude dependse on the in-medium baryon masses and chemical potentials. In the MFA the effective baryon masses are : $m_B^* = m_B - g_\sigma \sigma_0$. The baryon number conservation gives $\rho = n_n + n_p$ where ρ is the total baryon number density and n_n and n_p are the number densities of the neutrons and protons respectively. The asymetry in the numbers of neutrons and protons is given by $x = (n_n - n_p)/(n_n + n_p)$ where x is the asymetry factor. In this paper, where a $Pb - Pb$ system has been considered, the asymmetry is 44/208. From the above relations one can find out the medium dependence of the baryon masses and chemical potentials.

We can now feed these in-medium baryon masses and chemical potentials in equation(11). After calculating the total polarisation function numerically we have plotted the absolute value of the mixing amplitude $|\langle\rho|H|\omega\rangle|$ at $q^2 = m_\omega^2$ as a function of temperature, for different densities, in figure 2. The modulus of the mixing amplitude increases with temperature for $\rho = 0fm^{-3}$. At $T = 0MeV$ the amplitude is about $4300MeV$ and at $T = 200MeV$ it is about $5000MeV$. For $\rho = 0.15fm^{-3}$, *i.e.* at nuclear matter density, the amplitude first decreases from $30000MeV$, at $T = 0MeV$, to about $15000MeV$, at $T = 180MeV$. Then it increaess slightly. At $\rho = 0.3fm^{-3}$ the modulus of the mixing amplitude decreases form $100000MeV$ to $36000MeV$ as the temperature is varried from $T = 0$ to $T = 200MeV$. In fig.3 we have plotted the density dependence of the mixing amplitude for different temperatures. The amplitude increases sharply with density. But, with increase in temperature it decreases. The dominant contribution to the mixing amplitude, at low densities, comes from the vacuum part of the polarisation function which depends on temperature/density through the baryon masses only. At higher densities it comes from the part of the polarisation function which directly dependence on temperature/density through the distribution functions.

To conclude, we have calculated the temperature and density depenence of the $\rho - \omega$ mixing amplitude starting from a purely hadronic model. The mixing was assumed to be generated solely by $N\bar{N}$ loops and thus driven by the difference in the contribution from n and p loops. The mixing amplitude is found to change substantially. This can have a strong bearing on the QGP diagnostics. The emission rate of dileptons (l^+l^-) may get modified due to this mixing. One can, however, use the temperature and density dependence of baryon masses and chemical potentials from other models. This may change the results quantitatively to some extent but, the qualitative behaviour should not change. Studies in these directions are in progress.

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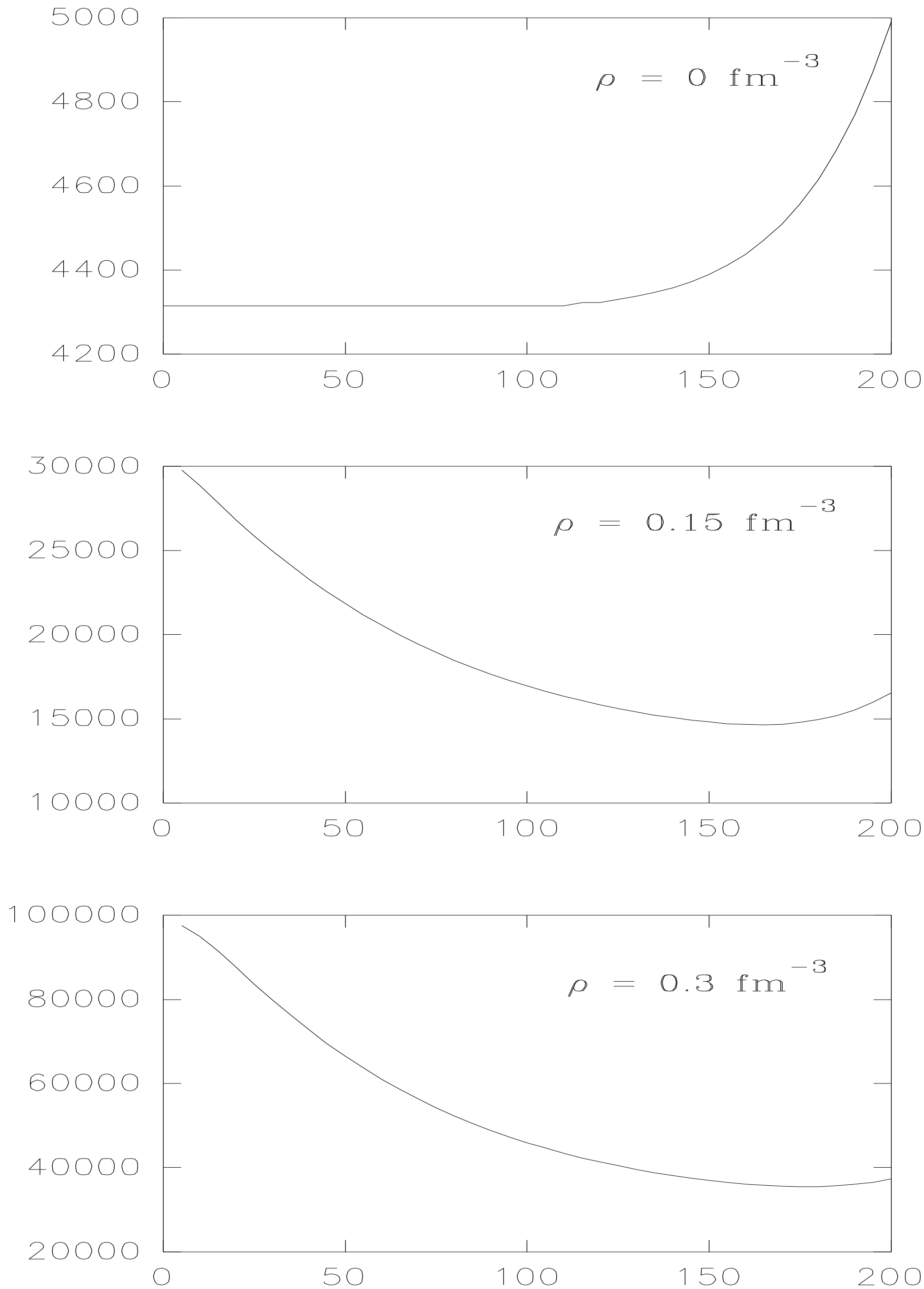

Fig. 2 : Temperature dependence of the modulus of the $\rho - \omega$ mixing amplitude ($|\langle\rho|H|\omega\rangle|$).

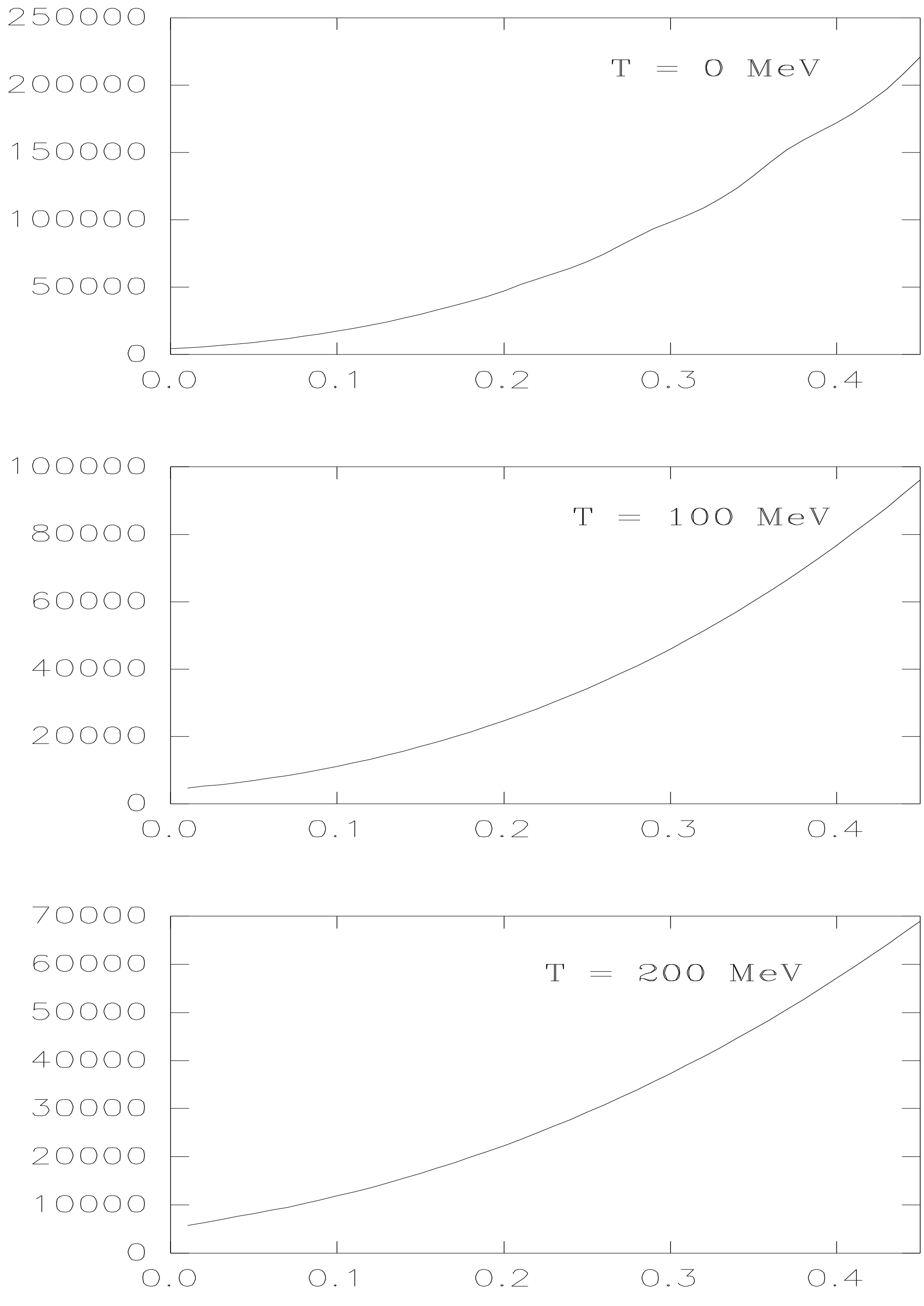

Fig. 3 : Density dependence of the modulus of the $\rho-\omega$ mixing amplitude ($|\langle\rho|H|\omega\rangle|$).

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